Research Article

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Computational examination of Jeffrey nanofluid through a stretchable surface employing Tiwari and Das model

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Abstract: In this research, we analyze the magnetohydrodynamics heat act of a viscous incompressible Jeffrey nanoliquid, which passed in the neighborhood of a linearly extending foil. As a process, we employ alumina (Al_2O_3) as nanoparticles, assuming that the base fluid is ethylene glycol. In this involvement, we consider the heating by Joule effect and viscous dissipation. We select the passable transformations, motion, and temperature formulas converting into non-linear differential equation arrangement. We solved the system by using a Keller-box method. Then, we provide a graphical description of

outcomes according to the selected control parameters. Higher values of dissipation parameter cause a surge in temperature field as well as strengthen width of the heat boundary layer. The velocity, drag coefficient, and heat transfer (HT) rate for the base fluid are comparatively greater than that of the Al_2O_3 –ethylene glycol nanofluid, although the temperature is embellished by the inclusion of nanoparticles. Moreover, we report depreciation in surface drag as well as HT by the virtue of amplification in the Deborah number. The proclaimed outcomes are advantageous to boost the incandescent light bulb's, cooling and heating processes, filament emitting light, energy generation, multiple heating devices, etc.

Keywords: alumina nanoparticles, magnetic field, Keller-box method, stretching sheet, viscous dissipation

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Nomenclature

U, V	velocity components
c_p	specific heat
C_f	skin friction coefficient
Υ_w	temperature
$f'(\zeta)$	dimensionless velocity
Υ_∞	environmental temperature
C_f	surface drag coefficient
L	sheet length
U_w	stretching speed
a	stretching rate
Nu	Nusselt number
q_w	heat flux at the wall
ν	kinematic viscosity
k	thermal based conductivity
Υ	temperature
s	solid phase
f	fluid phase
nf	nanofluid

ρ	density of fluid
λ_1	relaxation to retardation times
λ_2	retardation time
ϕ	volume fraction of nanoparticles
ς	similarity variable
θ	dimensionless form of temperature
ψ	stream function
α	thermal diffusivity
μ	dynamic viscosity
σ	electric based conductivity
τ_w	wall shear stress
β	Deborah number

1 Introduction

A nanofluid is a special type of fluid dealing with nanometer-sized atoms, named nanoparticles. These fluids are manufactured colloidal adjournments of nanoparticles in a base fluid. The nanoparticles in nanofluids are designated oxides, metals, carbon, or carbide nanotubes. Collective base fluids contain liquid, ethylene glycol, and emollient. Nanofluids have innovative properties that mark them theoretically and practically helpful in numerous applications such as heat transfer (HT) [1], micro-electronics [2], fuel cells [3], pharmaceutical procedures [4], hybrid-powered engines [5], engine cooling/vehicle heat organization, local icebox [6], heat exchanger, and air heat reduction [7]. Nanofluids correspondingly have larger acoustical possessions and in ultrasonic areas show extra shear-wave reform of an incidence compressional effort. Khan *et al.* [8] offered a data to determine the fluid procedure of HT between couples of parallel plates. Razzaq *et al.* [9] employed HT to investigate the Brinkman-type fractional nanofluid over a vertical spongy plate. Qi *et al.* [10] focused on backward-facing step stream and HT features of mixed nanofluids. Different advanced approaches utilizing HT and nanofluid are presented in refs [11,12]. More detailed information can be seen in refs [13–28].

Recently, investigations in the movement of non-Newtonian have been presented by many scientists because of their outstanding status in numerous manufacturing and biochemical accomplishments, nuclear manufactures, bio-engineering, geophysics, and physical processing. Consequently, the prominence of non-Newtonian fluids is still wondering, and therefore, various non-Newtonian projects are formulated in the efforts to validate their multiple rheological perspectives. In view of these collections, the Jeffrey fluid paradigm (JFP) is a linear viscoelastic fluid that positively stands the significant article of the relation of the reduction and obstruction times and owns time derivatives as an alternative of convective results. The memory and elastic article

guesses related to the organic and weak material resources can be indicated by utilizing this paradigm. Examples of Jeffrey fluid model are paints, DNA suspensions, biological fluids, and dilute polymer solution. The application areas are scaffolds for tissue engineering, drug delivery system, jet fuels, polymer extrusion, viscoelastic blood flow, *etc.* Due to such interesting performance, numerous researchers have considered the stream structures in JFP under many features. For example, Hayat *et al.* [29] reviewed the effects of variable heat conductivity in 3D-JFP over a strained surface with heat radiation structures. Shehzad *et al.* [30] discovered the thermophoresis and Brownian motion physiognomies in JFP through nanoparticles in the occurrence of heat radiation. Hayat *et al.* [31] exposed the HT structures by consuming the isolated heat flux philosophy in JFP over a revolving geometry. The shared effects of JFP and viscous degeneracy in conducting JFP were pictured by Kumar *et al.* [32]. Tlili [33] offered the impact of thermal group on electrically conducting JFP through characteristics bio-convection case. The distorted model is mathematically presented by the well-known finite difference arrangement. Ahmad [34] systematically examined the HT structures in a time-dependent, 3D-JFP tempted by a tottering strained surface. Shehzad *et al.* [35] methodically expounded the Brownian signal and thermophoresis features in JFP over a multi-directional stressed design. Aleem *et al.* [36] examined the HT structures in the magnetohydrodynamics (MHD) movement of JFP by modifying boundary conditions, and particular solution is realized for speed and heat fields by the Laplace transform method.

MHD (or known as hydro-magnetics) is the investigation of changing aspects in the occurrence of magnetic features and effect of electrically conducting fluids, which has major applications in industrial and biomedical skills. Fluid metals, plasma, electrolytes, and salt aquatic are the samples of MHD. It movements in much structure major to engineering skills and its applications. The requests of MHD hold the researchers and experts to grow novel mathematical demonstrating in the area of fluid mechanics. Several investigators and expert have been stimulated to study MHD movement of viscous material over a strained surface through their good-looking thermo-physical features and HT presentation as well as huge appreciated operations in every day. The operations of MHD can arise in magnetic drug leveling, astrophysics devices, and manufacturing [37,38]. Precisely, the magnetic fields have significant roles in creating the actors. Based on these impacting compensations, the detectives and academics are unceasingly reviewing the MHD movements. Patel and Singh [39] presented MHD movement of micro polar material with thermo-apheresis, nonlinear energy, and Brownian

dispersion, and diverse convection, JFP warming, and convective boundary conditions over a permeable stretched sheet. HT and MHD movement of differential types of fluid subject to dissipation and non-uniform heat by a permeable average are studied by Metri *et al.* [40]. Mahanthesh *et al.* [41] discovered MHD time-dependent Eyring–Powell nanomaterial flow via convectively excited surface with JFP warming and radiated thermal flux. Karkera *et al.* [42] studied 2D-MHD edge sheet viscous fluid movement and the consequences are calculated through mathematical methodology through wavelets. Imtiaz *et al.* [43] examined radiated MHD dual-altered biochemical-type movement of second-grade fluid against arched surface. Khan *et al.* [44] explored heat slip MHD movement of viscous physical focus on radioactivity by a strained cylinder. Hayat *et al.* [45] decorated nonlinear centrifugally MHD overextended movement of different kinds of fluid in view of Newtonian warming. Seyyedi *et al.* [46] investigated entropy optimization. Selvi and Muthuraj [47] discussed MHD oscillatory JFP in a perpendicular permeable frequency with JFP heating and viscous dissipation. Rasheed *et al.* [48] figured out the salient countryside of HT effects on instable natural stream of nanofluids over a cylinder surface with magnetic fields and radiation. Babu *et al.* [49] utilized a numerical study to examine the JFP heating of hydromagnetic movement and HT effects on a JFP in view of a holey extending sheet. After that, many studies appear, considering various aspects of the problem [50–57], to list only a few.

Joule heating process (JHP) is the procedure by which the channel of an electric current through an electrode creates heat. The noticeable detail of JHP generally stands up in the procedures of chilling of metal sheets or electronic damages, power generation schemes, fluid metal, and freezing of nuclear devices. In opinion of all these features, Hayat *et al.* [58] described the influence of glutinous degeneracy and JHP effects on viscous liquid. Shah *et al.* [59] examined the ionic slip and hall current in a perpendicular plate. Shoaib *et al.* [60] labeled MHD of magnetic particles. Lund *et al.* [61] considered the HT in JFP with postponed nanoparticles. Muhammad *et al.* [62] reported a boundary layer study of fluid statistically with heat relevant to the heat source.

Among the numerical procedures for explaining problems, the Keller-box method (KBM) is an implicit technique for which a class of differential equations is condensed to the scheme of first-order differential equations [63]. Jamshed and Nisar [64] presented a computational single-phase comparative investigation of a Williamson nanofluid using KBM. Shabbir *et al.* [65] modeled a numerical simulation of a micropolar fluid over utilizing KBM. Zeeshan *et al.* [66] formulated a study of

MHD on a perpendicular wavy surface with viscous dissipation and Joule heating effects employing KBM. Faisal *et al.* [67] investigated nanofluid movement due to unsteady multi-directional extending surface using KBM. Salahuddin [68] systemized a fluid model toward a stretching cylinder based on KBF. Jamshed *et al.* [69] applied KBM on Casson nanofluid over an extending sheet.

Our key objective here is to provide numerical solutions for a Jeffrey alumina (Al_2O_3)–ethylene glycol nanofluid MHD flow on a stretch sheet employing Tiwari and Das model suggested by Nayak *et al.* [70]. The present mathematical model of nanofluid flow emulates prototype model of thermal solar collectors and tough glass manufacturing. Here model proposed an introduction of flat porous surface within the flow stream and its effect, i.e., magnetic field, viscous dissipation, and Joule effects. The transformed ordinary differential equations (ODEs) were solved digitally using the KBM. Detailed analysis and graphs of dimensionless parameters' impact on velocity and thermal fields were performed. In addition, an analysis was done about the impact of various dimensionless parameters on drag coefficient and local Nusselt number. The application areas are scaffolds for tissue engineering, drug delivery system, jet fuels, polymer extrusion, viscoelastic blood flow *etc.*

2 Mathematical modelling

The model is supposed to flow across a stretched sheet with increasing velocity proportional to the distance from the origin along the X -axis horizontally. The momentum and thermal boundary-layer thickness of the laminar, incompressible, and steady flow properties of Jeffrey nanofluid have been determined using explicit equations. We deliberate a Al_2O_3 –ethylene glycol nanofluid flowing through a stretching sheet, experiencing a magnetic field B_0 employed in Y -axis direction and flow is bounded in $Y > 0$, as presented in Figure 1. Select the Cartesian coordinate system such that the X -axis is along the stretching sheet and the flow is made by stretching the sheet linearly with velocity $U_w = aX$ such that keeping a as a positive constant. The plate wall temperature is Ψ_w having quadratic-type expansion at $Y = 0$, i.e., $\Psi_w = \Psi_\infty + A\left(\frac{X}{L}\right)^2$. The magnetic Reynolds number is considered to be very low, allowing the generated magnetic field to be ignored and no external electric field to be supplied. The rheology equation [71] for the Jeffrey model can be presented as follows:

Figure 1: Physical interpretation of flow geometry.

$$\boldsymbol{\tau} = -p\mathbf{I} + \frac{\mu}{(1 + \lambda_1)} \left[\mathbf{S}_1 + \lambda_2 \left(\frac{\partial \mathbf{S}_1}{\partial t} + \nabla \cdot \mathbf{V} \right) \mathbf{S}_1 \right], \quad (1)$$

where $\boldsymbol{\tau}$ is the Cauchy stress tensor, p is the pressure, and $\mathbf{S}_1$ is the Rivlin–Ericken tensor defined by $\mathbf{S}_1 = (\text{grad } \mathbf{V} + \text{grad } \mathbf{V}^t)$. Subject to these assumptions, the boundary layer equations [72] administering the nanofluid flow and the heat fields can be presented, in dimensional form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (2)$$

$$\left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = \frac{\nu_{nf} \lambda_2}{(1 + \lambda_1)} \left(U \frac{\partial^3 U}{\partial X \partial^2 Y} - \frac{\partial U}{\partial X} \frac{\partial^2 U}{\partial Y^2} + \frac{\partial U}{\partial Y} \frac{\partial^2 U}{\partial X \partial Y} + V \frac{\partial^3 U}{\partial Y^3} \right) - \frac{\nu_{nf}}{(1 + \lambda_1)} \frac{\partial^2 U}{\partial Y^2} - \frac{\sigma_{nf} B_0^2 U}{\rho_{nf}}, \quad (3)$$

$$U \frac{\partial \Psi}{\partial X} + V \frac{\partial \Psi}{\partial Y} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 \Psi}{\partial Y^2} + \frac{\mu_{nf}}{(\rho c_p)_{nf}} \left(\frac{\partial U}{\partial Y} \right)^2 + \frac{\sigma_{nf}}{(\rho c_p)_{nf}} B_0^2 U^2. \quad (4)$$

The corresponding endpoint conditions are as follows:

$$\left. \begin{aligned} Y = 0 : U = U_w, V = 0, \Psi = \Psi_w, \\ Y \rightarrow \infty : U \rightarrow 0, \frac{\partial U}{\partial Y} \rightarrow 0, \Psi \rightarrow \Psi_\infty. \end{aligned} \right\} \quad (5)$$

The thermophysical characteristics of nanofluids are described as follows [73] (Tables 1 and 2).

Table 1: Thermo-physical features of nanofluid

Properties	Nanofluid
Dynamic viscosity	$\mu_{nf} = \mu_f (1 - \phi)^{-2.5}$
Density	$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s$
Heat capacity	$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{[(k_s + (m-1)k_f) - (m-1)\phi(k_f - k_s)]}{[(k_s + (m-1)k_f) + \phi(m-1)(k_f - k_s)]}$
Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi}$

Initiate the following similarity variables which transforms equations (2)–(4) into the ODEs

$$\psi = -\sqrt{av}XF(\zeta), \quad \theta = \frac{\Psi - \Psi_\infty}{\Psi_w - \Psi_\infty}, \quad \zeta = \sqrt{\frac{a}{v}}Y, \quad (6)$$

and stream function ψ is expressed as follows:

$$U = \frac{\partial \psi}{\partial Y} \quad \text{and} \quad V = -\frac{\partial \psi}{\partial X}. \quad (7)$$

So we have

$$U = aXF'(\zeta), \quad V = -\sqrt{av}F(\zeta). \quad (8)$$

Employing equations (6) and (8), equations (2)–(4) are turned into

$$\begin{aligned} F''' - \frac{K_2}{K_1}(1 + \lambda_1)[(F')^2 - FF''] + \beta[(F'')^2 - FF^{iv}] \\ - (1 + \lambda_1)\frac{K_5}{K_1}MF' = 0, \end{aligned} \quad (9)$$

$$\begin{aligned} \theta'' + \frac{K_3}{K_4}\text{Pr}(F\theta' - 2\theta F') + \frac{K_1}{K_4}\text{PrEc}F'' \\ + \frac{K_5}{K_4}\text{EcPr}M(F')^2 = 0, \end{aligned} \quad (10)$$

subjected to boundary conditions (BCs)

$$\left. \begin{aligned} \zeta = 0 : F(0) = 0, F'(0) = 1, \theta(0) = 1 \\ \zeta \rightarrow \infty : F'(\zeta) \rightarrow 0, \theta(\zeta) \rightarrow 0. \end{aligned} \right\} \quad (11)$$

Table 2: Values of thermophysical features of using materials at 293 K [74]

Physical properties	ρ (kg/m ³)	C_p (J/kgK)	k (W/m)	σ (S.M) ⁻¹
Al ₂ O ₃	3,970	765	40	35×10^6
Ethylene glycol	1,114	2,415	0.252	5.5×10^{-6}

Various dimensionless parameters occurring in equations (9) and (10) are delineated as

$$\left. \begin{aligned} \beta &= a\lambda_2 \text{ (Deborah number)}, \\ M &= \frac{\sigma_f B_0^2}{a\rho_f} \text{ (MHD parameter)}, \\ \text{Pr} &= \frac{\mu_f(c_p)_f}{k_f} \text{ (Prandtl number)}, \\ \text{Ec} &= \frac{a^2 l^2}{(\Psi_w - \Psi_\infty)(c_p)_f} \text{ (Eckert number)}, \\ K_1 &= (1 - \phi)^{-2.5}, \quad K_2 = \left[(1 - \phi) + \phi \frac{\rho_s}{\rho_f} \right], \\ K_3 &= \left[(1 - \phi) + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f} \right], \\ K_4 &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + (k_f - k_s)}, \\ K_5 &= 1 + \frac{3\left(\frac{\rho_s}{\rho_f} - 1\right)\phi}{\left(\frac{\rho_s}{\rho_f} + 2\right) - \left(\frac{\rho_s}{\rho_f} - 1\right)\phi}. \end{aligned} \right\} \quad (12)$$

The expressions regarding important surface drag C_f and HT Nu_X are manifested by

$$C_f = \frac{2\tau_w}{\rho_f U_w^2}, \quad \text{Nu}_X = \frac{Xq_w}{k_f(\Psi_w - \Psi_\infty)}, \quad (13)$$

expressions regarding shear as well as heat flux at the wall are manifested by $\tau_w = \mu_{\text{nf}}\left(\frac{\partial U}{\partial Y}\right)$ and $q_w = -k_{\text{nf}}\left(\frac{\partial \Psi}{\partial Y}\right)$, respectively, and moreover equation (13) can be transformed likewise

$$\left. \begin{aligned} C_f \text{Re}^{1/2} &= (1 - \phi)^{-2.5} F''(0), \\ \text{Re}_X^{-1/2} \text{Nu}_X &= -\frac{k_{\text{nf}}}{k_f} \theta'(0), \end{aligned} \right\} \quad (14)$$

whereas the expression regarding Reynolds number is $\text{Re}_X = \frac{U_X}{\nu_f}$.

3 Solution methodology

The dimensionless system of ODEs (9)–(10) alongside (11) can be handled numerically with reliable numerical scheme called KBM [75]. The flow chart of mechanisms of this scheme is displayed below in order to achieve numerical outcomes (Figure 2).

Figure 2: Flow chart illustrating KBM.

4 Numerical procedure

We introduce dependent variables $\Gamma\tilde{u}$, $\Gamma\tilde{v}$, $\Gamma\tilde{w}$, and $\Gamma\tilde{t}$ such that

$$\frac{dF}{d\zeta} = \Gamma\tilde{u}, \quad \frac{d\Gamma\tilde{u}}{d\zeta} = \Gamma\tilde{v}, \quad \frac{d\Gamma\tilde{v}}{d\zeta} = \Gamma\tilde{w}, \quad \frac{d\theta}{d\zeta} = \Gamma\tilde{t}. \quad (15)$$

So that equations (9) and (10) can be written as

$$\begin{aligned} -\beta F \frac{d\Gamma\tilde{w}}{d\zeta} + \Gamma\tilde{w} - \frac{K_2}{K_1}(1 + \lambda_1)[\Gamma\tilde{u}^2 - F\Gamma\tilde{v}] + \beta\Gamma\tilde{v}^2 \\ - \frac{1}{K_1}(1 + \lambda_1)M\Gamma\tilde{u} = 0 \end{aligned} \quad (16)$$

and

$$\begin{aligned} \frac{d\Gamma\tilde{t}}{d\zeta} + \frac{K_3}{K_4}Pr(F\Gamma\tilde{t} - 2\Gamma\tilde{u}\theta) + \frac{K_1}{K_4}PrEc\Gamma\tilde{v}^2 \\ + \frac{1}{K_4}MPrEc\Gamma\tilde{u}^2 = 0, \end{aligned} \quad (17)$$

and dimensionless BCs are enumerated by

$$\left. \begin{aligned} F(0) &= 0, \quad \Gamma\tilde{u}(0) = 1, \quad \theta(0) = 1, \\ \Gamma\tilde{u} &\rightarrow 0, \quad \theta \rightarrow 0 \text{ as } \zeta \rightarrow \infty. \end{aligned} \right\} \quad (18)$$

The domain of the system can be discretized with the following nodes (Figure 3):

Figure 3: Typical grid structure for difference approximations.

$\zeta_0 = 0$, $\zeta_j = \zeta_{j-1} + h_j$, $j = 0, 1, 2, 3, \dots, J$, $\zeta_J = \zeta_\infty$, whereas the term h_j indicates step-size. Equations (15)–(17) with the utilization central difference can be approximated at midpoint $\zeta_{j-1/2}$, likewise

$$\frac{F_j - F_{j-1}}{h_j} = \frac{\Gamma \tilde{u}_j + \Gamma \tilde{u}_{j-1}}{2}, \quad (19)$$

$$\frac{\Gamma \tilde{u}_j - \Gamma \tilde{u}_{j-1}}{h_j} = \frac{\Gamma \tilde{v}_j + \Gamma \tilde{v}_{j-1}}{2}, \quad (20)$$

$$\frac{\Gamma \tilde{v}_j - \Gamma \tilde{v}_{j-1}}{h_j} = \frac{\Gamma \tilde{w}_j + \Gamma \tilde{w}_{j-1}}{2}, \quad (21)$$

$$\frac{\theta_j - \theta_{j-1}}{h_j} = \frac{\Gamma \tilde{t}_j + \Gamma \tilde{t}_{j-1}}{2}, \quad (22)$$

$$\left. \begin{aligned} & \frac{\Gamma \tilde{w}_j + \Gamma \tilde{w}_{j-1}}{2} - \frac{K_2}{K_1}(1 + \lambda_1) \left[\left(\frac{\Gamma \tilde{u}_j + \Gamma \tilde{u}_{j-1}}{2} \right)^2 \right. \\ & \quad \left. - \left(\frac{F_j + F_{j-1}}{2} \right) \left(\frac{\Gamma \tilde{v}_j + \Gamma \tilde{v}_{j-1}}{2} \right) \right] + \beta \left[\left(\frac{\Gamma \tilde{v}_j + \Gamma \tilde{v}_{j-1}}{2} \right)^2 \right. \\ & \quad \left. - \left(\frac{F_j + F_{j-1}}{2} \right) \left(\frac{\Gamma \tilde{w}_j - \Gamma \tilde{w}_{j-1}}{h_j} \right) \right] \\ & \quad \left. - \frac{1}{K_1} M(1 + \lambda_1) \left(\frac{\Gamma \tilde{u}_j + \Gamma \tilde{u}_{j-1}}{2} \right) \right] = 0 \end{aligned} \right\}, \quad (23)$$

$$\left. \begin{aligned} & \frac{\Gamma \tilde{t}_j + \Gamma \tilde{t}_{j-1}}{h_j} + \frac{K_3}{K_4} \Pr \left(\frac{F_j + F_{j-1}}{2} \right) \left(\frac{\Gamma \tilde{t}_j + \Gamma \tilde{t}_{j-1}}{2} \right) \\ & - 2 \frac{K_3}{K_4} \Pr \left(\frac{\Gamma \tilde{u}_j + \Gamma \tilde{u}_{j-1}}{2} \right) \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \\ & + \frac{A_1}{A_5} \Pr \text{Ec} \left(\frac{\Gamma \tilde{v}_j + \Gamma \tilde{v}_{j-1}}{2} \right)^2 + \frac{A_3}{A_5} M \Pr \text{Ec} \left(\frac{\Gamma \tilde{u}_j + \Gamma \tilde{u}_{j-1}}{2} \right)^2 \\ & = 0. \end{aligned} \right\} \quad (24)$$

Equations (19)–(24) can be further linearized with the help of well-known scheme called Newton's method by introducing the substitutions underneath:

$$\left. \begin{aligned} F_j^{n+1} &= F_j^n + \delta F_j^n, \quad \Gamma \tilde{u}_j^{n+1} = \Gamma \tilde{u}_j^n + \delta \Gamma \tilde{u}_j^n, \\ \Gamma \tilde{v}_j^{n+1} &= \Gamma \tilde{v}_j^n + \delta \Gamma \tilde{v}_j^n, \\ \Gamma \tilde{w}_j^{n+1} &= \Gamma \tilde{w}_j^n + \delta \Gamma \tilde{w}_j^n, \quad \Gamma \tilde{t}_j^{n+1} = \Gamma \tilde{t}_j^n + \delta \Gamma \tilde{t}_j^n, \\ \theta_j^{n+1} &= \theta_j^n + \delta \theta_j^n. \end{aligned} \right\} \quad (25)$$

Putting these expressions in (19)–(24) and dropping terms having higher powers in terms of δ to get the system of equation mentioned below

$$\delta F_j - \delta F_{j-1} - \frac{h_j}{2} (\delta \Gamma \tilde{u}_j + \delta \Gamma \tilde{u}_{j-1}) = (r_1)_j, \quad (26)$$

$$\delta \Gamma \tilde{u}_j - \delta \Gamma \tilde{u}_{j-1} - \frac{h_j}{2} (\delta \tilde{v}_j + \delta \tilde{v}_{j-1}) = (r_2)_j, \quad (27)$$

$$\delta \Gamma \tilde{v}_j - \delta \Gamma \tilde{v}_{j-1} - \frac{h_j}{2} (\delta \Gamma \tilde{w}_j + \delta \Gamma \tilde{w}_{j-1}) = (r_3)_j, \quad (28)$$

$$\delta \theta_j - \delta \theta_{j-1} - \frac{h_j}{2} (\delta \Gamma \tilde{t}_j + \delta \Gamma \tilde{t}_{j-1}) = (r_4)_j, \quad (29)$$

$$\begin{aligned} & (\xi_1)_j \delta \Gamma \tilde{w}_j + (\xi_2)_j \delta \Gamma \tilde{w}_{j-1} + (\xi_3)_j \delta F_j + (\xi_4)_j \delta F_{j-1} \\ & + (\xi_5)_j \delta \Gamma \tilde{v}_j + (\xi_6)_j \delta \Gamma \tilde{v}_{j-1} + (\xi_7)_j \delta \Gamma \tilde{u}_j \\ & + (\xi_8)_j \delta \Gamma \tilde{u}_{j-1} = (r_5)_j, \end{aligned} \quad (30)$$

$$\begin{aligned} & (\varepsilon_1)_j \delta \Gamma \tilde{t}_j + (\varepsilon_2)_j \delta \Gamma \tilde{t}_{j-1} + (\varepsilon_3)_j \delta F_j + (\varepsilon_4)_j \delta F_{j-1} + (\varepsilon_5)_j \delta \Gamma \tilde{u}_j \\ & + (\xi_6)_j \delta \Gamma \tilde{u}_{j-1} + (\varepsilon_7)_j \delta \theta_j + (\varepsilon_8)_j \delta \theta_{j-1} + (\varepsilon_9)_j \delta \Gamma \tilde{v}_j \\ & + (\varepsilon_{10})_j \delta \Gamma \tilde{v}_{j-1} = (r_6)_j, \end{aligned} \quad (31)$$

where

$$\left. \begin{aligned}
 (\xi_1)_j &= -\frac{\beta}{2}(F_j + F_{j-1}) + \frac{h_j}{2}, \quad (\xi_2)_j = \frac{\beta}{2}(F_j + F_{j-1}) + \frac{h_j}{2}, \\
 (\xi_3)_j &= -\frac{\beta}{2}(\Gamma\tilde{w}_j + \Gamma\tilde{w}_{j-1}) + \frac{K_2}{K_1}\frac{h_j}{4}(1 + \lambda_1)(\Gamma\tilde{v}_j + \Gamma\tilde{v}_{j-1}) = (\xi_4)_j, \\
 (\xi_5)_j &= \frac{K_2}{K_1}\frac{h_j(1 + \lambda_1)(F_j + F_{j-1})}{4} + \frac{\beta h_j(\Gamma\tilde{v}_j + \Gamma\tilde{v}_{j-1})}{2} = (\xi_6)_j, \\
 (\xi_7)_j &= -\frac{K_2}{K_1}\frac{h_j(1 + \lambda_1)(\Gamma\tilde{u}_j + \Gamma\tilde{u}_{j-1})}{2} - \frac{1}{K_1}\frac{Mh_j(1 + \lambda_1)}{2} = (\xi_8)_j, \\
 (r_5)_j &= \frac{(\Gamma\tilde{w}_j + \Gamma\tilde{w}_{j-1})}{2}(\beta(F_j + F_{j-1}) - h_j) - \frac{K_2}{K_1}\frac{h_j(1 + \lambda_1)(F_j + F_{j-1})(\Gamma\tilde{v}_j + \Gamma\tilde{v}_{j-1})}{4} \\
 &\quad - \frac{\beta h_j(\Gamma\tilde{v}_j + \Gamma\tilde{v}_{j-1})^2}{4} + \frac{K_2}{K_1}\frac{h_j(1 + \lambda_1)(\Gamma\tilde{u}_j + \Gamma\tilde{u}_{j-1})^2}{4} + \frac{1}{K_1}\frac{Mh_j(1 + \lambda_1)(\Gamma\tilde{u}_j + \Gamma\tilde{u}_{j-1})}{2},
 \end{aligned} \right\} \quad (32)$$

$$\left. \begin{aligned}
 (\varepsilon_1)_j &= 1 + \frac{K_3}{K_4}\frac{\text{Pr}h_j(F_j + F_{j-1})}{4}, \quad (\varepsilon_2)_j = (\varepsilon_1)_j - 2, \\
 (\varepsilon_3)_j &= \frac{K_3}{K_4}\frac{\text{Pr}h_j(\Gamma\tilde{t}_j + \Gamma\tilde{t}_{j-1})}{4} = (\varepsilon_4)_j, \\
 (\varepsilon_5)_j &= -\frac{K_3}{K_4}\frac{\text{Pr}h_j(\theta_j + \theta_{j-1})}{2} + \frac{1}{K_4}\frac{M\text{Pr}Ech_j(\Gamma\tilde{u}_j + \Gamma\tilde{u}_{j-1})}{2} = (\varepsilon_6)_j, \\
 (\varepsilon_7)_j &= -\frac{K_3}{K_4}\frac{\text{Pr}h_j(\Gamma\tilde{u}_j + \Gamma\tilde{u}_{j-1})}{2} = (\varepsilon_8)_j, \\
 (\varepsilon_9)_j &= \frac{K_1}{K_4}\frac{\text{Pr}Ech_j(\Gamma\tilde{v}_j + \Gamma\tilde{v}_{j-1})}{2} = (\varepsilon_{10})_j, \\
 (r_6)_j &= -\frac{K_3}{K_4}\frac{\text{Pr}h_j(F_j + F_{j-1})(\Gamma\tilde{t}_j + \Gamma\tilde{t}_{j-1})}{4} + (\Gamma\tilde{t}_{j-1} - \Gamma\tilde{t}_j) \\
 &\quad - \frac{K_1}{K_4}\frac{\text{Pr}Ech_j(\Gamma\tilde{v}_j + \Gamma\tilde{v}_{j-1})^2}{4} - \frac{1}{K_4}\frac{M\text{Pr}Ech_j(\Gamma\tilde{u}_j + \Gamma\tilde{u}_{j-1})^2}{4}.
 \end{aligned} \right\} \quad (33)$$

After linearizing process the block tridiagonal matrix mentioned below is achieved.

$$A\delta = R,$$

(34)

$$[A_1] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ -0.5h_1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -0.5h_1 & 0 & 0 & -0.5h_1 & 0 \\ 0 & 0 & -0.5h_1 & 0 & 0 & -0.5h_1 \\ (\xi_6)_1 & (\xi_2)_1 & 0 & (\xi_3)_1 & (\xi_1)_1 & 0 \\ (\varepsilon_{10})_1 & 0 & (\varepsilon_2)_1 & (\varepsilon_3)_1 & 0 & (\varepsilon_1)_1 \end{bmatrix},$$

where

$$A = \begin{bmatrix} [A_1] & [C_1] \\ [B_2] & [A_2] & [C_2] \\ & \ddots & \ddots \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{bmatrix},$$

$$\delta = \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} [R_1] \\ [R_2] \\ \vdots \\ [R_{J-1}] \\ [R_J] \end{bmatrix},$$

where the elements defined in equation (34) are

$$[A_j] = \begin{bmatrix} -0.5h_j & 0 & 0 & 1 & 0 & 0 \\ -1 & -0.5h_j & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -0.5h_j & 0 \\ 0 & 0 & -1 & 0 & 0 & -0.5h_j \\ (\xi_8)_j & (\xi_6)_j & 0 & (\xi_3)_j & (\xi_1)_j & 0 \\ (\varepsilon_6)_j & (\varepsilon_{10})_j & (\varepsilon_8)_j & (\varepsilon_3)_j & 0 & (\varepsilon_1)_j \end{bmatrix}, \quad 2 \leq j \leq J$$

$$[B_j] = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5h_j & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5h_j \\ 0 & 0 & 0 & (\xi_4)_j & (\xi_2)_j & 0 \\ 0 & 0 & 0 & (\varepsilon_4)_j & 0 & (\varepsilon_2)_j \end{bmatrix}, \quad 2 \leq j \leq J$$

$$[C_j] = \begin{bmatrix} -0.5h_j & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.5h_j & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ (\xi_7)_j & (\xi_5)_j & 0 & 0 & 0 & 0 \\ (\varepsilon_5)_j & (\varepsilon_9)_j & (\varepsilon_7)_j & 0 & 0 & 0 \end{bmatrix}, 1 \leq j \leq J-1.$$

Now we factorize A as

$$A = LU, \quad (35)$$

where

$$L = \begin{bmatrix} [\alpha_1] & & & & & \\ [B_2] & [\alpha_2] & & & & \\ & & \ddots & & & \\ & & & [\alpha_{J-1}] & & \\ & & & [B_J] & [\alpha_J] & \end{bmatrix},$$

$$U = \begin{bmatrix} [I] & [\Gamma_1] & & & & \\ & [I] & [\Gamma_2] & & & \\ & & \ddots & \ddots & & \\ & & & [I] & [\Gamma_{J-1}] & \\ & & & & [I] & \end{bmatrix},$$

where A is the block tridiagonal matrix of order $J \times J$ with each block of size 6×6 and $[I]$ represents unit block of order 6. Equation (34) can be tackled with LU factorization scheme in order to obtain the numerical results of δ . Since physical domain of the problem is unbounded, while computational domain must be finite, we use far field boundary conditions such that $\zeta_{\max} = 9$. In order to achieve the numerical results the step size as well as error tolerance level can be adjusted at $h_j = 0.01$ and 10^{-6} . Reliability of upcoming results can be checked by taking a healthy comparison with already available literature Hayat *et al.* [76] and Ishak *et al.* [77] in the case of $\theta'(0)$ by keeping $\beta = \phi = \text{Ec} = M = 0$. In the case of $\phi = 0$ makes the problem identical with Ahmad and Ishak [78].

5 Results and discussions

The primary goal of this research is to explore the flow and heat transport characteristics of nanofluid due to a stretching sheet. For the sake of numerical simulation of the problem, the fix values of dimensionless parameters are utilized in particular $\phi = 0.05$, $\beta = 0.5$, $M = 0.5$, $\text{Ec} = 0.3$, $\text{Pr} = 6$ and $\lambda_1 = 0.1$. The values of surface drag as well as HT in terms of Al_2O_3 -ethylene glycol nanofluid are computed and displayed in Table 3. Magnification in ϕ and magnetic term M depreciate surface drag effect, but elevates for the case of positive variation

Table 3: $\theta'(0)$ comparison by keeping $\beta = \phi = \text{Ec} = M = 0$

Pr	Ishak <i>et al.</i> [77]	Hayat <i>et al.</i> [76]	Present study
0.72	0.8086	0.8086314	0.8062
1	1.0000	1.0000000	1.0100
3	1.9237	1.92359132	1.9211
10	3.7207	3.7215968	3.7204

in Deborah number β . Amplification in ϕ , β , and Prandtl number Pr amplifies HT phenomenon, whereas depreciates for the case of magnification in M , and Eckert number Ec .

Figures 4–19 manifest the impact of ϕ , β , M , Pr , and Ec on the flow and HT properties. Figure 4 presents the impacts of β and M on velocity flow $F'(\zeta)$. The field $F'(\zeta)$ is noticeably enhanced for the rising β and this outcome in the augmentation of momentum boundary layer thickness. As β depends upon the retardation time λ_2 , so higher λ_2 increases the fluid flow because of which the velocity field is elevated. The effect of M is monitored from Figure 4 and it is discovered that the existence of M diminishes the thickness of the boundary layer as well as the velocity field. An intensification in the magnetic parameter generates a drag force (Lorentz force), which lessens the flow velocity $F'(\zeta)$. Figure 5 displays β effect on the thermal field $\theta(\zeta)$. Here thermal field is a decreasing function of β . Hence, β causes a decrease in the molecular movements, which eventually decreases the temperature profile of the fluid. From Figure 5, it is quite clear that a magnification in M amplifies temperature profile. Bigger values of M means increase in Lorentz

Figure 4: Impact of β and M on $F(\zeta)$.

Figure 5: Impact of β and M on $\theta(\zeta)$. Figure 8: Impact of β on $F(\zeta)$. Figure 6: Impact of ϕ on $F(\zeta)$. Figure 9: Impact of β and Pr on $\theta(\zeta)$. Figure 7: Impact of ϕ on $\theta(\zeta)$. Figure 10: Impact of Ec and β on $\theta(\zeta)$.

Figure 11: Impact of β and Ec on $-\theta'(0)$. Figure 13: Impact of β and M on $F''(0)$. Figure 12: Impact of β and M on $-\theta'(0)$. Figure 14: Impact of ϕ and M on $F''(0)$.

force that is a resistive force. Hence, thermal profile enhances.

Figure 6 reflects the influence of nanoparticles concentration ϕ on velocity profile. It is noticed that the velocity profile $F'(\zeta)$ decreases with a boost in ϕ . Physically, ethylene glycol-based Jeffrey nanofluid is more dense and has a higher viscosity compared to conventional base fluid which in turn depreciates velocity $F'(\zeta)$ of the nanofluid getting slower as compared with the base fluid. Figure 7 shows that the nanoparticles concentration ϕ increases the fluid temperature. Physically, increasing ϕ brings about an enhancement in thermal conductivity phenomenon which elevates thermal profile. The dynamics of velocity profile for β is illustrated in Figure 8. It is

observed that $F'(\zeta)$ escalates on behalf of an enrichment in β , as an increase in β reduces the resistance of fluid motion.

Figure 9 reflects the performance of Prandtl number Pr in temperature field for both viscous nanofluid ($\beta = 0$) and Jeffrey nanofluid ($\beta = 1$), respectively. It can be noted from the figure that the fluid temperature reduces with escalating Pr . This is because a decrement in Pr amplifying thermal conductivities, hence heat can diffuse away from the heated surface more quickly compared to larger values of Pr . Consequently in case of lesser Pr , the HT rate is reduced. It is observed that the introduction of β reduces the thermal profile. Figure 10 reflects Eckert number Ec effect on thermal field in the case of $\beta = 0$ and $\beta = 1$. It is apparent that a magnification in Ec boosts

Figure 15: Impact of ϕ and β on $F''(0)$. Figure 17: Impact of ϕ and β on $-\theta'(0)$. Figure 16: Impact of ϕ and M on $-\theta'(0)$. Figure 18: Impact of ϕ and Ec on $-\theta'(0)$.

temperature field. Physically, the energy is accumulated in nanofluid because of the frictional heating.

Figure 11 depicts the rate of HT $-\theta'(0)$ with Ec for diverse values of β . It is clear that $-\theta'(0)$ enhances with β , where as it decreases with Eckert number Ec . An augmentation in the Eckert number Ec renders an increment in the temperature profile, which leads to a swift diminution in the heat transport rate. Figure 12 highlights $-\theta'(0)$ with β for distinguished values of M . A magnification in β magnifies $-\theta'(0)$, but lessens in the case of M . Figure 13 portrays the influence of Deborah number β on surface drag $F''(0)$ with a change in M . It is noted that the drag coefficient enhances with respect to β . Also, by increasing the value of magnetic parameter M , there is a decrease in

the drag coefficient. Figure 14 shows $F''(0)$ with ϕ in the case of diverse values of M . Drag coefficient detracts with rise in the nanoparticle concentration and magnetic parameter. The influence of nanoparticle concentration ϕ on the skin friction for a change in the values of β is depicted in Figure 15. It is seen that $F''(0)$ escalates for the case of an improvement in β . As a result, $F''(0)$ lessens by intensifying ϕ . The increase in the concentration of nanoparticles ϕ and the magnetic parameter M leads to the decline of dimensionless flow velocity, this yields a decrease in the skin friction.

It can be noticed from Figure 16 that HT rate $-\theta'(0)$ enhances with a rise in ϕ . However, Figures 17–19 indicate that the HT rate $-\theta'(0)$ reduces as ϕ grows. It is

Figure 19: Impact of ϕ and Pr on $-\theta'(0)$.

Table 4: Numerical outcomes for surface drag coefficient and HT rate for diverse values of sundry dimensionless parameters

ϕ	β	M	Pr	Ec	$C_f Re^{1/2}$	$Nu_x Re_x^{-1/2}$
0.0	0.5	0.5	0.7	0.3	-1.000029	0.957719
0.05					-1.076747	0.959386
0.1					-1.123221	0.962465
0.05	0.0	0.2	0.7	0.3	-0.963904	1.122354
	0.5				-0.825632	1.290590
	1				-0.715303	1.429025
0.05	0.2	0.0	0.7	0.3	-0.860352	1.324460
		0.5			-1.014064	0.956235
		1			-1.150166	0.650645
0.05	0.5	0.5	0.5	0.3	-0.907052	0.943686
			1		-0.907052	1.250629
			1.5		-0.907052	1.502400
0.05	2	0.5	0.7	0.5	-0.641585	1.006540
				1	-0.641585	0.090899
				1.5	-0.641585	-0.824742

identified from the Figures 16–19 that the HT rate $-\theta'(0)$ enhances for rising values of β and Pr ; however, it diminishes for escalating values of M and Ec . Physically, for greater value of Pr , thermal conductivity is reduced, consequently their heat conduction ability lessens. Heat transport rate amplifies near the wall. Amplification in M lessens HT phenomenon (Table 4).

6 Conclusion

In the present research, the problem of heat transport boundary-layer flow in a viscous, incompressible nanofluid

past a linearly stretched moving flat sheet in the presence of Joule heating and viscous dissipation using the Tiwari and Das model is reviewed. The governing partial differential equations are converted to ODEs by employing a suitable similarity variable and are then tackled numerically via KBM (MATLAB software). The major findings for this work are summed up as follows:

- An increment in the concentration ϕ of Al_2O_3 nanoparticles yields a decrement in the nanofluid's temperature, which leads to a rapid reduction in the skin friction.
- The increase in the concentration ϕ of Al_2O_3 nanoparticles leads to the increase in dimensionless temperature profile, which yields a rise in the heat transport rates at the interface.
- Velocity profile is escalating function of Deborah number; however, opposite impact is observed for magnetic parameter.
- The thermal field enhances with escalating values of Pr , ϕ , Ec , M , and β .
- Al_2O_3 -ethylene glycol nanofluid possesses small drag coefficient as well as HT rate in contrast to base fluid.
- The drag reduces with a rise in magnetic parameter M , however, enhances by escalating the Deborah number β .
- The HT rate enhances by a rise in the Prandtl number Pr and Deborah number β however reduces by boosting Ec and M .

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