Research Article

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Construction of abundant novel analytical solutions of the space–time fractional nonlinear generalized equal width model *via* Riemann–Liouville derivative with application of mathematical methods

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Abstract: The space–time fractional generalized equal width (GEW) equation is an imperative model which is utilized to represent the nonlinear dispersive waves, namely, waves flowing in the shallow water strait, one-dimensional wave origination escalating in the nonlinear dispersive medium approximation, gelid plasma, hydro magnetic waves, electro magnetic interaction, *etc.* In this manuscript, we probe advanced and broad-spectrum wave solutions of the formerly betokened model with the Riemann–Liouville fractional derivative *via* the prosperously implementation of two mathematical methods: modified elongated auxiliary equation mapping and amended simple equation methods. The nonlinear fractional differential equation (NLFDE) is renovated into ordinary differential equation by the composite function derivative and the chain rule putting together along with the wave transformations. We acquire several types of exact soliton solutions by setting specific values of the personified parameters. The proposed schemes are expedient, influential, and computationally viable to scrutinize notches of NLFDEs.

Keywords: fractional order GEW model, Riemann–Liouville fractional derivative, mathematical methods, exact and solitary solutions

1 Introduction

Fractional calculus and henceforth the fractional-order nonlinear evolution equations (FNLEEs) have magnetized immensely colossal interest regarding numerous analyses and are capable of depicting the interior component of the authentic world quandaries. Nonlinear fractional partial differential equations (FPDEs) currently play a chief role in applied sciences, signal processing, control hypothesis, framework identification, and in materials science [1–4]. Moreover, they are employed in social sciences, for example, food complements, atmosphere, finance, and financial issues. In physics, wave propagation process and heat flow are modeled by the FNLEEs. In environmental science, different types of population models are functioned by FNLEEs. Additionally, the modeling of gas dynamics evaluating the relationship among the nominal and real interest rates under inflation and numerous other fields can be well elected through FNLEEs.

Consequently, numerous pieces of literature have been provided to cultivate precise systems of fractional ordinary differential equations and fractional partial differential equations of physical cognizance. In order to find exact and approximation solutions of FDEs, various advanced and reliable methods have been suggested, the homotopy analysis and perturbation methods [5–8], the variation iteration scheme [9,10], differential transformation technique [11,12], iterative Laplace transformation method [13,14], the iterative algorithm [15,16], fractional subequation schemes [9,17,18], the fixed point technique [20,21], scheme of (G'/G) -expansion [22,23], Adomian decomposition technique [24,25], the exp-function technique [26,27], the generalized exponential rational function scheme [28–30], the planner system method [31], the adaptive control scheme [32], (ψ'/ψ) -expansion method [33], the double $(G'/G, 1/G)$ -expansion method [34], *etc.*

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The apparent generalized equal width (GEW) model has been scrutinized for exact analytic solutions through the Kudryashov scheme [35], the collocation technique [36,37], the homogeneous balance scheme [38], and many more. The impartial of this work is to investigate the more general and some solitary wave solutions to the acclaimed nonlinear space–time fractional model by means of the suggested schemes. The results constructed in this manuscript are associated with the existing results accessible in the literature and have shown that the achieved solutions are standard and further inclusive. Hence, it is to be anticipated that the reputation of the obtained solutions may be supported in the literature.

The rest of the article is planned as follows: In Section 2, definition and basics formulae are illustrated. In Section 3, proposed schemes have been portrayed. In Section 4, we have established the exact solutions to the space–time fractional GEW model by the proposed techniques, see details in refs [39,40]. In Section 5, the conclusion is given.

2 Definition and primers

Modified Riemann–Liouville derivative was presented by Jumarie. With the assistance of some helpful techniques, such types of fractional derivative, by using the variable transformation mentioned in ref. [41], the fractional differential equations are changed into integer-order differential equations. We will first stretch a couple of definitions and trademarks of the modified Riemann–Liouville derivative. In this research work, all these trademarks and definitions are employed for provision. Let $F : R \rightarrow R$, $x \rightarrow F(x)$ denote a continuous, however, not really differentiable function. The mathematical form of Jumarie's modified Riemann–Liouville derivative of order p is as follows:

$$D_x^p F(x) = \frac{1}{\Gamma(-p)} \int_0^x (x-\xi)^{-p-1} (F(\xi) - F(0)) d\xi, \quad p < 0, \\ \frac{1}{\Gamma(1-p)} \frac{d}{dx} \int_0^x (x-\xi)^{-p} (F(\xi) - F(0)) d\xi, \quad 0 < p < 1 \\ (F^{(n)}(x))^{(a-n)}, \quad n \leq p \leq n+1, \quad n > 1. \quad (1)$$

Furthermore,

$$D_t^p t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-p)} t^{\gamma-p}, \quad \gamma > 0, \quad (2)$$

$$D_t^p (aF(t) + bG(t)) = aD_t^p F(t) + bD_t^p G(t), \quad (3)$$

$$D_t^p F[L(x)] = F_L^p(L) D_x^p U(x), \quad (4)$$

$$D_t^p F[L(x)] = D_L^p F(L) (L'(x))^p, \quad (5)$$

$$d^p x(t) = \Gamma(1+p) dx(t), \quad (6)$$

where $L(x)$ is non-differentiable function for (3), (4) gradually and differentiable for (5). $L(x)$ is non-differentiable, $F(L)$ is differentiable in (4) and non-differentiable in (5). The elucidation equations (3), (4), and (5) should be used mindfully.

3 Formation of proposed schemes

Consider nonlinear nonlinear partial differential equation of fractional order,

$$R(U, D_t^p U, D_x^\beta U, D_x^\gamma U, \dots, \\ D_t^p D_t^p U, D_t^p D_x^\beta U, D_x^\beta D_x^\beta U, D_x^\beta D_x^\gamma U, \dots) = 0, \quad (7) \\ 0 < p, \beta, \gamma < 1.$$

Consider fractional transformation,

$$U = U(\xi), \quad \xi = \frac{kt^p}{\Gamma(p+1)} + \frac{lx^\beta}{\Gamma(\beta+1)} + \frac{my^\gamma}{\Gamma(\gamma+1)}. \quad (8)$$

Put equation (8) into equation (7),

$$T(U, U', U'', U''', \dots) = 0. \quad (9)$$

3.1 Modified extended auxiliary equation mapping scheme

Let solution (9):

$$U = \sum_{i=0}^N A_i \Psi^i + \sum_{i=-1}^{-N} B_{-i} \Psi^i + \sum_{i=2}^N C_i \Psi^{i-2} \Psi' + \sum_{i=1}^N D_i \left(\frac{\Psi'}{\Psi} \right)^i \quad (10)$$

Suppose Ψ satisfies the following:

$$\Psi' = \sqrt{\beta_1 \Psi^2 + \beta_2 \Psi^3 + \beta_3 \Psi^4}. \quad (11)$$

Put (10) with (11) in (9), solve obtained equations for the required destination of equation (7).

3.2 Improved simple equation scheme

Let (9) has solution,

$$U(\xi) = \sum_{i=-N}^N A_i \Psi^i(\xi). \quad (12)$$

Let Ψ satisfy,

$$\Psi' = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3. \quad (13)$$

Put (12) with (13) in (9). Solving the systems of equations for the required solution of (7).

4 Applications

4.1 Space–time fractional GEW equation

The space–time fractional GEW model is a nonlinear wave model summed up the EW equation was first established as a model for lavishness long waves on the exterior of the water in a channel by Peregrine and Benjamin [42–44]. Let us consider the fractional GEW model [35] as,

$$D_t^\alpha V + pD_x^\alpha V^{m+1} - qD_{xx}^{2\alpha} V - rD_{xxt}^{3\alpha} V = 0, \quad m > 1. \quad (14)$$

Consider the travelling wave transformation,

$$V(\xi) = V, \quad \xi = \frac{kx^\alpha}{\Gamma(\alpha+1)} - \frac{\lambda t^\alpha}{\Gamma(\alpha+1)}. \quad (15)$$

Putting (15) into (14),

$$-\lambda V' + pk(V^{m+1})' - qk^2V'' + r\lambda k^2V''' = 0. \quad (16)$$

After integration of (16) with taking constant of integration zero.

$$-\lambda V + pk(V^{m+1}) - qk^2V' + r\lambda k^2V'' = 0. \quad (17)$$

By balancing the order derivative V'' nonlinear term V^{m+1} from (17), we have $N = \frac{2}{m}$.

Now applying the $V = U^{2/m}$ in (17) yields

$$(U')^2(4\lambda rk^2 - 2k^2\lambda mr) + km^2pU^4 + (-\lambda)m^2U^2 - 2mqk^2UU' + 2\lambda rmk^2UU'' = 0. \quad (18)$$

4.2 Application of modified extended auxiliary equation mapping scheme

Let solution of (18) have the following form as:

$$U = A_1\Psi + A_0 + \frac{B_1}{\Psi} + \frac{D_1\Psi'}{\Psi}. \quad (19)$$

Put (19) with (11) in (18),

$$\begin{aligned} A_0 &= -\frac{\left(-\frac{1}{2}\right)^{3/4}(m+4)p\sqrt{q}(\beta_1(-m+2))r^{3/2}}{(m+2)^{5/4}r(\beta_1^2(m+4)^2p^2r)^{3/4}}, \\ A_1 &= -\frac{\sqrt[4]{-1}\sqrt{\beta_3}\sqrt[4]{m+2}\sqrt{q}}{2^{3/4}\sqrt{\beta_1^2(m+4)^2p^2r}}, \\ D_1 &= -\frac{\sqrt[4]{-1}\sqrt[4]{m+2}\sqrt{q}}{\sqrt[4]{8\beta_1^2m^2p^2r + 64\beta_1^2mp^2r + 128\beta_1^2p^2r}}, \\ B_1 &= 0, \quad k = -\frac{m}{\sqrt{2}\sqrt{\beta_1(-m)r - 2\beta_1r}}, \\ \lambda &= -\frac{2i\sqrt{2}\beta_1m\sqrt{m+2}pq}{\sqrt{\beta_1(-m)r - 2\beta_1r}\sqrt{8\beta_1^2m^2p^2r + 64\beta_1^2mp^2r + 128\beta_1^2p^2r}} \end{aligned} \quad (20)$$

Substituting (20) in (19), we have the following solution cases as follows (Figure 1):

Figure 1: (a) and (b) Soliton solution of U_1 is plotted as $\alpha = 1$, $\beta_1 = 1$, $\beta_2 = 2$, $\beta_3 = 1$, $m = 1.2$, $p = -0.5$, $q = -4$, $r = -5$, $\xi_0 = 0.1$, $\varepsilon = 1$.

Case 1.

$$U_1 = -\frac{\left(-\frac{1}{2}\right)^{3/4}(m+4)p\sqrt{q}(\beta_1(-(m+2))r)^{3/2}}{(m+2)^{5/4}r(\beta_1^2(m+4)^2p^2r)^{3/4}} - \frac{\sqrt[4]{-1}\sqrt{\beta_3}\sqrt[4]{m+2}\sqrt{q}}{2^{3/4}\sqrt[4]{\beta_1^2(m+4)^2p^2r}} - \frac{\beta_1\left(\varepsilon \coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)}{\beta_2} \\ - \left(\frac{\sqrt[4]{-1}\sqrt[4]{m+2}\sqrt{q}}{\sqrt[4]{8\beta_1^2m^2p^2r+64\beta_1^2mp^2r+128\beta_1^2p^2r}}\right) \left(\frac{\beta_1^{3/2}\varepsilon \operatorname{csch}\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)}{(2\beta_2)\left(-\beta_1\left(\varepsilon \coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)}\right), \quad \beta_1 > 0, \beta_2^2 - 4\beta_1\beta_3 = 0. \quad (21)$$

$$V_1 = (U_1)^{2/m}, \quad \beta_1 > 0, \beta_2^2 - 4\beta_1\beta_3 = 0. \quad (22)$$

Case 2.

$$U_2 = -\frac{\left(-\frac{1}{2}\right)^{3/4}(m+4)p\sqrt{q}(\beta_1(-(m+2))r)^{3/2}}{(m+2)^{5/4}r(\beta_1^2(m+4)^2p^2r)^{3/4}} - \frac{\sqrt[4]{-1}\sqrt{\beta_3}\sqrt[4]{m+2}\sqrt{q}}{2^{3/4}\sqrt[4]{\beta_1^2(m+4)^2p^2r}} \\ \times \left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\varepsilon \sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right) - \left(\frac{\sqrt[4]{-1}\sqrt[4]{m+2}\sqrt{q}}{\sqrt[4]{8\beta_1^2m^2p^2r+64\beta_1^2mp^2r+128\beta_1^2p^2r}}\right) \\ \times \left(\frac{\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\varepsilon \cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta} - \frac{\sqrt{\alpha_1}\varepsilon \sinh^2(\sqrt{\beta_1}(\xi+\xi_0))}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta)^2}\right)}{2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\varepsilon \sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)}\right), \quad \beta_1 > 0, \beta_3 > 0, \beta_2 = (4\beta_1\beta_3)^{1/2}. \quad (23)$$

$$V_2 = (U_2)^{2/m}, \quad \beta_1 > 0, \beta_3 > 0, \beta_2 = (4\beta_1\beta_3)^{1/2}. \quad (24)$$

Case 3.

$$U_3 = -\left(\frac{\left(-\frac{1}{2}\right)^{3/4}(m+4)p\sqrt{q}(\beta_1(-(m+2))r)^{3/2}}{(m+2)^{5/4}r(\beta_1^2(m+4)^2p^2r)^{3/4}}\right) - \left(\frac{\sqrt[4]{-1}\sqrt{\beta_3}\sqrt[4]{m+2}\sqrt{q}}{2^{3/4}\sqrt[4]{\beta_1^2(m+4)^2p^2r}}\right) \\ \times \left(\frac{\beta_1\left(\frac{\varepsilon(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+p_1)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p_1^2+1}}+1\right)}{\beta_2}\right) - \left(\frac{\sqrt[4]{-1}\sqrt[4]{m+2}\sqrt{q}}{\sqrt[4]{8\beta_1^2m^2p^2r+64\beta_1^2mp^2r+128\beta_1^2p^2r}}\right) \\ \times \left(\frac{\beta_1\left(\frac{\sqrt{\beta_1}\varepsilon \cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p_1^2+1}} - \frac{\sqrt{\beta_1}\varepsilon \sinh(\sqrt{\beta_1}(\xi+\xi_0))(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+p_1)}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p_1^2+1})^2}\right)}{\beta_2\left(-\beta_1\left(\frac{\varepsilon(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+p_1)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p_1^2+1}}+1\right)\right)}\right), \quad \beta_1 > 0. \quad (25)$$

$$V_3 = (U_3)^{2/m}, \quad \beta_1 > 0. \quad (26)$$

4.3 Application of improved simple equation scheme

Let solution of (18),

$$U = \frac{A_{-1}}{\psi} + A_1\psi + A_0. \quad (27)$$

Put (27) with (13) in (18) (Figure 2).

Case 1. $c_3 = 0$,

Family I

$$\begin{aligned} \lambda &= \frac{mq}{\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}, \quad k = \frac{im}{\sqrt{2}\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}}, \quad A_{-1} = 0, \\ A_0 &= \frac{(c_1(m+4)r - \sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2})\sqrt{-\frac{ic_2^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{2^{3/4}c_2(m+4)\sqrt{p}r}, \\ A_1 &= \frac{\sqrt[4]{2}\sqrt{-\frac{ic_2^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{\sqrt{p}}. \end{aligned} \quad (28)$$

Put (28) in (27) (Figure 3),

$$\begin{aligned} U_4 &= \left(\frac{(c_1(m+4)r - \sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2})\sqrt{-\frac{ic_2^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{2^{3/4}c_2(m+4)\sqrt{p}r} \right) \\ &\quad - \left(\frac{\sqrt[4]{2}\sqrt{-\frac{ic_2^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{\sqrt{p}} \right) \left(\frac{1}{2} \left(c_1 - \sqrt{4c_2c_0 - c_1^2} \tan \left(\frac{1}{2} \sqrt{4c_2c_0 - c_1^2} (\xi + \xi_0) \right) \right) \right), \quad 4c_0c_2 > c_1^2. \end{aligned} \quad (29)$$

$$V_4 = (U_4)^{2/m}, \quad 4c_0c_2 > c_1^2. \quad (30)$$

Figure 2: (a) and (b) Soliton solution of U_3 is plotted as $\alpha = 1$, $\beta_1 = 1.3$, $\beta_2 = 2$, $\beta_3 = 1$, $\eta = 1$, $m = 3$, $p = -0.01$, $p_1 = 1$, $q = -0.1$, $r = -0.4$, $\xi_0 = 0.001$, $\varepsilon = -1$.

Figure 3: (a) and (b) Soliton solution of U_4 is plotted as $c_0 = -2$, $\alpha = 1$, $c_1 = 1.5$, $c_2 = -1$, $m = 2$, $p = 0.01$, $q = 0.1$, $r = -0.1$, $\xi_0 = -2.5$.

Family II

$$\begin{aligned}
 A_0 &= \frac{(\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2} - c_1(m+4)r)\sqrt{-\frac{ic_0^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{2^{3/4}c_0(m+4)\sqrt{p}r}, \\
 A_{-1} &= -\frac{\sqrt[4]{2}\sqrt{-\frac{ic_0^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{\sqrt{p}}, \quad k = -\frac{im}{\sqrt{2c_1^2mr - 8c_0c_2mr + 4c_1^2r - 16c_0c_2r}}, \\
 A_1 &= 0, \quad \lambda = -\frac{nq}{\sqrt{c_1^2m^2r^2 - 4c_0c_2m^2r^2 + 8c_1^2mr^2 - 32c_0c_2mr^2 + 16c_1^2r^2 - 64c_0c_2r^2}}.
 \end{aligned} \tag{31}$$

Substitute (31) in (27),

$$\begin{aligned}
 U_5 &= -\frac{\sqrt[4]{2}\sqrt{-\frac{ic_0^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{\frac{1}{2}\sqrt{p}\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)} \\
 &\quad + \left(\frac{(\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2} - c_1(m+4)r)\sqrt{-\frac{ic_0^2(m+2)qr}{\sqrt{(c_1^2 - 4c_0c_2)(m+2)r}\sqrt{(c_1^2 - 4c_0c_2)(m+4)^2r^2}}}}{2^{3/4}c_0(m+4)\sqrt{p}r}\right), \quad 4c_0c_2 > c_1^2
 \end{aligned} \tag{32}$$

$$V_5 = (U_5)^{2/m}, \quad 4c_0c_2 > c_1^2. \tag{33}$$

Case 2. $c_0 = 0$, $c_3 = 0$ (Figure 4),

$$A_1 = -\frac{\sqrt[4]{2}\sqrt{-\frac{c_2^2(m+2)q}{\sqrt{c_1^2(-m+2)r}}}}{\sqrt{c_1(m+4)p}}, \quad A_0 = 0, \quad A_{-1} = 0, \quad k = \frac{m}{\sqrt{2}\sqrt{c_1^2(-m)r - 2c_1^2r}}, \quad \lambda = \frac{mq}{c_1(m+4)r}. \tag{34}$$

Put (34) in (27),

Figure 4: (a) and (b) Soliton solution of U_5 is plotted as $c_0 = 2$, $\alpha = 1$, $c_1 = -0.5$, $c_2 = 1$, $m = 2$, $p = 1.01$, $q = 4.1$, $r = -6.1$, $\xi_0 = -2.5$.

$$U_6 = - \left(\frac{(c_1 \exp(c_1(\xi + \xi_0))) \left(\sqrt[4]{2} \sqrt{-\frac{c_2^2(m+2)q}{\sqrt{c_1^2(-(m+2))r}}} \right)}{(1 - c_2 \exp(c_1(\xi + \xi_0))) \sqrt{c_1(m+4)p}} \right), \quad c_1 > 0, \quad (35)$$

$$V_6 = (U_6)^{2/m}, \quad 4c_0c_2 > c_1^2. \quad (36)$$

$$U_7 = - \left(\frac{(-c_1 \exp(c_1(\xi + \xi_0))) \left(\sqrt[4]{2} \sqrt{-\frac{c_2^2(m+2)q}{\sqrt{c_1^2(-(m+2))r}}} \right)}{(c_2 \exp(c_1(\xi + \xi_0)) + 1) \sqrt{c_1(m+4)p}} \right), \quad c_1 < 0. \quad (37)$$

$$V_7 = (U_7)^{2/m}, \quad 4c_0c_2 > c_1^2. \quad (38)$$

Case 3. $c_1 = 0$, $c_3 = 0$,

Family I

$$\lambda = - \left(\frac{4c_0m\sqrt{m+2}pq}{\sqrt{-8c_0c_2mr - 16c_0c_2r\sqrt{8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r}}} \right),$$

$$A_1 = \frac{(-1)^{3/4} \sqrt[4]{c_2} \sqrt[4]{m+2} \sqrt[4]{q}}{\sqrt[4]{8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r}}, \quad k = \frac{im}{\sqrt{-8c_0c_2mr - 16c_0c_2r}}, \quad A_{-1} = 0, \quad (39)$$

$$A_0 = \frac{-\frac{32(-1)^{3/4}c_0^2c_2^{3/2}(n+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2nr - 16c_0c_2r(8c_0^2n^2p^2r + 64c_0^2np^2r + 128c_0^2p^2r)^{3/4}}} - \frac{8(-1)^{3/4}c_0^2c_2^{3/2}n(n+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2nr - 16c_0c_2r(8c_0^2n^2p^2r + 64c_0^2np^2r + 128c_0^2p^2r)^{3/4}}}}{c_2q}.$$

Put (39) in (27) (Figure 5),

Figure 5: (a) and (b) Soliton solution of U_6 is plotted as $\alpha = 1$, $c_1 = 3.5$, $c_2 = -3.13$, $m = 2$, $p = 1.01$, $q = 1.1$, $r = 7.01$, $\xi_0 = -0.005$.

$$U_8 = \frac{-\frac{32(-1)^{3/4}c_0^2c_2^{3/2}(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r)^{3/4}}} - \frac{8(-1)^{3/4}c_0^2c_2^{3/2}m(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r)^{3/4}}}}{c_2q} + \left(\frac{((-1)^{3/4}\sqrt{c_2}\sqrt{m+2}\sqrt{q})(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0)))}{c_2^4\sqrt{8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r}} \right), \quad c_0c_2 > 0, \quad (40)$$

$$V_8 = (U_8)^{2/m}, \quad c_0c_2 > 0. \quad (41)$$

$$U_9 = \frac{-\frac{32(-1)^{3/4}c_0^2c_2^{3/2}(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r)^{3/4}}} - \frac{8(-1)^{3/4}c_0^2c_2^{3/2}m(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r)^{3/4}}}}{c_2q} + \left(\frac{(-\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0))) + \frac{(-1)^{3/4}\sqrt{c_2}\sqrt{m+2}\sqrt{q}}{\sqrt{8c_0^2m^2p^2r + 64c_0^2mp^2r + 128c_0^2p^2r}}}{c_2} \right), \quad c_0c_2 < 0. \quad (42)$$

$$V_9 = (U_9)^{2/m}, \quad c_0c_2 < 0. \quad (43)$$

Family II

$$A_0 = -\frac{-\frac{32(-1)^{3/4}c_2^2c_0^{3/2}(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r)^{3/4}}} - \frac{8(-1)^{3/4}c_2^2c_0^{3/2}m(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r)^{3/4}}}}{c_0q}, \quad (44)$$

$$A_{-1} = \frac{(-1)^{3/4}\sqrt{c_0}\sqrt{m+2}\sqrt{q}}{\sqrt{8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r}}, \quad k = \frac{im}{\sqrt{-8c_0c_2mr - 16c_0c_2r}}, \quad A_1 = 0,$$

$$\lambda = -\frac{4c_2m\sqrt{m+2}pq}{\sqrt{-8c_0c_2mr - 16c_0c_2r}\sqrt{8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r}}.$$

Put (44) in (27) (Figure 6),

$$U_{10} = \frac{-\frac{32(-1)^{3/4}c_2^2c_0^{3/2}(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r)^{3/4}}} - \frac{8(-1)^{3/4}c_2^2c_0^{3/2}m(m+2)^{3/4}pq^{3/2}r}{\sqrt{-8c_0c_2mr - 16c_0c_2r(8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r)^{3/4}}}}{c_0q} + \left(\frac{(-1)^{3/4}\sqrt{c_0}\sqrt{m+2}\sqrt{q}}{\sqrt{8c_2^2m^2p^2r + 64c_2^2mp^2r + 128c_2^2p^2r}} \right) \left(\frac{1}{\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0))}{c_2}} \right), \quad c_0c_2 > 0, \quad (45)$$

Figure 6: (a) and (b) Soliton solution of U_9 is plotted as $\alpha = 1$, $c_0 = -0.00001$, $c_2 = 0.13$, $m = 4$, $p = 3.01$, $q = 0.5$, $r = -1.001$, $\xi_0 = -4.5$.

$$V_{10} = (U_{10})^{2/m}, \quad c_0 c_2 > 0. \quad (46)$$

$$U_{11} = - \frac{\frac{32(-1)^{3/4} c_2^2 c_0^{3/2} (m+2)^{3/4} p q^{3/2} r}{\sqrt{-8c_0 c_2 m r - 16c_0 c_2 r (8c_2^2 m^2 p^2 r + 64c_2^2 m p^2 r + 128c_2^2 p^2 r)^{3/4}}} - \frac{8(-1)^{3/4} c_2^2 c_0^{3/2} m (m+2)^{3/4} p q^{3/2} r}{\sqrt{-8c_0 c_2 m r - 16c_0 c_2 r (8c_2^2 m^2 p^2 r + 64c_2^2 m p^2 r + 128c_2^2 p^2 r)^{3/4}}}}{c_0 q} + \left(\frac{(-1)^{3/4} \sqrt{c_0} \sqrt[4]{m+2} \sqrt{q}}{\sqrt[4]{8c_2^2 m^2 p^2 r + 64c_2^2 m p^2 r + 128c_2^2 p^2 r}} \right) \left(- \frac{1}{\frac{\sqrt{-c_0 c_2} \tanh(\sqrt{-c_0 c_2}(\xi + \xi_0))}{c_2}} \right), \quad c_0 c_2 < 0. \quad (47)$$

$$V_{11} = (U_{11})^{2/m}, \quad c_0 c_2 < 0. \quad (48)$$

Family III

$$\begin{aligned} A_1 &= \frac{\left(-\frac{1}{2}\right)^{3/4} c_2 (m+2)^{3/4} (m+4) p \sqrt{q} r}{2\sqrt{c_0 c_2} (-(m+2)) r ((m+4)^2 p^2 r)^{3/4}}, \\ A_0 &= - \frac{(-1)^{3/4} \sqrt[4]{m+2} \sqrt{q}}{\sqrt[4]{8m^2 p^2 r + 64mp^2 r + 128p^2 r}}, \\ A_{-1} &= - \frac{\left(-\frac{1}{2}\right)^{3/4} c_0 (m+2)^{3/4} (m+4) p \sqrt{q} r}{2\sqrt{c_0 c_2} (-(m+2)) r ((m+4)^2 p^2 r)^{3/4}}, \\ k &= \frac{im}{\sqrt{-32c_0 c_2 m r - 64c_0 c_2 r}}, \\ \lambda &= \frac{4c_2 m \sqrt{m+2} p q}{4\sqrt{c_0 c_2} (-(m+2)) r \sqrt{(m+4)^2 p^2 r}}. \end{aligned} \quad (49)$$

Put (49) in (27) (Figure 7),

$$\begin{aligned} U_{12} &= - \frac{(-1)^{3/4} \sqrt[4]{m+2} \sqrt{q}}{\sqrt[4]{8m^2 p^2 r + 64mp^2 r + 128p^2 r}} + \left(\frac{\left(-\frac{1}{2}\right)^{3/4} c_2 (m+2)^{3/4} (m+4) p \sqrt{q} r}{2\sqrt{c_0 c_2} (-(m+2)) r ((m+4)^2 p^2 r)^{3/4}} \right) \\ &\quad \times \left(\frac{\sqrt{c_0 c_2} \tan(\sqrt{c_0 c_2}(\xi + \xi_0))}{c_2} \right) - \left(\frac{\left(-\frac{1}{2}\right)^{3/4} c_0 (m+2)^{3/4} (m+4) p \sqrt{q} r}{2\sqrt{c_0 c_2} (-(m+2)) r ((m+4)^2 p^2 r)^{3/4}} \right) \\ &\quad \times \left(\frac{1}{\frac{\sqrt{c_0 c_2} \tan(\sqrt{c_0 c_2}(\xi + \xi_0))}{c_2}} \right), \quad c_0 c_2 > 0, \end{aligned} \quad (50)$$

$$V_{12} = (U_{12})^{2/m}, \quad c_0 c_2 > 0, \quad (51)$$

Figure 7: (a) and (b) Soliton solution of U_{11} is plotted as $\alpha = 1$, $c_0 = -0.00001$, $c_2 = 0.13$, $m = 4$, $p = 3.01$, $q = 0.5$, $r = -5$, $\xi_0 = -4.5$.

$$\begin{aligned}
 U_{13} = & -\frac{(-1)^{3/4}\sqrt{m+2}\sqrt{q}}{\sqrt[4]{8m^2p^2r+64mp^2r+128p^2r}} \\
 & + \left(\frac{\left(-\frac{1}{2}\right)^{3/4}c_2(m+2)^{3/4}(m+4)p\sqrt{q}r}{2\sqrt{c_0c_2}(-(m+2)r((m+4)^2p^2r)^{3/4})} \right) \\
 & \times \left(-\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0))}{c_2} \right) \\
 & \times \left(\frac{\left(-\frac{1}{2}\right)^{3/4}c_0(m+2)^{3/4}(m+4)p\sqrt{q}r}{2\sqrt{c_0c_2}(-(m+2)r((m+4)^2p^2r)^{3/4})} \right) \\
 & \times \left(-\frac{1}{\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0))}{c_2}} \right), \quad c_0c_2 < 0. \\
 V_{13} = & (U_{13})^{2/m}, \quad c_0c_2 < 0.
 \end{aligned} \tag{52}$$

$$\tag{53}$$

too much convincing and it might be employed to solve several different NLPDEs in mathematical physics and engineering sciences.

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5 Conclusion

In this article, we have pondered the space–time fractional GEW equation and derived different form of solutions like trigonometric, hyperbolic, and exponential function solutions with the assistance of Riemann–Liouville fractional derivative via two novel mathematical methods. The constructed results have been confirmed with computational software Mathematica by keeping them back into NLPDE to found accuracy. To study the physical behavior of the concern GEW model, some established solutions are plotted graphically in 2D and 3D by throwing the specific values to the parameters. Hence, the suggested schemes are effective,

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