Research Article

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Modal characteristics of harmonic gear transmission flexspline based on orthogonal design method

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Abstract: The flexspline features several modal characteristics that are related to its structural parameters, which could impose an impact on the stability and reliability of the system of harmonic gear transmission. In this article, we have integrated the method of orthogonal design with the approach of finite element, so as to study the modal characteristics of flexspline under varying structural parameters. Furthermore, we have analyzed the ways of how varying structural parameters could impose an impact on the first eight-order natural frequencies of flexspline. In addition, by adopting the method of range and variance analysis for the obtained data, we have examined the influence imposed by varying structural parameters on the modal characteristics of flexspline, which could lay a foundation for the optimal design of the structural parameters of flexspline.

Keywords: flexspline, modal characteristics, structural parameter, natural frequency, variance analysis

1 Introduction

The transmission of harmonic gear is based on the theory of coordination of elastic deformation. Unlike the conventional approach to gear transmission, the method of

harmonic gear transmission has its unique advantages due to its compact structure, high speed ratio, optimal accuracy and efficiency of transmission, smooth operation, less noise, and optimal coaxiality [1–3]. As a result, the method has been widely applied in spatial technology, military science, robotics, and optical instruments [4–6].

During the harmonic gear transmission, as a thin-wall member, the flexspline is expected to generate a vibration response under the action of periodic excitation, which is prone to undermine the kinetic accuracy of the mechanism and compromise the transmission performance. In particular, when the excitation frequency is close to the natural frequency, local or overall resonance will occur, resulting in more serious consequences. Therefore, it is necessary to study the modal characteristics of flexspline to improve the dynamic characteristics of the system. During their study, Pan and Yang examined the dynamic characteristics of a certain type of harmonic reducer, and obtained the first four-order modal shapes of flexspline, laying a foundation for the optimal design of flexspline [7]. Oh adopted a sort of composite of steel and reinforced fiber to fabricate flexspline, facilitating improvement in the dynamic properties of flexspline [8]. Based on the sensitivity analysis, Deng identified the mechanism of how varying structural parameters of the flexspline may impose an impact on the fatigue strength coefficient as well as their degree of sensitivity [9].

Judging from our study, the structural parameters of flexspline serve as critical factors contributing to the modal characteristics of flexspline, whereas varying structural parameters could impose different impacts on the modal characteristics accordingly. However, among existing studies, few scholars have carried out research on the mechanism of how structural parameters could influence the modal characteristics of flexspline. By integrating the method of orthogonal design and the approach of finite element, we have analyzed the inherent law of how different structural parameters could affect the modal characteristics of flexspline, so as to lay a foundation for selection of optimal structural parameters [10,11].

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2 Modal theory

The method of modal analysis is an analytical approach used in structural dynamics. At present, the modal analysis method has evolved into a vital approach to address issues of engineering vibration. Major methods related to the modal analysis include the subspace iteration method, the iterative Lanczos method, and the iterative Ritz vector method. In the software of finite element analysis, the iterative Lanczos method has found extensive applications. Specifically, the basic step of this approach is to select the initial vector and form m Lanczos vectors through multiple times of inverse iterations, orthogonalization, and normalization. The orthogonal and normalized coefficients could form a tridiagonal matrix, whose eigensolution features certain relations to the first higher order eigensolutions of the original generalized eigenproblem. By leveraging such relations, we are able to obtain the first higher order eigensolutions of the original generalized eigenproblem [12,13].

With respect to the generalized eigenvalue of the n order:

$$K\phi = \lambda M\phi. \quad (1)$$

In terms of the structure, suppose that the stiffness K is positive definite, then $S = K^{-1}M$. First, we have selected a proper initial vector q^1 to satisfy the equation of $(q^1)^T M q^1 = 1$. Subsequently, suppose that k refers to the number of iterations, then we have conducted the following iteration of $k = 1, 2, \dots, m$:

(1) Inverse iteration of vector:

$$\bar{q}^{k+1} = S q^k. \quad (2)$$

(2) Orthogonalization of vector. We have performed the orthogonalization of $\bar{q}^{k+1}$ and two Lanczos vectors, with orthogonal coefficients of α^k and β^k , as specified in the following formulas:

$$\alpha^k = \bar{q}^{k+1} - \alpha^k q^k - \beta^k q^{k-1}, \quad (3)$$

$$\beta^k = (\bar{q}^{k+1})^T M q^{k-1}, \quad (4)$$

$$\beta^1 = 0. \quad (5)$$

(3) Normalization vector. We have performed normalization of $\bar{q}^{k+1}$ to form the $(k+1)$ th Lanczos vector, as specified in the following formula:

$$q^{k+1} = \bar{q}^{k+1} / \gamma^{k+1}, \quad (6)$$

where,

$$\gamma^{k+1} = \sqrt{(\bar{q}^{k+1})^T M \bar{q}^{k+1}}. \quad (7)$$

According to the calculation of generalized inner product, after we have completed the aforementioned procedures of orthogonalization and normalization, we have formed m Lanczos vectors $q^i (i = 1, 2, \dots, m)$, accordingly. The tridiagonal matrix is specified in the following formula:

$$T_m = \begin{bmatrix} \alpha^1 & \beta^2 & & & \\ \gamma^2 & \alpha^2 & \beta^3 & & \\ & \gamma^3 & 0 & 0 & \\ & & 0 & 0 & \\ & & & \alpha^{m-1} & \beta^m \\ & & & \lambda^m & \alpha^m \end{bmatrix}. \quad (8)$$

Solving the tridiagonal eigenvalue,

$$T_m z^i = \rho z^i, \quad (9)$$

we have obtained the approximate solutions of the first m -order eigenvalues of the original generalized eigenproblem [12,13].

$$\bar{\lambda}^i = \frac{1}{\rho^i}, \quad i = 1, 2, \dots, m. \quad (10)$$

Let

$$Q_m = [q^1 \ q^2 \ \dots \ q^m]. \quad (11)$$

Then, the approximate solution of the first m -order eigenvector of the original problem is,

$$\bar{\phi}^i = Q_m z^i, \quad i = 1, 2, \dots, m. \quad (12)$$

For dynamic analysis, only the first m order eigenvalues are often required. Therefore, the solution of the m -order tridiagonal matrix T_m eigenvalue problem is much simpler than the original generalized eigenvalue problem.

3 Orthogonal design

The structural parameters of flexspline include: the length L , tooth width B , tooth root transition fillet R , tooth ring thickness δ , and wall thickness δ_1 , and d_z is the diameter of the wave generator, (as illustrated in Figure 1), which could jointly impose an impact on the inherent properties of flexspline. Given that there is a large number of structural parameters, and that each parameter amounts to varying levels, one separate analysis for each level of combination of parameters will increase the time of analysis [14,15]. Without affecting the conclusion, the orthogonal

Figure 1: Structural parameters of flexspline.

design method can reduce the analysis time to a maximum extent and improve the efficiency. Therefore, we have adopted the method of orthogonal design to perform the finite element modal analysis [16].

We have opted for five discrete values for each structural parameter of flexspline in a certain harmonic reducer. On this basis, we have selected the table of $L_{25}(5^6)$ orthogonal design, while meeting the condition of statistical accuracy [17]. Moreover, these five structural parameters featured five discrete values, whereas the remaining one factor was adopted for error analysis for 25 times. The table of orthogonal design generated through the software of SPSS is specified in Table 1.

4 Analysis of modal characteristics

4.1 Analytical model

There are rather intricate conditions in the boundary of flexspline. In case the established contact model features tooth characteristics, then the grid, memory occupancy,

Table 1: Orthogonal design table

Model No.	Length [L] (mm)	Tooth ring thickness [δ] (mm)	Wall thickness [δ_1] (mm)	Tooth width [B] (mm)	Tooth root transition fillet [R] (mm)	Error column
1	19	0.75	0.5	12	4	2
2	44	0.75	0.65	9	8	4
3	19	0.65	0.45	9	0	1
4	25	0.85	0.65	13	4	1
5	19	0.8	0.65	11	6	5
6	38	0.75	0.55	11	2	1
7	44	0.8	0.55	13	0	2
8	38	0.65	0.5	13	8	5
9	25	0.75	0.6	10	0	5
10	31.5	0.85	0.5	11	0	4
11	31.5	0.75	0.45	13	6	3
12	19	0.85	0.55	10	8	3
13	38	0.85	0.6	9	6	2
14	44	0.85	0.45	12	2	5
15	44	0.7	0.5	10	6	1
16	38	0.7	0.65	12	0	3
17	25	0.7	0.45	11	8	2
18	44	0.65	0.6	11	4	3
19	31.5	0.8	0.6	12	8	1
20	19	0.7	0.6	13	2	4
21	38	0.8	0.45	10	4	4
22	31.5	0.7	0.55	9	4	5
23	31.5	0.65	0.65	10	2	2
24	25	0.65	0.55	12	6	4
25	25	0.8	0.5	9	2	3

Figure 2: Grid division of no. 10 flexspline.

and calculation time will be subject to significant increases. Therefore, we need to ensure that the treatments of the tooth of flexspline are equivalent. By adopting the equivalent method, we intend to reduce the tooth ring of flexspline to a smooth ring with a certain equivalent thickness, whereas the equivalent tooth ring thickness δ amounted to $\sqrt[3]{1.67}$ [18–20] of the smooth ring wall thickness at the tooth root. In this study, we have applied the ABAQUS software into

our analysis of the modal of flexspline, and the grid division of no. 10 flexspline is illustrated in Figure 2.

4.2 Natural frequency analysis

We have opted for the 42CrMo as the material of flexspline. In terms of the ABAQUS material properties, we suppose that elastic modulus $E = 2.06 \times 10^5$ MPa, Poisson's ratio $\mu = 0.3$, and density $\rho = 7,850$ kg/m³. After dividing the hexahedral grid, we have set the type of programs as the frequency analysis, and we have imposed constraints on the bottom end surface of the flexspline cup. We are able to carry out the natural frequency analysis after submitting the task. Based on our analysis of each model, we have obtained the first eight-order natural frequencies of each model (as specified in Table 2) and the first eight-order modal diagrams of Model 1 (as illustrated in Figure 1).

Judging from Table 2, it could be noted that the natural frequency of different models varied to a significant extent, indicating that the structural parameters could impose a huge impact on the modal characteristics of flexspline. Judging from Figure 3, it could be noted that

Table 2: The first eight order natural frequencies of each model

Model No.	Modal 1 (Hz)	Modal 2 (Hz)	Modal 3 (Hz)	Modal 4 (Hz)	Modal 5 (Hz)	Modal 6 (Hz)	Modal 7 (Hz)	Modal 8 (Hz)
1	777	785	874	874	1,165	1,165	1,419	1,419
2	333	335	508	508	785	785	899	899
3	356	356	391	433	454	945	945	1,596
4	644	645	798	798	1,247	1,247	1,351	1,351
5	918	919	992	992	1,549	1,549	1,556	1,556
6	156	156	253	331	340	809	809	1,433
7	294	297	535	535	673	673	964	964
8	322	325	494	494	734	734	865	865
9	627	629	767	767	1,259	1,259	1,274	1,274
10	401	403	683	683	815	815	1,198	1,222
11	384	389	651	651	754	754	1,100	1,147
12	772	773	1,002	1,002	1,352	1,352	1,651	1,651
13	381	383	622	622	873	873	1,104	1,104
14	274	277	536	536	574	574	964	1,044
15	242	246	284	284	450	450	636	636
16	394	396	524	524	905	905	916	916
17	482	485	690	690	927	927	1,175	1,175
18	312	315	459	459	781	781	797	797
19	196	196	290	293	297	702	702	1,236
20	977	982	1,156	1,156	1,709	1,709	1,747	1,747
21	312	316	592	592	640	640	1,044	1,065
22	444	447	611	611	993	993	1,039	1,039
23	511	513	578	578	948	948	1,118	1,118
24	237	237	308	393	409	898	898	1,482
25	528	530	770	770	1,013	1,013	1,331	1,331

Figure 3: The first eight-order modals of Model 1. (a) Modal 1, (b) Modal 2, (c) Modal 3, (d) Modal 4, (e) Modal 5, (f) Modal 6, (g) Modal 7, and (h) Modal 8.

the natural frequency of each order of Model 1 was the equivalent, whereas the difference lies in the amplitude and direction of vibration. Specifically, the vibration deformation of the first six order took place at the tooth ring, and the vibration deformation of the last second order took place at the bottom of the cup, all of which were obviously symmetric. The modals of the remaining models showed similar characteristics, which we would not elaborate further in this article.

4.3 Impact imposed by varying parameters on the modal characteristics of flexspline

4.3.1 Influence imposed by the length on the modal characteristics

The discrete value of the length L amounted to 19, 25, 31.5, 38, and 44 mm (length-to-diameter ratio reaching 0.3, 0.4, 0.5, 0.6, and 0.7, respectively). After taking the mean value of the varying L values of the same level, we have performed the polynomial fitting of the varying levels and mean values of L to identify the rules of how the values of L could affect the natural frequency of each

order, as illustrated in Figure 4. Judging from Figure 4, the first eight-order natural frequencies of flexspline experienced declines with the increase in L . In addition, they decreased significantly in the range of 19–31.5 mm, and decreased slightly in the range of 31.5–44 mm, showing a similar trend of variation. Moreover, the first-order natural

Figure 4: Influence imposed by the length on the natural frequency of each order.

frequency and the second-order natural frequency were close to each other, whereas the third-order natural frequency and the fourth-order natural frequency were different.

4.3.2 Influence imposed by the tooth ring thickness on the modal characteristics

The discrete value of the tooth ring thickness δ amounted to 0.65, 0.70, 0.75, 0.80, and 0.85 mm. The relationship between the δ and the natural frequency of each order is illustrated in Figure 5. Judging from Figure 5, the δ and the natural frequency curve of each order were not monotonic. In addition, the first 7-order natural frequencies experienced gradual increase in the range of 0.65–0.70 mm, decline in the range of 0.70–0.80 mm, and increase again in the range of 0.80–0.85 mm. The trends of variation in the first seven-order natural frequencies were similar, which were opposite to those of the first eight-order natural frequencies.

4.3.3 Influence imposed by the wall thickness on the modal characteristics

The discrete value of the wall thickness δ_1 amounted to 0.45, 0.50, 0.55, 0.60, and 0.65 mm. The relationship between the δ_1 and the natural frequency of each order is illustrated in Figure 6. Judging from Figure 6, the trends of variation in the first four-order natural frequencies were similar, whereas the trends of variation in the last four-order natural frequencies were different. In addition, the trend of variation in the fifth-order natural frequency was contrary to that of the eighth-order natural frequency,

Figure 5: Influence imposed by the tooth ring thickness on the natural frequency of each order.

Figure 6: Influence imposed by the wall thickness on the natural frequency of each order.

whereas the sixth-order natural frequency revealed an increasing trend. Moreover, the seventh-order natural frequency experienced slight changes.

4.3.4 Influence imposed by the tooth width on the modal characteristics

The discrete value of the tooth width B amounted to 9, 10, 11, 12, and 13 mm. The relationship between B and the natural frequency of each order is illustrated in Figure 7. Judging from Figure 7, the trends of variation in the first seven-order natural frequencies were similar, all of which experienced increase first, then decrease, and then

Figure 7: Influence imposed by the tooth width on the natural frequency of each order.

Figure 8: Influence imposed by the tooth root transition fillet on the natural frequency of each order.

increase again. Furthermore, the maximum value of the first seven-order natural frequencies appeared when B was approaching 10 mm, and the minimum value of the first seven-order natural frequencies appeared when B was approaching 12 mm. The trend of variation in the eighth-order natural frequency was contrary to that of the seventh-order natural frequency, whereas the sixth-order natural frequency and the seventh-order natural frequency experienced slight changes.

4.3.5 Influence imposed by the tooth root transition fillet on the modal characteristics

The discrete value of the tooth root transition fillet R amounted to 0, 2, 4, 6, and 8 mm. The relationship between the values of R and the natural frequency of each order is illustrated in Figure 8. Judging from Figure 8, the trends of variation in the natural frequencies of each order were similar, all of which experienced slow increase first, then slight decrease, and then slight increase again. In addition, the natural frequencies of each order experienced slight fluctuations, reaching the peak value when the value of R was approaching 2 mm.

Figure 9: Curve of ranges.

Based on the trends of how the aforementioned five parameters imposed an impact on the natural frequency of each order of flexspline, we may conclude that the mechanisms of how the values of L and R could affect the natural frequency of each order were similar, but the range of variation in the natural frequency of each order was subject to significant impact of the value of L , and slightly influenced by the value of R . In addition, the influencing mechanisms of the values of δ and B on the natural frequency of each order were also similar. For instance, the trends of variation in the first seven-order natural frequencies were similar, whereas the trends of variation in the eighth-order natural frequency were contrary to those of the first seven-order natural frequency. In addition, the influence imposed by the value of δ_1 on the natural frequency of each order showed greater variations.

5 Correlation analysis

The value of each structural parameter could impose an impact on the natural frequency of flexspline. To study the influence imposed by each structural parameter on the natural frequency of flexspline, the authors have

Table 3: Ranges of the first eight-order natural frequencies at varying levels of structural parameters

Parameter	First order range	Second order range	Third order range	Fourth order range	Fifth order range	Sixth order range	Seventh order range	Eighth order range
L	469	469	419	427	593	691	612	726
δ	160	162	282	257	332	136	329	172
δ_1	198	197	138	106	417	319	122	219
B	148	149	220	203	353	175	226	88
R	84	85	95	86	158	111	135	200

Table 4: F value obtained through variance analysis

F value	First order	Second order	Third order	Fourth order	Fifth order	Sixth order	Seventh order	Eighth order
L	6.12	6.13	4.74	4.86	9.99	12.2	9.70	12.4
δ	0.66	0.67	1.80	1.46	2.90	0.47	2.28	0.75
δ_1	1.13	1.12	0.56	0.36	4.78	3.23	0.38	1.09
B	0.61	0.62	1.10	0.93	2.89	0.71	1.20	0.19
R	0.26	0.27	0.33	0.32	0.84	0.37	0.62	0.99

conducted the correlation analysis, which is oftentimes related to the methods of range analysis and variance analysis. To facilitate the calculation, the effects imposed by the interaction between structural parameters have been excluded. The authors have calculated the mean value of the natural frequency of each order at varying levels of structural parameters by using the data of Table 2. The ranges of the first eight-order natural frequencies at varying levels of structural parameters are specified in Table 3.

The curve of ranges illustrated based on Table 3 is shown in Figure 9. Judging from Figure 9, at the natural frequencies of all orders, the range of the value of L was the maximum, whereas the range of the value of R was the minimum, and the range of the values of δ_1 , δ , and B intertwined with each other. The range in the order from large to small was L , δ_1 , δ , B , and R in the scope of the first order and the second order; L , δ , B , δ_1 , and R in the scope of the third order and the fourth order; and L , δ_1 , B , δ , and R in the scope of the fifth order and sixth order.

The variance analysis was carried out by using the data listed in Table 2, and the obtained value of F was specified as shown in Table 4. The curve of F illustrated according to Table 4 is shown in Figure 10. The value of F in the order from large to small was L , δ_1 , δ , B , and R in the scope of the first order, the second order, and the fifth order; L , δ , δ_1 , B , and R in the scope of the third order and the fourth order; the value of F was the maximum in the scope of the sixth order, the seventh order, and the eighth

order, and showed no evident trend in the scope of the remaining parameters. For the natural frequencies of all orders, in case $\alpha = 0.10$ was set, we could obtain $F_\alpha(4, 4) = 4.11$ by looking up the table, and it could be noted that the F value of L obtained through variance analysis exceeded 4.11. Therefore, the value of L could impose a significant impact on the natural frequency of flexspline. The F value of the other parameters was less than 4.11, which could impose no significant impact on the natural frequency of flexspline. The F value of each parameter could reveal the influencing degree on the natural frequency of flexspline.

6 Conclusion

By integrating the method of orthogonal design and the method of finite element, the mechanism of how varying structural parameters could impose an impact on the natural frequencies of each order of flexspline, was analyzed. The authors performed the correlation analysis and came to the following conclusions:

- (1) The modal characteristics of the flexspline show regular vibration deformation in the radial direction and are evenly distributed in the circumferential direction, mostly in the sixth natural frequency. In the axial direction, the vibration deformation shows the inclination of the cup bottom plane, which mostly occurs above the sixth natural frequency. The damage of the flexspline mostly occurs at the maximum deformation in the radial direction of the ring gear and the axial direction of the cup body.
- (2) The value of L and the natural frequency of all orders experienced monotonical declines, whereas the value of L could impose a significant impact on the natural frequency of all orders.
- (3) The values of δ_1 , δ , B , and R could impose no significant impact on the natural frequency of all orders, whereas the trends of variation revealed a certain degree of randomness.

Figure 10: Curve of F value through variance analysis.

- (4) The first-order natural frequency of flexspline was rather low. During the design of varying parameters, we need to calculate the first-order natural frequency so as to prevent the natural frequency from approaching the working frequency as much as possible and to avoid resonance.

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