Research Article

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Irreversibility analysis in time-dependent Darcy–Forchheimer flow of viscous fluid with diffusion-thermo and thermo-diffusion effects

https://doi.org/10.1515/phys-2022-0136
received February 25, 2021; accepted March 21, 2022

Abstract: In this article, we analyze the entropy analysis in unsteady hydromagnetic flow of a viscous fluid over a stretching surface. The energy attribute is scrutinized through dissipation, heat source/sink, and radiation. Furthermore, diffusion-thermo and thermo-diffusion behaviors are analyzed. The physical description of the entropy rate is discussed through the second law of thermodynamics. Additionally, a binary chemical reaction is considered. Partial differential equations are transformed into ordinary ones by adequate variables. Here, we used an optimal homotopy analysis method (OHAM) to develop a convergent solution. The influence of flow variables on velocity, Bejan number, thermal field, concentration, and entropy rate is examined through graphs. The physical performance of drag force, Sherwood number, and temperature gradient versus influential variables is studied. A similar effect holds for velocity through variation of porosity and magnetic variables. An increment in thermal field and entropy rate is noted through radiation. A reverse trend holds for the Bejan number and thermal field through a magnetic variable. An augmentation in the Soret number enhances the concentration. An amplification in drag force is noted through the Forchheimer number. Higher estimation of radiation corresponds to a rise in the heat transfer rate.

Keywords: Darcy–Forchheimer model, entropy generation, viscous dissipation, heat source/sink, chemical reaction, thermal radiation and Soret and Dufour effects

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>ρ</td>
<td>density</td>
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<tr>
<td>μ</td>
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<td>J_w</td>
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Radiation has a considerable impact on the heat transit phenomenon in electrically driven flows over any surface. Radiation is regarded as a decisive parameter in controlling the heat transfer rate used by processes involving high temperatures. On the other hand, because of its comprehensive applications, the Joule heating effect, which occurs due to interactions between fluid particles, has maintained prominence. Due to its resistive heating property, Joule heating is utilized in nuclear engineering, electrical appliances, iron soldering, glycol vaporizing, and many more applications. In the manufacturing industry, the flow of radiation heat transfer is critical for the design of reliable machinery, gas turbines, nuclear power plants, and a variety of propulsion technologies, such as, satellites, aircraft, and space vehicles. Mahanthesh et al. [14] worked on radiation analysis of a hybrid Al₂O₃–H₂O nanoliquid by a vertical plate. The forced convective hydromagnetic flow of hybrid nanomaterials with the radiation effect was illuminated by Sulochana et al. [15]. Numerous researchers [16–30] elaborated, in their studies, on the significance of radiation and its effect on fluid flow.

Nowadays, the essential concern of engineers and researchers is to determine the mechanism that can manage the consumption of good energy. It is a well-known fact that all thermal devices work on the thermodynamics principle and produce an irreversibility phenomenon. Entropy minimization is necessary to enhance efficiency of thermodynamical systems such as refrigerators, power plants, thermal storage devices, environmental control of aircraft, heat exchanger design, and electronic device cooling systems. Irreversibility analysis problems have gained more consideration due to astonishing applications in power collectors, fuel cells, slider bearings, geothermal processes, engineering phenomena, geothermal energy systems, and advanced nanotechnology. Entropy generation occurs through the Joule–Thomson effect, fluid friction, thermal flux, Joule heating, molecular vibration, mass flux, radiation, and many other effects. Bejan [31,32] discussed theoretical work on entropy problems in fluid flow with thermal transportation. Khan et al. [33] performed the entropy and melting analysis for the hydromagnetic flow of nanoliquid with radiation over a stretchable surface. Irreversibility analysis of the Darcy–Forchheimer flow of CNT-based nanomaterials with Lorentz force over a porous surface was studied by Seth et al. [34]. Entropy analysis of the hydromagnetic flow of a power-law fluid with Dufour and Soret behaviors in a permeable cavity was highlighted by Kefayati [35]. Some important studies in this field are highlighted in refs. [35–45].

The above-mentioned evaluations indicate that no effort has been made to investigate the effect of entropy
on time-dependent Darcy–Forchheimer flow of a viscous fluid with Lorentz force over a permeable surface. Yet, in recent times, numerous researchers have scrutinized the Soret and Dufour effects in viscous liquid with entropy rate over a permeable surface. Heat communication is discussed with dissipation heat source/sink and radiation. Furthermore, Soret and Dufour behaviors are also addressed. The physical description of irreversibility analysis is given. The first-order reaction is considered. Ordinary differential systems are obtained through adequate variables. Here, we used the optimal homotopy analysis method (OHAM) to construct a convergent solution [46,47]. Significant impacts of sundry variables on entropy rate, velocity field, thermal field, Bejan number, and concentration are graphically discussed. The influence of flow variables on drag force, concentration gradient and Nusselt number are studied. A comparison study with published studies is highlighted in Table 1, which shows an excellent agreement.

2 Methodology

Consider time-dependent hydromagnetic Darcy–Forchheimer flow of a viscous fluid over a permeable surface. Dissipation, heat source/sink, and radiation are considered in the heat expression. Thermo-diffusion and diffusion-thermo effects are also addressed. The physical feature of entropy analysis is discussed through the second law of thermodynamics. The first-order reaction rate is also taken into account. The magnetic force of strength \(B_0\) is incorporated. Let us suppose that \(u\left(=u_w = \frac{cx}{(1-\lambda t)}\right)\) is the stretching velocity with a positive rate constant. The induced magnetic field is neglected due to the low magnetic Reynolds number. The flow sketch is highlighted in Figure 1.

![Flow diagram](Image)

The governing equation satisfies

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_i^2 u}{\rho} - \frac{v}{K} u - Fu^2, \tag{2}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{(\rho c_p)} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{16}{3} k^2(\rho c_p) \frac{\partial^2 T}{\partial y^2}, \tag{3}
\]

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k T}{T_m} \frac{\partial^2 T}{\partial y^2} - k_{d}(C - C_{\infty}). \tag{4}
\]

For \(t > 0\), we have

\[
\begin{align*}
    u &= u_w, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \\
    u &\to 0, \quad T = T_{\infty}, \quad C = C_{\infty} \quad \text{when} \quad y \to \infty.
\end{align*}
\]

Considering

\[
\begin{align*}
    u &= f'/(1-\lambda t), \quad v = -\sqrt{\frac{\nu}{(1-\lambda t)^{3/2}}} f, \\
    T &= T_{\infty} + T_m \left[1 + \frac{cx}{2(1-\lambda t)^{3/2}} \theta \right], \\
    C &= C_{\infty} + C_m \left[1 + \frac{cx}{2(1-\lambda t)^{3/2}} \phi \right],
\end{align*}
\]

one obtains

\[
f'''' + f'' - f'^2 - \frac{A}{2} f'' f'' - (M + \lambda) f' - F f'' = 0, \tag{7}
\]

### Table 1: Comparative results of heat transfer rate with [46,47]

<table>
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<td>70.00</td>
<td>6.4622</td>
<td>6.4622</td>
<td>6.4623</td>
</tr>
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</table>
(1+Rd)\theta' + Pr f\theta' - Pr f'\theta - \frac{A}{2} Pr \eta \theta' - 2Pr A\theta + Pr Du \phi' + Pr Ec \phi'^2 + Pr Q\theta = 0, \tag{8}

\phi'' + Sce \phi' - Sce \phi - \frac{A}{2} Sce \eta \phi' \tag{9}

f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1

f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \tag{10}

Here, dimensionless variables are \( A\left(-\frac{1}{c}\right), \quad \lambda\left(\frac{v(1-\Lambda)}{Kc}\right), \quad M\left(\frac{\alpha R(1-\Lambda)}{\eta c p}\right), \quad D_f\left(\frac{D_c K_c c}{\nu c_p T_c}\right), \quad f\left(\frac{\alpha x}{\nu c}\right), \quad \text{Fr}\left(\frac{c x}{K^2}\right), \quad \text{Ec}\left(\frac{2 c x}{\nu c_p T_c}\right), \quad \text{Str}\left(\frac{D_c K_c c}{\nu c_p T_c}\right). \)

### 2.1 Entropy generation

Entropy generation is defined as \([33–39]\)

\begin{align*}
S_G &= \frac{k}{\tau_m} \left(1 + \frac{16 \sigma T_1}{3 K}\right) \left(\frac{\alpha T}{\nu c}\right)^2 + \frac{\mu}{\tau_m} \left(\frac{\alpha u}{\nu c}\right)^2 + \frac{\mu}{\nu c m} u^2 \right) , \tag{11}

&= \frac{Rc}{\tau o} \left(\frac{\alpha T}{\nu c}\right) + \frac{Rc}{\tau o} \left(\frac{\alpha C}{\nu c}\right)^2 .
\end{align*}

One can find

\begin{align*}
N_G &= \alpha_i \text{Re}(1+Rd) \theta' + \frac{1}{2} \text{Br} \phi'^2 + \frac{1}{2} \text{Br} \phi'^2 + L \text{Re} \theta' \phi' + L \frac{\alpha_i}{\alpha_i} \phi'^2 . \tag{12}

\end{align*}

The Bejan number (Be) is mathematically written as follows:

\begin{align*}
\text{Be} &= \frac{\text{Heat and mass transfer irreversibility}}{\text{Total irreversibility}} , \tag{13}

\end{align*}

or

\begin{align*}
N_G &= \alpha_i \text{Re}(1+Rd) \theta' + L \text{Re} \theta' \phi' + L \frac{\alpha_i}{\alpha_i} \phi'^2 + \frac{1}{2} \text{Br} \phi'^2 + \frac{1}{2} \text{Br} \phi'^2 . \tag{14}

\end{align*}

in which dimensionless parameters are \( N_G\left(\frac{-S_c \phi'^2(1-\Lambda)}{K c x T_c}\right), \quad \alpha_i\left(\frac{\tau o}{c x}\right), \quad \alpha_i\left(\frac{c x}{T_c}\right), \quad \text{and} \quad L\left(\frac{-R e (c x - \alpha)}{c x}\right). \)

### 2.2 Quantities of interest

#### 2.2.1 Surface drag force

Surface drag force is defined by

\[ C_f = \frac{\tau_w}{\rho u^2} , \tag{15} \]

\( \tau_w \) shear stress satisfy

\[ \tau_w = \frac{\partial u}{\partial y} \bigg|_{y=0} . \tag{16} \]

We have

\[ C_f \text{Re}_1^2 = f''(0) . \tag{17} \]

#### 2.2.2 Heat transfer rate

Mathematically

\[ \text{Nu}_x = \left(\frac{2v}{c}\right)^2 \left(\frac{1-\Lambda}{kT_o}\right) \left(\frac{2v}{c}\right) q_w , \tag{18} \]

\( q_w \) heat flux is given by

\[ q_w = -k \left(\frac{\partial T}{\partial y}\right) - \frac{16}{3} \frac{\sigma T^3}{k^*} \left(\frac{\partial T}{\partial y}\right) , \tag{19} \]

one can find

\[ \text{Nu}_x \text{Re}_{1/2} = -(1+Rd)\phi'(0) . \tag{20} \]

#### 2.2.3 Mass transfer rate

Mathematically

\[ \text{Sh}_x = \left(\frac{2v}{c}\right)^2 \left(\frac{1-\Lambda}{D_m c_{in}}\right) \left(\frac{2v}{c}\right) j_w , \tag{21} \]

\( j_w \) mass flux is

\[ j_w = -D_m \left(\frac{\partial C}{\partial y}\right) \tag{22} \]

or

\[ \text{Sh}_x \text{Re}_{1/2} = -\phi'(0) . \tag{23} \]
2.3 Solutions

Linear operators and initial guesses for OHAM satisfy

\[
\begin{align*}
    f_0(\eta) &= 1 - e^{-\eta}, \\
    \theta_0(\eta) &= e^{-\eta}, \\
    \phi_0(\eta) &= e^{-\eta}, \\
    L_f &= \frac{\partial^3 f(\eta; \eta)}{\partial \eta^3}, \\
    L_\theta &= \frac{\partial^2 \theta(\eta; \eta)}{\partial \eta^2}, \\
    L_\phi &= \frac{\partial^2 \phi(\eta; \eta)}{\partial \eta^2}.
\end{align*}
\]

with

\[
L_f = [a_0 + a_1 e^{\eta} + a_2 e^{-\eta}], \quad L_\theta = [a_3 e^{\eta} + a_4 e^{-\eta}],
\]

(26)

\[
L_\phi = [a_5 e^{\eta} + a_6 e^{-\eta}],
\]

here \( a_i \) (\( i = 0, 1, 2, 3, 4, 5, 6 \)) signify the arbitrary constants.

Suppose that \( h_f \), \( h_\theta \), and \( h_\phi \) are auxiliary variables and \( q \in [0, 1] \) the embedding variable.

2.3.1 Zeroth-order deformation problems

It is given by

\[
\begin{align*}
(1 - p) L_1[F(\eta; p) - f_0(\eta)] &= ph_f R_f, \\
(1 - p) L_2[\theta(\eta; p) - \theta_0(\eta)] &= ph_\theta R_\theta, \\
(1 - p) L_3[\phi(\eta; p) - \phi_0(\eta)] &= ph_\phi R_\phi, \\
F'(0; p) &= 1, \quad F(0; p) = 0, \quad F'(\infty; p) = 0, \\
\theta(0; p) &= 1, \\
\theta(\infty; p) &= 0, \quad \phi(0; p) = 1, \quad \phi(\infty; p) = 0.
\end{align*}
\]

Linear operators are defined as

\[
\begin{align*}
L_f &= \frac{\partial^3 f(\eta; \eta)}{\partial \eta^3} + F(\eta; p) \frac{\partial^3 F(\eta; \eta)}{\partial \eta^3} - \frac{A}{2} \frac{\partial^2 F(\eta; \eta)}{\partial \eta^2} - A \frac{\partial F(\eta; \eta)}{\partial \eta}, \\
L_\theta &= \frac{\partial^2 \theta(\eta; \eta)}{\partial \eta^2} + \text{Rd} \frac{\partial^2 \theta(\eta; \eta)}{\partial \eta^2} + \text{Pr} \left( F(\eta; p) \frac{\partial \theta(\eta; \eta)}{\partial \eta} \right) - \text{Pr} \left( \theta(\eta; p) \frac{\partial \eta(\eta; \eta)}{\partial \eta} \right) - \frac{A}{2} \text{Pr} \frac{\partial \eta(\eta; \eta)}{\partial \eta}, \\
L_\phi &= \frac{\partial^2 \phi(\eta; \eta)}{\partial \eta^2} + \text{Pr} \frac{\partial \phi(\eta; \eta)}{\partial \eta} + \text{Pr} \left( F(\eta; p) \frac{\partial \phi(\eta; \eta)}{\partial \eta} \right) - 2 \text{Pr} A \frac{\partial \eta(\eta; \eta)}{\partial \eta} + \text{Pr} Q \frac{\partial \eta(\eta; \eta)}{\partial \eta}.
\end{align*}
\]

(30)

\[
\begin{align*}
L_f &= \frac{\partial^3 f(\eta; \eta)}{\partial \eta^3} + \text{Rd} \frac{\partial^3 \theta(\eta; \eta)}{\partial \eta^3} + \text{Pr} \left( F(\eta; p) \frac{\partial \phi(\eta; \eta)}{\partial \eta} \right) - 2 \text{Pr} A \frac{\partial \eta(\eta; \eta)}{\partial \eta} + \text{Pr} Q \frac{\partial \eta(\eta; \eta)}{\partial \eta}, \\
L_\theta &= \frac{\partial^2 \theta(\eta; \eta)}{\partial \eta^2} + \text{Pr} \frac{\partial \theta(\eta; \eta)}{\partial \eta} + \text{Pr} \left( F(\eta; p) \frac{\partial \eta(\eta; \eta)}{\partial \eta} \right) - \frac{A}{2} \text{Pr} \frac{\partial \eta(\eta; \eta)}{\partial \eta}, \\
L_\phi &= \frac{\partial^2 \phi(\eta; \eta)}{\partial \eta^2} + \text{Pr} \frac{\partial \phi(\eta; \eta)}{\partial \eta} + \text{Pr} \left( F(\eta; p) \frac{\partial \phi(\eta; \eta)}{\partial \eta} \right) - 2 \text{Pr} A \frac{\partial \eta(\eta; \eta)}{\partial \eta} + \text{Pr} Q \frac{\partial \eta(\eta; \eta)}{\partial \eta}.
\end{align*}
\]

(32)
2.4 Convergence analysis

Initially, Liao [44] gives the concept of residual errors

\[
\varepsilon_m^f = \frac{1}{k+1} \sum_{i=0}^{k} \left[ N_i \left( \sum_{j=0}^{m} f(\zeta) \right) \right]^2,
\]

(42)

\[
\varepsilon_m^{\theta} = \frac{1}{k+1} \sum_{i=0}^{k} \left[ N_i \left( \sum_{j=0}^{m} \theta(\zeta) \right) \right]^2,
\]

(43)

\[
\varepsilon_m^{\phi} = \frac{1}{k+1} \sum_{i=0}^{k} \left[ N_i \left( \sum_{j=0}^{m} \phi(\zeta) \right) \right]^2,
\]

(44)

The total squared residual error is given by [45]

\[
\varepsilon_m^t = \varepsilon_m^f + \varepsilon_m^{\theta} + \varepsilon_m^{\phi},
\]

(45)

here, \(\varepsilon_m^t\) signifies a total squared residual error.

Figure 2 is drafted to analyze the total squared residual error. Computational results for an individual averaged squared residual error are demonstrated in Table 2.

Here, the obtained results indicate an excellent agreement.

3 Discussion

The physical impact of influential variables on the velocity field, entropy rate, thermal field, concentration, and Bejan number is scrutinized. The influence of flow variables on physical quantities is graphically studied.

3.1 Velocity

The influence of velocity on the variation of the porosity variable is shown in Figure 3. A manifestation in the porosity variable augments the viscous force, which enhances resistance in the flow region. Thus, the velocity diminishes. The physical feature of the velocity against the Forchheimer number is examined in Figure 4. Here, the velocity decreases with a higher Forchheimer number. An increase in the magnetic variable rises the Lorentz force, which improves disturbance to liquid flow, and consequently, declines the velocity (Figure 5). Figure 6 presents the influence of the unsteadiness variable on velocity. One can find that velocity is the decaying function of \((A)\).

<table>
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</tr>
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</table>

Figure 2: Total residual error.

Figure 3: \(f'(\eta)\) via \(\lambda\).
3.2 Temperature

Prominent effects of influential variables like Rd, Du, M, and Ec on the thermal field are demonstrated in Figures 7–10. The impact of thermal field on radiation is portrayed in Figure 7. In fact, radiation is the combined effect of heat and thermal radiation transfer rates. Thus, an increase in radiation augments temperature. The prominent effect of M on the thermal field is drafted in Figure 8. Physically, an amplification in magnetic variable produces more resistance, which rises collision between liquid particles. Thus, an improvement in temperature is seen. A physical description of temperature versus Dufour number is disclosed in Figure 9. Clearly, temperature boosts up for a higher Dufour number. The thermal field performance against the Eckert number is shown in Figure 10. An increase in Eckert's number increases the kinetic energy, which enhances temperature.

3.3 Concentration

Variation of flow variables like Sc, γ, and Sr on concentration are displayed in Figures 11–13. The influence of
the Schmidt number on $\phi(\eta)$ is shown in Figure 11. A reduction occurs in mass diffusivity with the Schmidt number, which declines the concentration. Higher approximation of reaction variables diminishes the concentration (Figure 12). The prominent variation in the concentration against the Soret number is disclosed in Figure 13. An increase in the Soret number corresponds to a decline in the concentration.

3.4 Entropy optimization and Bejan number

The influence of radiation on Be and $N_G$ is shown in Figures 14 and 15. An intensification in both Bejan number and entropy rate is noticed with radiation. In fact, an increment in radiation increases the emission of radiation, which enhances disordering in the thermal system. Therefore, the entropy rate enhances. Figures 16 and 17 sketch the influence of the porosity variable on (Be) and ($N_G$). A reverse trend holds for the Bejan number and entropy rate.
through the porosity variable. Figures 18 and 19 interpret the Brinkman number effect on Be and $N_G$. An opposite effect is noted for (Be) and ($N_G$) versus the Brinkman number. An increase in the Brinkman number increases viscous force, which improves collision between liquid particles. Thus, the entropy rate enhances.

3.5 Physical quantities

The influence of sundry variables on drag force, gradient of temperature, and Sherwood number is studied.

3.5.1 Skin friction

The influence of porosity and magnetic variables on drag force is demonstrated in Figure 20. An increment in drag force is seen with variations in magnetic and porosity variables.

3.5.2 Nusselt number

Figures 21 and 22 elucidate the performance of the Nusselt number via involved variables. An increase in
heat transfer rate is observed under magnetic and radiation effects. A reverse trend holds for the temperature gradient with the Prandtl number and Brinkman numbers.

3.5.3 Sherwood number

Figure 23 shows the effect of Soret and Schmidt numbers on the Sherwood number. An improvement in the mass transfer rate is seen with Sr and Sc.

4 Conclusions

The main points of the present study are listed below:

- A reduction occurs in the velocity profile via unsteadiness and porosity variables.
- The velocity profile decreases with the Forchheimer number.
- An opposite effect on thermal field and velocity is noted through the magnetic variable.
An increase in the thermal field is seen through radiation.

A higher Dufour number boosts up the thermal field.

An increment in the Eckert number improves the thermal field.

Concentration reduces with the Schmidt number.

A reduction in the concentration occurs for reaction variables.

An increase in the Soret number decreases the concentration.

Higher radiation improves Bejan number.

An augmentation in entropy rate is noticed through porosity variable.

An opposite effect on the Bejan number and entropy rate is noted through the Brinkman number.

An increase in drag force is noticed through magnetic variable.

Higher radiation increases the heat transfer rate.

Mass transfer rate increases with a higher Soret number.

**Acknowledgments:** The authors are grateful to Deanship of Scientific Research (DSR) at King Abdulaziz University (KAU), Jeddah, Saudi Arabia for funding this project, under grant no. (RG-4-130-43).

**Funding information:** The Deanship of Scientific Research (DSR) at King Abdulaziz University (KAU), Jeddah, Saudi Arabia has funded this project, under grant no. (RG-4-130-43).

**Author contributions:** All authors have accepted responsibility for the entire content of this manuscript and approved its submission.
Conflict of interest: The authors state no conflict of interest.

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