Research Article

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Abundant stable novel solutions of fractional-order epidemic model along with saturated treatment and disease transmission

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Abstract: This article proposes and analyzes a fractional-order susceptible, infectious, susceptible (SIS) epidemic model with saturated treatment and disease transmission by employing four recent analytical techniques along with a novel fractional operator. This model is computationally handled by extended simplest equation method, sech–tanh expansion method, modified Khater method, and modified Kudryashov method. The results' stable characterization is investigated through the Hamiltonian system's properties. The analytical solutions are demonstrated through several numerical simulations.

Keywords: SIS fractional model, computational simulations, stable property

1 Introduction

Epidemiology is recently considered one of the most interesting factors that have attracted the whole world's attention because it evaluates the diseases in populations and causes health outcomes [1,2]. The epidemiology name consists of three Greek parts (epi, demos, and logos) that refer, respectively, to on or upon, people, and studying of health and disease conditions [3,4]. Consequently,

this branch of science is defined by a systematic studying of data driven for determinants (causes, risk factors) of health-related states, the distribution (frequency, pattern), and events (not just diseases) in specified populations (global, country, state, school, neighborhood) [5,6]. It usually refers to investigating what befalls a population [7]. Collection, analysis, and interpretation of systematic data unbiased approach to the collection are the main factors of this science [8]. Additionally, careful focus and use of valid comparison groups evaluate the accuracy of observing the number of cases of the disease in a particular area during a specific period [9]. These factors shows that the frequency of exposure among persons with disease differs from what might be expected [10].

Therefore, epidemiological science depends on biostatistics and informatics, with biological, behavioral sciences, social, and economics that make the standard definition of this branch of science the basic science of public health [11,12]. This quantitative discipline science depends on sound research methods, statistics, and working knowledge of probability [13]. Developing and testing new hypotheses grounded in such scientific fields as physics, behavioral sciences, biology, and ergonomics to explain health-related behaviors, states, and events [14]. Thus, It is an integral component of public health where it provides the general ideas and foundation for directing practical and appropriate public health action [15].

In this context, many epidemic models are mathematically formulated in fractional forms such as Covide-19, SIR model, HIV model, etc. [16–18]. This model is given by refs [19,20]

$$\begin{cases} \mathcal{D}_t^q S = -\frac{r_1}{r_2} SI, \\ \mathcal{D}_t^q I = \frac{r_1}{r_2} SI - r_3 I, \\ \mathcal{D}_t^q R = I, \end{cases} \quad (1)$$

where $0 < q < 1$, S , I , R represent, respectively, the stock of susceptible population, the stock of infected, and the stock of removed population while r_1 , r_2 , r_3 refer to the

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average number of contacts per person per time, the total population, and the transition rate. System (1) is given by William Ogilvy Kermack and Anderson Gray McKendrick [21,22]. Additionally, this system is considered as a special case of Kermack–McKendrick theory, which is interested in prediction of the total number infected, the duration of an epidemic, and the disease spreads to estimate various epidemiological parameters such as the reproductive number [23,24]. Thus, system (1) describes how different public health interventions may affect the outcome of the epidemic [25]. System (1) is also considered as a general model in epidemic science such as SIS model, SIRD model, MSIR model, EIR model, SEIS model, MSEIR model, and MSEIRS model [25–27].

This article focuses on studying the analytical solutions of the fractional SIS model, which is given by Kermack and McKendrick in 1927 in the following form:

$$\begin{cases} \mathcal{D}_t^\varrho S = r_1 - r_2 SI + \left(r_3 + \frac{r_4 r_5}{1 + r_5 r_6 I}\right) I - r_7 S, \\ \mathcal{D}_t^\varrho I = r_2 SI - \left(r_3 + \frac{r_4 r_5}{1 + r_5 r_6 I}\right) I - (r_7 + r_8) I, \end{cases} \quad (2)$$

where $S(t_0) = S_0 \geq 0$, $I(t_0) = I_0 \geq 0$, while S, I represent, respectively, infected and population. Additionally, r_k , ($k = 1, 2, \dots, 7, 8$) represent, respectively, the newly recruited population, disease transmission rate, the per capita recovery rate, the parameter measuring the effectiveness of the treatment, the treatment control for the infected populations, the side effect or overdose of the treatment control, and the natural death rate of both susceptible and infected population. Applying the Atangana–Baleanu fractional operator [28,29] with the next formula $[\mathfrak{P}] = \frac{\lambda(\varrho-1)t^{-2\varrho}}{\mathcal{B}(\varrho)\sum_{\eta=0}^m\left(\frac{\varrho}{1-\varrho}\right)^\eta \Gamma(1-\eta\varrho)}$ where λ, η are arbitrary constants, while $\mathcal{B}$ is a normalized function] to system (2) converts it into the following system:

$$\begin{cases} \lambda S = r_1 - r_2 SI + \left(r_3 + \frac{r_4 r_5}{1 + r_5 r_6 I}\right) I - r_7 S, \\ \lambda I = r_2 SI - \left(r_3 + \frac{r_4 r_5}{1 + r_5 r_6 I}\right) I - (r_7 + r_8) I. \end{cases} \quad (3)$$

Balancing the terms of the previous system with the auxiliary equations of the suggested analytical schemes leads to format the model's general solutions in the following form:

$$\begin{cases} \mathfrak{S}(\mathfrak{P}) = \sum_{i=-n}^n a_i f(\mathfrak{P})^i = \frac{a_{-1}}{f(\xi)} + a_1 f(\xi) + a_0, \\ \mathfrak{I}(\mathfrak{P}) = \sum_{i=-l}^l b_i f(\mathfrak{P})^i = \frac{b_{-1}}{f(\xi)} + b_1 f(\xi) + b_0, \\ \mathfrak{S}(\mathfrak{P}) = \sum_{i=1}^n \operatorname{sech}^{i-1}(\mathfrak{P})(a_i \operatorname{sech}(\mathfrak{P}) + b_i \tanh(\mathfrak{P})) \\ \quad + a_0 = a_1 \operatorname{sech}(\mathfrak{P}) + a_0 + b_1 \tanh(\mathfrak{P}), \\ \mathfrak{I}(\mathfrak{P}) = \sum_{i=1}^l \operatorname{sech}^{i-1}(\mathfrak{P})(p_i \operatorname{sech}(\mathfrak{P}) + q_i \tanh(\mathfrak{P})) \\ \quad + p_0 = p_1 \operatorname{sech}(\mathfrak{P}) + p_0 + q_1 \tanh(\mathfrak{P}). \end{cases} \quad (4)$$

$$\begin{cases} \mathfrak{S}(\mathfrak{P}) = \sum_{i=-n}^n a_i \mathcal{M}^{if(\mathfrak{P})} = a_{-1} \mathcal{M}^{-f(\mathfrak{P})} + a_1 \mathcal{M}^{f(\mathfrak{P})} + a_0, \\ \mathfrak{I}(\mathfrak{P}) = \sum_{i=-l}^l b_i \mathcal{M}^{if(\mathfrak{P})} = b_{-1} \mathcal{M}^{-f(\mathfrak{P})} + b_1 \mathcal{M}^{f(\mathfrak{P})} + b_0. \end{cases}$$

$$\begin{cases} \mathfrak{S}(\mathfrak{P}) = \sum_{i=0}^n a_i Q(\mathfrak{P})^i = a_1 Q(\mathfrak{P}) + a_0, \\ \mathfrak{I}(\mathfrak{P}) = \sum_{i=0}^n b_i Q(\mathfrak{P})^i = b_1 Q(\mathfrak{P}) + b_0, \end{cases}$$

where $a_{-1}, b_{-1}, a_0, b_0, a_1, b_1$ are real arbitrary constants, while the auxiliary equations of the above-mentioned analytical schemes are given in the following order: $[f'(\mathfrak{P}) = \beta_1 + \beta_3 f(\mathfrak{P})^2 + \beta_2 f^2(\mathfrak{P}), f'(\mathfrak{P}) = \frac{1}{\ln(\mathcal{M})} (d_2 + d_1 \mathcal{M}^{-f(\mathfrak{P})} + d_3 \mathcal{M}^{f(\mathfrak{P})}), Q'(\mathfrak{P}) = \ln(e)(Q(\mathfrak{P})^2 - Q(\mathfrak{P}))$ where $\beta_j, j, (j = 1, 2, 3)$ are real constants].

The rest of the article is organized as follows: Section 2 investigates the computational solutions of the fractional nonlinear SIS model by handling the converted ordinary differential system through four analytical schemes [30–43]. Section 4 shows the stability characterization of the constructed solutions through the Hamiltonian system's properties. Section 5 gives the epilogue of the whole article.

2 Computational versus numerical solutions

Employing the general steps of the suggested analytical schemes on the converted nonlinear ordinary differential system evaluates the above-mentioned arbitrary constants as follows.

2.1 Extended simplest equation (ESE) method's solitary wave solutions

Calculating the value of arbitrary constant through ESE method gives:

Group I

$$\begin{aligned}\lambda &= 1, \quad r_1 = r_7(a_0 + b_0), \quad r_2 = \frac{\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)} + b_0\beta_2}{2b_0^2}, \\ r_3 &= \frac{1}{2} \left(\frac{a_0\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)}}{b_0^2} + \frac{a_0\beta_2}{b_0} - \frac{\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)}}{b_0} + \beta_2 - 2r_7 \right), \\ r_4 = r_8 = a_1 = b_1 &= 0, \quad a_{-1} = \frac{1}{2} \left(\frac{\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)}}{\beta_3} - \frac{b_0\beta_2}{\beta_3} \right), \\ b_{-1} &= \frac{b_0\beta_2 - \sqrt{b_0^2\beta_2^2 - 4b_0^2\beta_1\beta_3}}{2\beta_3}.\end{aligned}$$

Group II

$$\begin{aligned}r_1 &= r_7(a_0 + b_0), \quad r_2 = \frac{2\beta_1\beta_3}{\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)} - b_0\beta_2}, \\ r_3 &= -\frac{1}{2b_0^2} (b_0(a_0\beta_2 - \sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)}) + a_0\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)} + b_0^2(\beta_2 + 2r_7)), \\ r_4 = r_8 = b_{-1} = a_{-1} &= 0, \quad \lambda = 1, \quad a_1 = \frac{1}{2} \left(\frac{\sqrt{b_0^2(\beta_2^2 - 4\beta_1\beta_3)}}{\beta_1} - \frac{b_0\beta_2}{\beta_1} \right), \\ b_1 &= \frac{b_0\beta_2 - \sqrt{b_0^2\beta_2^2 - 4b_0^2\beta_1\beta_3}}{2\beta_1}.\end{aligned}$$

Consequently, the solitary wave solutions of the investigated system are represented by

For $\beta_2 = 0, \beta_1\beta_3 < 0$, we get

$$\begin{aligned}S_{I,1}(t) &= S_{II,1}(t) - b_0 \left(\cosh(2\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2}) \operatorname{csch} \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \operatorname{sech} \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \right) \\ &= a_0 - b_0 \coth \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right),\end{aligned}\tag{5}$$

$$\begin{aligned}S_{I,2}(t) &= S_{II,2}(t) - b_0 \left(\coth \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) + \tanh \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \right) \\ &= a_0 - b_0 \tanh \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right),\end{aligned}\tag{6}$$

$$\begin{aligned}I_{I,1}(t) &= I_{II,1}(t) + b_0 \left(\cosh(2\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2}) \operatorname{csch} \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \operatorname{sech} \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \right) \\ &= b_0 \left(1 + \coth \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \right),\end{aligned}\tag{7}$$

$$\begin{aligned}I_{I,2}(t) &= I_{II,2}(t) + b_0 \left(\coth \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) + \tanh \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \right) \\ &= b_0 \left(1 + \tanh \left(\sqrt{-\beta_1\beta_3}\xi \mp \frac{\log(\Xi)}{2} \right) \right).\end{aligned}\tag{8}$$

For $\beta_1 = 0$, $\beta_2 < 0$, we get

$$S_{1,3}(t) = \frac{1}{2\beta_3^2} (2a_0\beta_3^2 - \sqrt{b_0^2\beta_2^2} e^{\beta_2(-(\xi+\Xi))} + b_0\beta_2(e^{\beta_2(-(\xi+\Xi))} + \beta_3) - \beta_3\sqrt{b_0^2\beta_2^2}), \quad (9)$$

$$\begin{aligned} I_{1,3}(t) = & -\frac{\sqrt{b_0^2\beta_2^2} \sinh(\beta_2\xi + \beta_2\Xi)}{2\beta_3^2} + \frac{b_0\beta_2 \sinh(\beta_2\xi + \beta_2\Xi)}{2\beta_3^2} + \frac{\sqrt{b_0^2\beta_2^2} \cosh(\beta_2\xi + \beta_2\Xi)}{2\beta_3^2} \\ & - \frac{b_0\beta_2 \cosh(\beta_2\xi + \beta_2\Xi)}{2\beta_3^2} + \frac{\sqrt{b_0^2\beta_2^2}}{2\beta_3} - \frac{b_0\beta_2}{2\beta_3} + b_0, \end{aligned} \quad (10)$$

$$\text{where } \xi = \frac{(\varrho-1)t^{-2\varrho}}{\mathcal{B}(\varrho) \sum_{\eta=0}^m \left(-\frac{\varrho}{1-\varrho}\right)^\eta \Gamma(1-\eta\varrho)}.$$

2.2 Sech-tanh expansion (STE) method's explicit wave solutions

Calculating the value of arbitrary constant through the sech-tanh method gives:

$$\begin{aligned} p_0 &= q_1, \quad r_1 = r_7(a_0 + q_1), \quad r_2 = \frac{1}{q_1}, \quad r_3 = \frac{a_0 - q_1 r_7 - q_1}{q_1}, \\ \lambda &= 1, \quad r_4 = r_8 = a_1 = p_1 = 0, \quad b_1 = -q_1. \end{aligned}$$

Consequently, the solitary wave solutions of the investigated system are represented by

$$S(t) = \operatorname{sech}(\xi)(a_0 \cosh(\xi) - q_1 \sinh(\xi)), \quad (11)$$

$$I(t) = -\frac{e^{-\xi} q_1}{e^{-\xi} + e^{\xi}} + \frac{e^{\xi} q_1}{e^{-\xi} + e^{\xi}} + q_1, \quad (12)$$

$$\text{where } \xi = \frac{(\varrho-1)t^{-2\varrho}}{\mathcal{B}(\varrho) \sum_{\eta=0}^m \left(-\frac{\varrho}{1-\varrho}\right)^\eta \Gamma(1-\eta\varrho)}.$$

2.3 MKhat method's soliton wave solutions

Calculating the value of arbitrary constant through MKhat method gives:

Group I

$$\begin{aligned} r_1 &= r_7(a_0 + b_0), \quad r_2 = \frac{\sqrt{b_0^2(d_2^2 - 4d_1d_3)} + b_0d_2}{2b_0^2}, \\ r_3 &= \frac{1}{2b_0^2} (a_0(\sqrt{b_0^2(d_2^2 - 4d_1d_3)} + b_0d_2) + b_0(b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)})) - r_7, \\ r_4 &= 0, \quad \lambda = 1, \quad r_8 = 0, \quad a_{-1} = \frac{\sqrt{b_0^2(d_2^2 - 4d_1d_3)} - b_0d_2}{2d_3}, \quad a_1 = 0, \\ b_{-1} &= \frac{b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)}}{2d_3}, \quad b_1 = 0. \end{aligned} \quad (13)$$

Group II

$$\begin{aligned}
 r_1 &= r_7(a_0 + b_0), \quad r_2 = \frac{2d_1d_3}{\sqrt{b_0^2(d_2^2 - 4d_1d_3)} - b_0d_2}, \\
 r_3 &= -\frac{1}{2b_0^2}(b_0(a_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)}) + a_0\sqrt{b_0^2(d_2^2 - 4d_1d_3)} \\
 &\quad + b_0^2(d_2 + 2r_7)), \\
 r_4 &= 0, \quad r_8 = 0, \quad a_{-1} = 0, \quad \lambda = 1, \quad a_1 = \frac{\sqrt{b_0^2(d_2^2 - 4d_1d_3)} - b_0d_2}{2d_1}, \quad b_{-1} = 0, \quad b_1 = \frac{b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)}}{2d_1}.
 \end{aligned} \tag{14}$$

Consequently, the solitary wave solutions of the investigated system are represented by

For $d_2^2 - 4d_1d_3 > 0$, $d_3 \neq 0$, we get

$$S_{I,1}(t) = a_0 + \frac{b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)}}{\sqrt{d_2^2 - 4d_1d_3} \tanh\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2}, \tag{15}$$

$$S_{I,2}(t) = a_0 + \frac{b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)}}{\sqrt{d_2^2 - 4d_1d_3} \coth\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2}, \tag{16}$$

$$S_{II,1}(t) = a_0 + \frac{(b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)})}{4d_1d_3} \times \left(\sqrt{d_2^2 - 4d_1d_3} \tanh\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2 \right), \tag{17}$$

$$S_{II,2}(t) = a_0 + \frac{(b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)})}{4d_1d_3} \times \left(\sqrt{d_2^2 - 4d_1d_3} \coth\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2 \right), \tag{18}$$

$$I_{I,1}(t) = \frac{b_0\sqrt{d_2^2 - 4d_1d_3} \tanh\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + \sqrt{b_0^2(d_2^2 - 4d_1d_3)}}{\sqrt{d_2^2 - 4d_1d_3} \tanh\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2}, \tag{19}$$

$$I_{I,2}(t) = \frac{b_0\sqrt{d_2^2 - 4d_1d_3} \coth\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + \sqrt{b_0^2(d_2^2 - 4d_1d_3)}}{\sqrt{d_2^2 - 4d_1d_3} \coth\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2}, \tag{20}$$

$$I_{II,1}(t) = \frac{(\sqrt{b_0^2(d_2^2 - 4d_1d_3)} - b_0d_2)}{4d_1d_3} \times \left(\sqrt{d_2^2 - 4d_1d_3} \tanh\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2 \right) + b_0, \tag{21}$$

$$I_{II,2}(t) = b_0 - \frac{(b_0d_2 - \sqrt{b_0^2(d_2^2 - 4d_1d_3)})}{4d_1d_3} \times \left(\sqrt{d_2^2 - 4d_1d_3} \coth\left(\frac{1}{2}\sqrt{d_2^2 - 4d_1d_3}\xi\right) + d_2 \right). \tag{22}$$

For $d_1d_3 < 0$, $d_1 \neq 0$, $d_3 \neq 0$, $d_2 = 0$, we get

$$S_{I,3}(t) = a_0 - \frac{\sqrt{-b_0^2d_1d_3}}{\sqrt{-d_1d_3}} \coth(\sqrt{-d_1d_3}\xi), \tag{23}$$

$$S_{I,4}(t) = a_0 - \frac{\sqrt{-b_0^2d_1d_3}}{\sqrt{-d_1d_3}} \tanh(\sqrt{-d_1d_3}\xi), \tag{24}$$

$$S_{II,3}(t) = \frac{1}{\sqrt{-d_1d_3}} (\operatorname{sech}(\sqrt{-d_1d_3}\xi)(a_0\sqrt{-d_1d_3} \times \cosh(\sqrt{-d_1d_3}\xi) + \sqrt{-b_0^2d_1d_3} \sinh(\sqrt{-d_1d_3}\xi))), \tag{25}$$

$$S_{II,4}(t) = a_0 + \frac{\sqrt{-b_0^2 d_1 d_3} (e^{\sqrt{-d_1 d_3}(-\xi)} + e^{\sqrt{-d_1 d_3} \xi})}{\sqrt{-d_1 d_3} (e^{\sqrt{-d_1 d_3} \xi} - e^{\sqrt{-d_1 d_3}(-\xi)})}, \quad (26)$$

$$\begin{aligned} I_{I,3}(t) = & \operatorname{csch}(\sqrt{-d_1 d_3} \xi) \left(\frac{1}{2} a_0 \sinh(\sqrt{-d_1 d_3} \xi) + \frac{1}{2} a_0 \cosh(\sqrt{-d_1 d_3} \xi) \right) \\ & + \operatorname{csch}(\sqrt{-d_1 d_3} \xi) \left(\frac{1}{2} a_0 \sinh(\sqrt{-d_1 d_3} \xi) - \frac{1}{2} a_0 \cosh(\sqrt{-d_1 d_3} \xi) \right) + \operatorname{csch}(\sqrt{-d_1 d_3} \xi) \\ & \times \left(\frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \sinh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} + \frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \cosh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} \right) \\ & + \operatorname{csch}(\sqrt{-d_1 d_3} \xi) \left(\frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \cosh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} - \frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \sinh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} \right), \end{aligned} \quad (27)$$

$$\begin{aligned} I_{I,4}(t) = & \operatorname{sech}(\sqrt{-d_1 d_3} \xi) \left(\frac{1}{2} a_0 \sinh(\sqrt{-d_1 d_3} \xi) + \frac{1}{2} a_0 \cosh(\sqrt{-d_1 d_3} \xi) \right) + \operatorname{sech}(\sqrt{-d_1 d_3} \xi) \\ & \times \left(\frac{1}{2} a_0 \cosh(\sqrt{-d_1 d_3} \xi) - \frac{1}{2} a_0 \sinh(\sqrt{-d_1 d_3} \xi) \right) + \operatorname{sech}(\sqrt{-d_1 d_3} \xi) \\ & \times \left(\frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \sinh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} + \frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \cosh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} \right) \\ & + \operatorname{sech}(\sqrt{-d_1 d_3} \xi) \left(\frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \sinh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} - \frac{\sqrt{-d_1 d_3} \sqrt{-b_0^2 d_1 d_3} \cosh(\sqrt{-d_1 d_3} \xi)}{2 d_1 d_3} \right) \end{aligned} \quad (28)$$

$$I_{II,3}(t) = b_0 - \frac{\sqrt{-b_0^2 d_1 d_3} (e^{\sqrt{-d_1 d_3} \xi} - e^{\sqrt{-d_1 d_3}(-\xi)})}{\sqrt{-d_1 d_3} (e^{\sqrt{-d_1 d_3}(-\xi)} + e^{\sqrt{-d_1 d_3} \xi})}, \quad (29)$$

$$I_{II,4}(t) = \frac{1}{\sqrt{-d_1 d_3}} (\operatorname{csch}(\sqrt{-d_1 d_3} \xi) (b_0 \sqrt{-d_1 d_3} \times \sinh(\sqrt{-d_1 d_3} \xi) - \sqrt{-b_0^2 d_1 d_3} \cosh(\sqrt{-d_1 d_3} \xi))), \quad (30)$$

$$\text{where } \xi = \frac{(\varrho - 1)t^{-2\varrho}}{\mathcal{B}(\varrho) \sum_{\eta=0}^m \left(\frac{\varrho}{1-\varrho} \right)^\eta \Gamma(1-\eta\varrho)}.$$

2.4 MKud method's soliton wave solutions

Calculating the value of arbitrary constant through MKud method gives:

Group I

$$\begin{aligned} r_1 = a_0 r_7, \quad r_2 = -\frac{1}{b_1}, \quad r_3 = \frac{-a_0 - b_1 r_7 + b_1}{b_1}, \quad \lambda = 1, \quad r_4 = 0, \\ r_8 = 0, \quad a_1 = -b_1, \quad b_0 = 0. \end{aligned}$$

Group II

$$\begin{aligned} r_1 = r_7(a_0 + b_0), \quad r_2 = \frac{1}{b_0}, \quad r_3 = \frac{a_0 - b_0 r_7}{b_0}, \quad r_4 = 0, \quad r_8 = 0, \\ \lambda = 1, \quad a_1 = b_0, \quad b_1 = -b_0. \end{aligned}$$

Consequently, the solitary wave solutions of the investigated system are represented by

$$S_{I,1}(t) = a_0 - \frac{b_1}{\pm \sinh(\xi) \pm \cosh(\xi) + 1}, \quad (31)$$

Figure 1: Analytical simulations in two-dimensional plot of equations (6), (9), and (10), respectively, in right, center, and left for $[a_0 = 8, \beta_1 = -5, \beta_3 = 3, b_0 = 7 \text{ and } a_0 = 7, b_0 = 8, \beta_2 = -5, \beta_3 = 9, \Xi = 10 \text{ and } a_0 = 0.2, b_0 = 0.3, \beta_2 = -0.4, \beta_3 = 0.5, \Xi = 0.1]$.

$$S_{II,1}(t) = a_0 + \frac{b_0}{1 \pm e^\xi}. \quad (32)$$

$$I_{1,1}(t) = \operatorname{sech}\left(\frac{\xi}{2}\right) \left(\pm \frac{1}{2} b_1 \cosh\left(\frac{\xi}{2}\right) \mp \frac{1}{2} b_1 \sinh\left(\frac{\xi}{2}\right) \right), \quad (33)$$

$$I_{II,1}(t) = b_0 - \frac{b_0}{\pm \sinh(\xi) \pm \cosh(\xi) + 1}, \quad (34)$$

where $\xi = \frac{(\varrho - 1)t^{-2\varrho}}{\mathcal{B}(\varrho) \sum_{\eta=0}^m \left(\frac{\varrho}{1-\varrho} \right)^\eta \Gamma(1-\eta\varrho)}$.

3 Figure interpretation

Here, we show the above-explained sketches as follows:

- Figure 1 shows two-dimensional (2D) plots of equations (6), (9), and (10) when $[a_0 = 8, \beta_1 = -5, \beta_3 = 3, b_0 = 7 \text{ and } a_0 = 7, b_0 = 8, \beta_2 = -5, \beta_3 = 9, \Xi = 10 \text{ and } a_0 = 0.2, b_0 = 0.3, \beta_2 = -0.4, \beta_3 = 0.5, \Xi = 0.1]$.
- Figure 2 demonstrates equations (11) and (12) in 2D plots for $[a_0 = 9, q_1 = 7 \text{ and } q_1 = 7]$.
- Figure 3 demonstrates (15) and (19) in 2D plots when $[a_0 = 7, b_0 = 6, d_2 = 5, d_1 = 3, d_3 = 2 \text{ and } b_0 = 7, d_2 = 8, d_3 = 5, d_1 = 2]$.
- Figure 4 shows 2D plots of equations (31) and (33) for $[a_0 = 5, b_1 = 9 \text{ and } b_1 = 9]$.

4 Investigation of solutions' stable

Investigating the solutions' stability by implementing the Hamiltonian system's properties gets the momentum of equations (6), (9), (11), and (31) in the following forms, respectively:

$$\mathcal{G}_{S_{1,2}(t)} = \frac{49}{\lambda^2} (\tanh^{-1}(\tanh(5\lambda^2)) - \tanh(5\lambda^2)) + 320, \quad (35)$$

$$\mathcal{G}_{S_{1,3}(t)} = \frac{1}{6561\lambda} (214,245\lambda - 3,312e^{50} \sinh(25\lambda) + 160e^{100} \sinh(50\lambda)), \quad (36)$$

$$\mathcal{G}_{S(t)} = \frac{1}{\lambda} (405\lambda + 49 \tanh^{-1}(\tanh(5\lambda)) - 49 \tanh(5\lambda)), \quad (37)$$

$$\mathcal{G}_{S_{1,1}(t)} = \frac{1}{\lambda} \left(290 \tanh^{-1} \left(\tanh \left(\frac{5\lambda}{2} \right) \right) + \frac{48 \tanh \left(\frac{5\lambda}{2} \right)}{\tanh^2 \left(\frac{5\lambda}{2} \right) - 25} - 48 \coth^{-1} \left(5 \coth \left(\frac{5\lambda}{2} \right) \right) \right). \quad (38)$$

Thus, equations (6), (9), (10), and (11) are stable when $\lambda = 0$, where $\left[\frac{\partial \mathcal{M}}{\partial \lambda} \right]_{\lambda=1} \geq 0$, for the above-tested equations].

Handling the other solutions by same steps, studies the stability property of all other results.

Figure 2: Analytical simulations in 2D plot of equations (11), (12), respectively, in right and left for $[a_0 = 9, q_1 = 7 \text{ and } q_1 = 7]$.

Figure 3: Analytical simulations in 2D plot of equations (15) and (19), respectively, in right and left for $[a_0 = 7, b_0 = 6, d_2 = 5, d_1 = 3, d_3 = 2]$ and $b_0 = 7, d_2 = 8, d_3 = 5, d_1 = 2]$.

Figure 4: Analytical simulations in 2D plot of equations (31) and (33), respectively, in right and left for $[a_0 = 5, b_1 = 9]$ and $b_1 = 9]$.

5 Conclusion

This manuscript analyzes the analytical solutions of the fractional SIS epidemic biological model by four recent computational schemes. The Atangana–Baleanu fractional operator is used to convert the system's fractional form into the ordinary system with an integer order. The analytical study was explained by the ESE, STE, MKhat, and MKud methods. Additionally, the solutions' stability is checked and demonstrated in some distinct plots.

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