Research Article

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Exact wave solutions of the nonlinear Rosenau equation using an analytical method

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Abstract: The purpose of the current study is to find exact travelling wave solutions of the Rosenau equation. By the use of the extended auxiliary equation method, various exact solutions are obtained in terms of Jacobi elliptic functions and exponential functions. Moreover, several solitary and periodic wave solutions are given as special cases. When the parameters take some values, some graphical illustrations are shown in order to understand the behaviour of these new solutions. Furthermore, we compare our solutions with some familiar solutions, which can be considered as special cases.

Keywords: Rosenau equation, exact solution, the extended auxiliary equation method

1 Introduction

The importance of nonlinear partial differential equations (PDEs) has increased in the previous few years. Nonlinear PDEs are widely used for modelling some phenomena in physics, engineering, mechanics, chemistry and biology; see ref. [1]. Thus, the exact solutions of these nonlinear PDEs can be useful in understanding these models. Many different approaches and techniques are used to find the solutions of these problems, such as the Backlund transform, the Hirota bilinear method, the inverse scattering transform, the Adomian decomposition method, the variational method and the homotopy perturbation method (see refs. [2–7]). Additionally, several works and methods have been carried out recently in finding exact solutions of nonlinear PDEs including tanh method [8], the (G'/G)

expansion method [9,10], the Riccati equation expansion method [11] and the (W/G) expansion method [12]. Other methods can be found in refs. [13–17]. The auxiliary equation method is a powerful method, which has been used widely; see refs. [18–20]. The motivation of this article is to find a series of exact solutions of the Rosenau equation using the auxiliary equation method.

The Rosenau equation was proposed to characterize the dynamical behaviour of dense discrete systems [21]. Park studied the existence of the Rosenau equation solution, and an exact solution was given explicitly [22]. Zuo used two different methods to find the solitons and periodic solutions of the Rosenau–KdV Eq. [23]. Based on the classical Lie group method, Gao and Tian studied the similarity reductions and exact solutions of the Rosenau Eq. [24]. Razborova *et al.* investigated the dynamics of perturbed soliton solutions for the Rosenau Eq. [25]. The numerical solutions of the generalized Rosenau equation are obtained by Avazzadeh *et al.* [26]. Also, other different numerical methods were used in order to solve the Rosenau equation; see refs. [27,28]. It can be seen that most of the previous studies used numerical procedures. In this article, we try to solve the Rosenau equation analytically.

This article is organized as follows. In Section 2, the main steps of the extended auxiliary equation method are given. In Section 3, we utilize this method to solve the Rosenau equation. The final section summarizes our key findings.

2 The extended auxiliary equation method

Here we outline the main steps of the extended auxiliary equation method. Let us have the following nonlinear PDE:

$$G(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, \dots) = 0. \quad (1)$$

Step 1. First, we use the transformation

$$u(x, t) = U(\xi), \quad \xi = c(x - kt), \quad (2)$$

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where k, c are undetermined constants. Eq. (2) converts Eq. (1) to the nonlinear ordinary differential equation (ODE) as follows:

$$P(U, U'', U''', \dots) = 0, \quad (3)$$

where P involves U and its derivatives with respect to ξ .

Step 2. The solution of Eq. (3) is assumed to take the form:

$$U(\xi) = \sum_{i=-m}^m a_i F^i(\xi), \quad (4)$$

where m is a positive integer, and a_i are arbitrary constants. The function $F(\xi)$ is assumed to be the solution of the following:

$$F'^2(\xi) = g_0 + g_1 F^2(\xi) + g_2 F^4(\xi), \quad (5)$$

where g_0, g_1, g_2 are arbitrary constants. Here we just mention that F can be assumed to satisfy other ODEs; see refs. [29,30]. The solutions of Eq. (5) depend on the values of g_0, g_1, g_2 , and these solutions are given by the following.

Case 1. If $g_0 = 1, g_1 = -(1 + n^2), g_2 = n^2$, then the solution of Eq. (5) is $F(\xi) = \text{sn}(\xi, n)$, where $\text{sn}(\xi, n)$ is the Jacobi elliptic sine function and n is the elliptic modulus such that $0 < n < 1$. It is known that $\text{sn}(\xi, n) = \sin(\xi)$ when $n \rightarrow 0$ and $\text{sn}(\xi, n) = \tanh(\xi)$ when $n \rightarrow 1$.

Case 2. If $g_0 = 1 - n^2, g_1 = 2n^2 - 1, g_2 = -n^2$, then the solution of Eq. (5) is $F(\xi) = \text{cn}(\xi, n)$, where $\text{cn}(\xi, n)$ is the Jacobi elliptic cosine function and n is the elliptic modulus where $0 < n < 1$. As a special case, $\text{cn}(\xi, n) = \cos(\xi)$ when $n \rightarrow 0$ and $\text{cn}(\xi, n) = \text{sech}(\xi)$ when $n \rightarrow 1$.

Case 3. If $g_0 = n^2 - 1, g_1 = 2 - n^2, g_2 = -1$, then the solution of Eq. (5) is $F(\xi) = \text{dn}(\xi, n)$, where $\text{dn}(\xi, n)$ is the Jacobi elliptic delta amplitude function and n is the elliptic modulus such that $0 < n < 1$. As a special case, $\text{dn}(\xi, n) = 1$ when $n \rightarrow 0$ and $\text{cn}(\xi, n) = \text{sech}(\xi)$ when $n \rightarrow 1$.

Case 4. If $g_0 = 0, g_1 > 0$, then the solution of Eq. (5) is given by

$$F(\xi) = 4 \left(\frac{g_1 e^{2\varepsilon \sqrt{g_1} \xi}}{(e^{2\varepsilon \sqrt{g_1} \xi} - 4g_2)^2} \right)^{\frac{1}{2}}, \quad (6)$$

where $\varepsilon = \pm 1$.

Step 3. By equating the highest-order derivatives and the highest nonlinear terms in Eq. (3), we may determine the value of m in Eq. (4).

Step 4. By substituting Eq. (4) along with Eq. (5) into Eq. (3) and setting the coefficients of $F(\xi)$ and its powers

to zero, we get a system of algebraic equations which can be used to calculate the values of k, c, a_i . Consequently, the solution of Eq. (1) is derived.

3 Exact solutions to the Rosenau equation

In this section, we utilize the extended auxiliary equation method to find exact solutions of the Rosenau equation. Consider the Rosenau equation in the form [22]

$$u_t + u_x + u_{xxxx} - 30u^2u_x + 60u^4u_x = 0. \quad (7)$$

Another form of the Rosenau equation can be found in ref. [26]. By the use of the travelling wave transformation,

$$u(x, t) = U(\xi), \quad \xi = c(x - kt), \quad (8)$$

where c, k are nonzero constants, equation (7) becomes

$$c(1 - k)U' - c^5kU^{(5)} - 30cU^2U' + 60cU^4U' = 0. \quad (9)$$

By balancing the highest order derivative $U^{(5)}$ and the highest nonlinear term U^4U' in equation (9), we find $m + 5 = 4m + m + 1$, which gives $m = 1$. Therefore, the solution of Eq. (9) takes the form:

$$U(\xi) = a_0 + a_1 F(\xi) + \frac{a_{-1}}{F(\xi)}. \quad (10)$$

As discussed in the previous section, we have the following cases:

Case 1. $g_0 = 1, g_1 = -(1 + n^2), g_2 = n^2$. Substituting (10) with (5) into (9) and setting the coefficients of like powers of $F(\xi)$ to be zero, we get a system of algebraic equations, which is not written here for the sake of brevity. Solving this system with the help of Mathematica, we obtain the following families:

Family 1.

$$\begin{aligned} a_0 &= 0, \quad a_1 = 0, \quad a_{-1} = \pm \frac{1}{\sqrt{n^2 + 1}}, \\ k &= \frac{n^4 - 10n^2 + 1}{2(n^2 + 1)^2}, \quad c = -\frac{1}{\sqrt[4]{n^4 - 10n^2 + 1}}. \end{aligned} \quad (11)$$

Inserting these values into Eq. (10), we get the exact solution of Eq. (7)

$$u(x, t) = \pm \frac{1}{\sqrt{n^2 + 1} \text{sn}(\xi, n)}. \quad (12)$$

We remark that when $n \rightarrow 0$, then the exact solution of Eq. (7) reduces to

$$u(x, t) = \pm \csc\left(\frac{1}{2}(t - 2x)\right). \quad (13)$$

Figure 1 represents the behaviour of the periodic wave solution of the Rosenau equation when $n = 0$ and within the interval $4 \leq x \leq 4$, $-3 \leq t \leq 3$.

Family 2.

$$\begin{aligned} a_0 = 0, \quad a_1 = \pm \frac{n}{\sqrt{n^2 + 1}}, \quad a_{-1} = 0, \\ k = \frac{n^4 - 10n^2 + 1}{2(n^2 + 1)^2}, \quad c = -\frac{1}{\sqrt[4]{n^4 - 10n^2 + 1}}. \end{aligned} \quad (14)$$

Thus, the exact solution of Eq. (7) is given by

$$u(x, t) = \pm \frac{n \operatorname{sn}(\xi, n)}{\sqrt{n^2 + 1}}. \quad (15)$$

The Jacobi elliptic periodic solution (15) is plotted when $n = 0.2$, $4 \leq x \leq 4$ and $-3 \leq t \leq 3$ in Figure 2a and $t = 0$ in Figure 2b.

Family 3.

$$\begin{aligned} a_0 = 0, \quad a_1 = 0, \quad a_{-1} = \pm \frac{1}{\sqrt{n^2 + 1}}, \\ k = \frac{n^4 - 10n^2 + 1}{2(n^2 + 1)^2}, \quad c = \frac{1}{\sqrt[4]{n^4 - 10n^2 + 1}}. \end{aligned} \quad (16)$$

In this case, the exact solution of Eq. (7) is given by

$$u(x, t) = \pm \frac{1}{\sqrt{n^2 + 1} \operatorname{sn}(\xi, n)}. \quad (17)$$

We mention that the solutions (17) and (12) are different since the values of k, c are different in these two cases.

Family 4.

$$\begin{aligned} a_0 = 0, \quad a_1 = \pm \frac{n}{\sqrt{n^2 + 1}}, \quad a_{-1} = 0, \\ k = \frac{n^4 - 10n^2 + 1}{2(n^2 + 1)^2}, \quad c = -\frac{1}{\sqrt[4]{n^4 - 10n^2 + 1}}. \end{aligned} \quad (18)$$

Therefore, the exact solution of Eq. (7) leads to

$$u(x, t) = \pm \frac{n \operatorname{sn}(\xi, n)}{\sqrt{n^2 + 1}}. \quad (19)$$

Family 5.

$$\begin{aligned} a_0 = 0, \quad a_1 = \pm f_1, \quad a_{-1} = \pm \frac{f_1}{n}, \\ k = \frac{n^4 - 36n^3 - 58n^2 - 36n + 1}{2(n^2 + 6n + 1)^2}, \\ c = -\frac{1}{\sqrt[4]{n^4 - 36n^3 - 58n^2 - 36n + 1}}, \end{aligned} \quad (20)$$

where

$$f_1 = \frac{\sqrt{n^2(n^4 - 6n^3 + 2n^2 - 6n + 1)}}{\sqrt{n^6 - 33n^4 - 33n^2 + 1}}.$$

Therefore, the solution of Eq. (7) is given by

$$u(x, t) = \pm \frac{f_1(n \operatorname{sn}(\xi, n)^2 + 1)}{n \operatorname{sn}(\xi, n)}. \quad (21)$$

Figure 3 depicts the Jacobi elliptic periodic solution (21) when $t = 0$, $n = 0.02$, $-10 \leq x \leq 10$.

Figure 1: The 3D (a) and the contour plots (b) of the positive solution of (13) when $n = 0$.

Figure 2: (a) The 3D plot of the positive solution of (15) and (b) the positive solution of (15) when $t = 0$ and $n = 0.2$.

Family 6.

$$\begin{aligned} a_0 &= 0, \quad a_1 = \pm f_1, \quad a_{-1} = \pm \frac{f_1}{n}, \\ k &= \frac{n^4 - 36n^3 - 58n^2 - 36n + 1}{2(n^2 + 6n + 1)^2}, \\ c &= \frac{1}{\sqrt[4]{n^4 - 36n^3 - 58n^2 - 36n + 1}}. \end{aligned} \quad (22)$$

In this case, we have

$$u(x, t) = \pm \frac{f_1(n \operatorname{sn}(\xi, n)^2 + 1)}{n \operatorname{sn}(\xi, n)}. \quad (23)$$

Case 2. Now, we consider case 2, i.e. when $g_0 = 1 - n^2$, $g_1 = 2n^2 - 1$, $g_2 = -n^2$. In this case, we obtain the following families:

Family 7.

$$\begin{aligned} a_0 &= 0, \quad a_1 = 0, \quad a_{-1} = \pm \frac{\sqrt{n^2 - 1}}{\sqrt{2n^2 - 1}}, \\ k &= \frac{-8n^4 + 8n^2 + 1}{2(1 - 2n^2)^2}, \quad c = -\frac{1}{\sqrt[4]{-8n^4 + 8n^2 + 1}}. \end{aligned} \quad (24)$$

Substituting these values into Eq. (10), the exact solution of Eq. (7) takes the form

$$u(x, t) = \pm \frac{\sqrt{n^2 - 1}}{\sqrt{2n^2 - 1} \operatorname{cn}(\xi, n)}. \quad (25)$$

The behaviour of the Jacobi elliptic periodic solution (25) is shown in Figure 4 when $n = 0.8$, $t = 1$, $-50 \leq x \leq 50$. When $n \rightarrow 1$, Eq. (25) becomes

Figure 3: The positive solution of (21) is represented when $t = 0$, $n = 0.02$.

Figure 4: The negative solution (25) is shown when $t = 1$, $n = 0.8$.

$$u(x, t) = \pm \sec\left(\frac{1}{2}(t - 2x)\right). \quad (26)$$

Family 8.

$$\begin{aligned} a_0 = 0, \quad a_1 &\rightarrow \pm \frac{n}{\sqrt{2n^2 - 1}}, \quad a_{-1} = 0, \\ k &= \frac{-8n^4 + 8n^2 + 1}{2(1 - 2n^2)^2}, \quad c = -\frac{1}{\sqrt[4]{-8n^4 + 8n^2 + 1}}. \end{aligned} \quad (27)$$

In this case, we get

$$u(x, t) = \pm \frac{n \operatorname{cn}(\xi, n)}{\sqrt{2n^2 - 1}}. \quad (28)$$

When $n \rightarrow 1$, the exact solution (28) reduces to

$$u(x, t) = \pm \operatorname{sech}\left(\frac{1}{2}(t - 2x)\right). \quad (29)$$

We note that the positive solution of (29) is the same solution obtained by ref. [22]. Figure 5 represents the solitary wave solution (29) when $n = 1$, $-4 \leq x \leq 4$ and $-3 \leq t \leq 3$.

Family 9.

$$\begin{aligned} a_0 = 0, \quad a_1 &= \pm \frac{n}{\sqrt{2n^2 - 1}}, \quad a_{-1} = 0, \\ k &= \frac{-8n^4 + 8n^2 + 1}{2(1 - 2n^2)^2}, \quad c = \frac{1}{\sqrt[4]{-8n^4 + 8n^2 + 1}}. \end{aligned} \quad (30)$$

The exact solutions of Eq. (7) can be written in this case as

$$u(x, t) = \pm \frac{n \operatorname{cn}(\xi, n)}{\sqrt{2n^2 - 1}}, \quad (31)$$

where c, k are given in Eq. (30). Figure 6 represents the Jacobi elliptic periodic solution (31) where $n = 0.8$, $-4 \leq x \leq 4$ and $-3 \leq t \leq 3$.

Family 10.

$$\begin{aligned} a_0 = 0, \quad a_1 = 0, \quad a_{-1} &= \pm \frac{\sqrt{n^2 - 1}}{\sqrt{2n^2 - 1}}, \\ k &= \frac{-8n^4 + 8n^2 + 1}{2(1 - 2n^2)^2}, \quad c = \frac{1}{\sqrt[4]{-8n^4 + 8n^2 + 1}}. \end{aligned} \quad (32)$$

Thus, the exact solutions of Eq. (7) can be written in this case as

$$u(x, t) = \pm \frac{\sqrt{n^2 - 1}}{\sqrt{2n^2 - 1} \operatorname{cn}(\xi, n)}. \quad (33)$$

Case 3. Now, we consider case 3, i.e. when $g_0 = n^2 - 1$, $g_1 = 2 - n^2$, $g_2 = -1$. In this case, we have the following.

Family 11.

$$\begin{aligned} a_{-1} = 0, \quad a_0 = 0, \quad a_1 &= -\frac{1}{\sqrt{2 - n^2}}, \\ k &= \frac{n^4 + 8n^2 - 8}{2(n^2 - 2)^2}, \quad c = -\frac{1}{\sqrt[4]{n^4 + 8n^2 - 8}}. \end{aligned} \quad (34)$$

Substituting these values into Eq. (10), we find that the exact solution of Rosenau Eq. (7) takes the form

Figure 5: (a) and (b) The 3D and the contour plots of the positive solution of (29) when $n = 1$.

Figure 6: (a) and (b) The 3D and the contour plots of the positive solution of (31) when $n = 0.8$.

$$u(x, t) = \pm \frac{\text{dn}(\xi, n)}{\sqrt{2 - n^2}}. \quad (35)$$

Family 12.

$$\begin{aligned} a_0 = 0, \quad a_1 = 0, \quad a_{-1} &= \pm \sqrt{\frac{n^2 - 1}{n^2 - 2}}, \\ k &= \frac{n^4 + 8n^2 - 8}{2(n^2 - 2)^2}, \quad c = -\frac{1}{\sqrt[4]{n^4 + 8n^2 - 8}}. \end{aligned} \quad (36)$$

By substituting these values into Eq. (10), the exact solution of Rosenau Eq. (7) takes the form

$$u(x, t) = \pm \sqrt{\frac{n^2 - 1}{n^2 - 2}} \frac{1}{\text{dn}(\xi, n)}. \quad (37)$$

Family 13.

$$\begin{aligned} a_{-1} = 0, \quad a_0 = 0, \quad a_1 &= -\frac{1}{\sqrt{2 - n^2}}, \\ k &= \frac{n^4 + 8n^2 - 8}{2(n^2 - 2)^2}, \quad c = \frac{1}{\sqrt[4]{n^4 + 8n^2 - 8}}. \end{aligned} \quad (38)$$

In this case, the exact solution of Rosenau Eq. (7) takes the form

$$u(x, t) = \pm \frac{\text{dn}(\xi, n)}{\sqrt{2 - n^2}}. \quad (39)$$

Family 14.

$$\begin{aligned} a_0 = 0, \quad a_1 = 0, \quad a_{-1} &= \pm \sqrt{\frac{n^2 - 1}{n^2 - 2}}, \\ k &= \frac{n^4 + 8n^2 - 8}{2(n^2 - 2)^2}, \quad c = \frac{1}{\sqrt[4]{n^4 + 8n^2 - 8}}. \end{aligned} \quad (40)$$

By substituting these values into Eq. (10), the exact solution of Rosenau Eq. (7) takes the form

$$u(x, t) = \pm \sqrt{\frac{n^2 - 1}{n^2 - 2}} \frac{1}{\text{dn}(\xi, n)}. \quad (41)$$

Case 4. Now, we consider case 4, i.e. when $g_0 = 0, g_1 > 0$. In this case, we have the following.

Family 15.

$$a_0 = 0, \quad a_{-1} = 0, \quad k = \frac{c}{2}, \quad g_1 = \frac{1}{c^2}, \quad g_2 = -\frac{a_1^2}{c^2}. \quad (42)$$

Figure 7: The solution (43) is represented when $\varepsilon = a_1 = c = 1$.

Figure 8: The solution (43) is represented when $\varepsilon = a_1 = c = t = 1$.

Therefore, the exact solution of Eq. (7) is given by

$$u(x, t) = \frac{4\sqrt{e^{\varepsilon(-t+2x)}}}{c\left(e^{\varepsilon(-t+2x)} + \frac{4a_1^2}{c^2}\right)}, \quad (43)$$

where c, a_1 are arbitrary constants and $\varepsilon = \pm 1$. Figures 7 and 8 illustrate the solitary wave solution (43) when $\varepsilon = a_1 = c = 1, 0 \leq t \leq 10, -10 \leq x \leq 10$ in Figure 7 and $\varepsilon = a_1 = c = t = 1, -10 \leq x \leq 10$ in Figure 8. All of our solutions and figures are obtained using the computer software Mathematica.

4 Conclusion

In this article, we have successfully used the extended auxiliary equation method to construct exact solutions of the Rosenau equation. As a result, a new set of exact solutions are obtained. The solutions in this article are given in terms of some Jacobi elliptic functions and the exponential functions. However, the solitary wave solutions and the periodic wave solutions are given when the elliptic modulus $n \rightarrow 1, n \rightarrow 0$, respectively. Other solutions can be obtained in similar manner if necessary. Also, graphical illustrations are given for some of our solutions. We note that the solution given by Park [22] can be viewed as a special case of our solutions. It is worth noting that the extended auxiliary equation method is a powerful tool in finding exact solutions of nonlinear PDEs. Thus, this method could be considered for unsolved problems in future investigations.

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