Coherent manipulation of bright and dark solitons of reflection and transmission pulses through sodium atomic medium

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Abstract: The coherent manipulation and control of bright and dark solitons through sodium atomic medium have been investigated in this manuscript. Dark soliton is reported for reflection and bright soliton is reported for transmission pulses with variation in position and driving field parameters through sodium atomic medium. Further the transmission pulse is periodic dark and bright solitonic behaviors and reflection pulse is periodic bright solitonic behavior with variation in the incident angle and Rabi frequency of the control field. Elliptical dark and bright solitons as well as breather types solitons are also investigated for reflection and transmission pulses. The dark soliton in reflection is due to slow light propagation and bright soliton is obtained due to fast light propagation of transmission through the medium. The modified results of the dark and bright solitons are useful for telecommunication and ultra-fast signal routing system.

Keywords: bright and darks solitons, breather solitons, sodium atomic medium

1 Introduction

The most stable localized wave packet generated from balancing of dispersion and nonlinearity is known as soliton. These self-sustained wavepackets are involved in many physical models and phenomenon [1]. Soliton has particle-like behavior and has been studied in Bose Einstein condensation [2], field theory [3], and nonlinear optics [4]. Soliton collides and annihilates like particle with interaction with other particles [5]. In nonlinear media, where the nonlinearity of the material balances dispersion, temporal solitons arise [6–8]. Solitons have useful applications in mathematics such as differential equations, Lie groups, algebras, and differential and geometrical algebra. Solitons also have potential and important applications in telecommunication and ultra-fast signal routing system [9], data bit streams over long distances [10], and data processing [11]. On the basis of the above application, soliton is an amusing field of study for experimental and theoretical researchers.

Ashkin and Bjorkholm [12] reported the first experimental study of optical spatial solitons using two-dimensional circularly symmetric beam in a nonlinear medium. Since then, many different experimental techniques have been used to analyze solitons in different models and geometries. Rohrmann et al. [13] experimentally investigated the bounded condition of two and three solitons in dispersive fibers. The soliton technology [14] is useful in radar and communication systems. Spectroscopic interferometry of the probes’ inner dynamics of femto-second soliton molecules was reported for rogue waves, breathers, and
soliton bound states [15]. Optical soliton generation in optical fiber network has been discussed by Sabirov et al. [16]. Poy et al. [17] discussed optomechanical interactions between topological and optical solitons. Tsatourian et al. [18] studied the polarization dynamics of vector solitons in molecules and mode locking in fiber lasers. The propagation theory of nonlinear weak wave is investigated in the presence of magnetic fields in fluid [19]. The propagation of three-dimensional nonlinear dust-ion-acoustic waves and the dynamic characteristics of the (3+1)-dimensional water waves are studied by previous researchers [20,21]. Arshad et al. [22] studied the spatio-temporal localized packets of waves travelling in the optical Kerr medium. Umer et al. [23] used three symbolic computing techniques to investigate the modified unstable dynamical model analytically. Tang et al. [24] reported higher-order polarization locking of solitons vector in fiber laser optics. Numerous studies have been carried out on Raman and Kerr solitons in Kerr resonating medium as well as optical cavities [25,26], Jang et al. [27] studied pairs of solitons interacting over a distance $8 \times 10^3$ times of their pulse width. Obrzud et al. [28] investigated the temporal solitons in micro-resonators by optical pulses in the resonating medium. Xue et al. [26] studied dispersionless Kerr solitons in confined optical cavities. Melnikov et al. [29] also studied forced soliton equation and semiclassical soliton form factors in their works. Lannig et al. [30] studied collisions of three vector solitons in Bose–Einstein condensates. Herring et al. [31] studied thermal noise in soliton microcombs using thermodynamic correlations induced by the balancing of nonlinearity and group velocity dispersion. He and Chen [32] studied the solitary wave and periodic wave solutions of a (2 + 1)-dimensional Korteweg–de Vries (KDV) equation. Seadawy et al. [33] investigated the stability and integrability of the governing model by using linear stability technique for solitons. Seadawy and Alsaedi [34,35] examined the two and three types of nonlinear Schrödinger equation (NLSE) using powerful and easy understandable techniques for stable soliton solution. The diverse range of travelling wave structures is obtained by using the extended $(G'/G1)$-expansion and two-variable $(G'/G, 1/G)$ expansion techniques [36,37]. Seadawy and Nuruddeen [38] introduced a new three-dimensional modified Benjamin–Bona–Mahony equations for solitons.

Studies of Raman and Kerr solitons in optical cavities and kerr resonating media have been conducted extensively. In this manuscript, dark and bright solitons of reflection and transmission pulses are coherently controlled and modified through sodium atomic medium driven by a weak probe and control fields. The dark and bright solitons of reflection and transmission pulses are controlled periodically in the medium. Contrast behaviors of solitons exist for reflection and transmission pulses with variation on position and applied field parameters. The novelty and significance of the results are clear in view of the other existing literature. In this study, the dark and bright solitons are simultaneously controlled in reflection and transmission as compared to the other existing results in the available literature. In the other literature, dark and bright solitons are obtained in transmission pulse but no work is done to control the dark and bright soliton simultaneously in reflection and transmission spectrums in sodium atomic medium. In this article, in the region of subluminal propagation of reflection of light, the dark soliton and in the superluminal propagation region of transmission spectrum, the bright solitons of light beam are simultaneously controlled and modified.

2 Atomic structure and how susceptibility is derived?

The sodium atomic medium model depicted in Figure 1 is utilized to control optical soliton through reflection and transmission processes. The atomic system represents the D-1 line of sodium atom. The $3S_{1/2}$ state and $3P_{1/2}$ state of sodium D-1 line are further split to level $|1, 2⟩$ and $|3, 4, 5, 6⟩$

![Figure 1: The suggested experimental setup for the ground state of the four-level sodium atomic system $|1⟩$, state $|2⟩$ and State $|3⟩$ resided below the excited state $|4⟩$. Two states the probe field having Rabi frequency $Ω_p$ couples the states $|1⟩$ and $|4⟩$ with detuning $Δ_p$.](https://example.com/figure1.png)
due to Hyperfine splitting. For simplicity, only four levels of $|1, 2, 3, 4\rangle$ are considered for optical responses and their controlling. The lower level $|1\rangle$ is coupled to upper exited state $|4\rangle$ by a probe field $E_p$ of Rabi frequency $G_p$ and detuning $\Delta_p$. The level $|2\rangle$ is coupled to level $|3\rangle$ by a control field $E_3$ of Rabi frequency $G_3$ and detuning $\Delta_3$. The level $|3\rangle$ is coupled to level $|4\rangle$ by a control field $E_2$ of Rabi frequency $G_2$ and detuning $\Delta_2$. Further, level $|2\rangle$ is coupled to level $|4\rangle$ by a control field $E_1$ of Rabi frequency $G_1$ and detuning $\Delta_1$. The corresponding decay rates are $\Gamma_{12}$, $\Gamma_{13}$, $\Gamma_{14}$, and $\Gamma_{15}$. In Figure 1, we present the procedure diagrammatically.

The unperturbed self-Hamiltonian of the model is shown as follows:

$$H_i = \sum_{i=1}^{4} \hbar \omega_i |i\rangle \langle i|.$$  \hfill (1)

The following is the interaction image Hamiltonian for the specified four-level model:

$$H_I = -\frac{\hbar}{2} G_p e^{-i\Delta_p} |1\rangle \langle 4| + G_4 e^{-i\Delta_4} |2\rangle \langle 4| + G_2 e^{-i\Delta_2} |3\rangle \langle 4| + G_4 e^{-i\Delta_4} |2\rangle \langle 3| + H.C.$$  \hfill (2)

The atomic model driven by coherent fields is solved by the following density matrix equation:

$$\frac{d\rho}{dt} = -\frac{1}{2} \sum_i \Gamma_i (-2k_{kp}^i + k^i k^j + \rho k^j k^i) - ih^i |H_i, \rho|,$$  \hfill (3)

where $\rho$ is the density-matrix operator, $k^i$ is the raising operator, and $k$ is the lowering operator, respectively, and $\Gamma_i$, where $(i, j = 1, 2, 3, 4)$, are decay rates. The following formula yields the rates of the following coupled equations that are explicitly independent of time:

$$\dot{\rho}_{14} = -\frac{i}{2} [G_2 \rho_{13} + G_4 \rho_{12} - G_4 (\rho_{44} - \rho_{14})] + P_3 \rho_{14},$$

$$\dot{\rho}_{13} = -\frac{i}{2} [G_2 \rho_{14} + G_4 \rho_{12} - G_4 (\rho_{43} - \rho_{13})] + P_3 \rho_{13},$$

$$\dot{\rho}_{12} = -\frac{i}{2} [G_4 \rho_{14} + G_2 \rho_{13} - G_2 (\rho_{32} - \rho_{12})] + P_3 \rho_{12},$$

where

$$P_1 = \frac{1}{2} \Gamma_{14} + i \Delta_p,$$  \hfill (4)

$$P_2 = \frac{1}{2} (\Gamma_{43} + \Gamma_{41}) + i (\Delta_p - \Delta_2),$$  \hfill (5)

$$P_3 = -\frac{1}{2} (\Gamma_{42} + \Gamma_{43} + \Gamma_{40}) + i (\Delta_p - \Delta_1).$$  \hfill (6)

Initially, the atoms are resided at the ground level $|1\rangle$, hence $\rho_{11}^{(0)} = 1$, while $\rho_{41,42,44}^{(0)} = 0$. The probe coherence term is calculated using first-order perturbation approximations and the following equation [39–41]:

$$W(t) = R^{-1} M,$$  \hfill (7)

where $R$ is a $3 \times 3$ matrix and $W(t)$ and $M$ are the column matrices. The probe coherence term $\rho_{14}$ is computed as follows:

$$\rho_{14} = \frac{-i (4P_3 P_4 + G_4 \rho_{43})}{2 (P_3 G_1^2 + P_3 G_3^2 + P_4 (4P_3 G_3^2 + G_4^2) + i G_1 G_4 \cos \varphi)}. \hfill (8)$$

The polarization is calculated from the probe coherence term $\rho_{14}$ using the relation $P = N \rho_{14} \overline{\rho}_{14}$, where $N$ is the atomic number density and $\varphi_{14}$ is the dipole matrix element between the states $|1\rangle$ and $|4\rangle$, while the Rabi frequency of the probe field is calculated using $\Omega_p = E \cdot \varphi_{14}/\hbar$. Comparing this equation to the general polarization equation given by $P = \epsilon_0 \chi E$. The electric susceptibility is calculated from the above two polarization equations. Hence, the complex electric susceptibility for the proposed four-level sodium-atomic system is determined as follows [42,43]:

$$\chi = \frac{2N \rho_{14}^{(1)} \overline{\rho}_{14}^{(2)}}{\epsilon_0 \hbar \rho_{14}}.$$

The dipole matrix element in this case is $\varphi_{14}^2$, the atomic number density is $N$, the permittivity of open space is $\epsilon_0$, and the reduced plank constant is $h$.

$$\varphi_{14} = \sqrt{\frac{3\hbar \omega_4 \epsilon_0^2}{2\omega_0^3}}. \hfill (9)$$

The calculated reflection and transmission coefficients are as follows [44]:

$$r = \frac{f_0 f_1^2}{f_1 f_2^2} \sin 2\alpha_0 \cos \alpha_1 + \sin \alpha_1,$$

where

$$t = \frac{2f_0 f_2^2}{f_1 f_2^2} \sin \alpha_1.$$

such that

$$u_1 = 2f_0 f_1 \cos 2\alpha_0 + (f_0^2 + f_1^2) \sin 2\alpha_0,$$

$$u_2 = f_1^2 (f_0^2 + f_2^2) \cos^2 \alpha_0 - (f_1^4 + f_0^2 f_2^2) \sin^2 \alpha_0,$$

$$u_3 = f_1^2 (f_0^2 + f_2^2) \cos^2 \alpha_0 - (f_1^4 - f_0^2 f_2^2) \sin^2 \alpha_0,$$

where $\alpha_2 = 1 + \chi$, $f_{0,1} = \sqrt{\epsilon_0 \alpha- \sin^2 \theta_1}$, $f_2 = \sqrt{\alpha_2 - \sin^2 \theta_1}$, $\alpha_0 = \frac{2}{\alpha} d_{1} \epsilon_1 \sin \theta_1$, and $\alpha_1 = \frac{2}{\alpha} d_2 f_2$. The incident light beam’s reflected and transmitted pulse components at the interior and outside interfaces are listed below as in the study by Uddin and Qamar [44]:
The general decay rate is \( \gamma = 2\pi \times \text{GHz} \) and other frequencies and decays are scaled to this \( \gamma \). Further constant parameters are written as follows: \( \hbar = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}, \quad \lambda = 2\pi c/\omega_0, \quad c = 3 \times 10^8 \text{ m/s}, \quad \omega_0 = 10^3 \text{ y}, \quad N = 10^{16} \text{ atom/cm}^3, \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}, \quad \epsilon_1 = 3.22, \quad d_1 = 1.5\lambda, \quad d_2 = 16\lambda, \quad w = 0.5\lambda, \quad z = 0.2\lambda, \) and \( k = 2\pi/\lambda \). In the previous studies on the governing model, bright optical soliton is studied in the transmission under Doppler broadening effect. In this study, the dark and bright optical solitons are controlled by reflection and transmission spectrum simultaneously in the mentioned sodium atomic model. The advantage of the selected methods is that we can simultaneously control both dark and bright solitons in reflection and transmission beams by the application of applied coupled fields in the medium. Hence, one can control the bright to dark and dark to bright solitons in reflection and transmission spectrum with variation in the control coupled fields detuning, Rabi frequency, and phases of control fields.

In Figure 2, the graphs plotted for reflection and transmission pulses through sodium medium vs \( y/w \) and probe field detuning \( \Delta_p/y \), where \( w \) is the pulse width in position domain and \( y \) is the general decay rate. The reflection and transmission pulses are the functions of probe field detuning \( \Delta_p/y \) and position \( y/w \). Here the position is normalized to pulse width \( w \) and probe detuning is normalized to decay rate \( y \). The reflection pulse has strong stable undistorted behaviors with variation in \( \Delta_p/y \) and \( y/w \). The reflection pulse shows dark soliton behaviors with variation in probe detuning \( \Delta_p/y \) and position \( y/w \). The reflection pulse has negative value and dark soliton behavior as shown in Figure 2(a). The transmission pulse has strong stable undistorted behaviors with variation in \( \Delta_p/y \) and \( y/w \) and has behaviors contrast to that of

\[
E^{(r)}(y, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} r(k_y) e^{i(k_yz+y)} V(k_y) dk_y,
\]

\[
E^{(t)}(y, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t(k_y) e^{i((k-y)d + k_yz)} V(k_y) dk_y.
\]

Here

\[
V(k_y) = \frac{w_y}{2\sqrt{\pi}} \exp\left(-\frac{w_y^2(k_y-k_y0)^2}{4}\right).
\]

where \( d = 2d_1 + d_2, w_y = w \sec \theta_i, \) and \( k_y = k \cos \theta_i \).

### 3 Findings and explanation

The works are presented for bright and dark solitons of reflection and transmission pulses in sodium atomic medium. The reflectance and transmission spectrum in sodium medium are written as follows: \( \hbar = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}, \quad \lambda = 2\pi c/\omega_0, \quad c = 3 \times 10^8 \text{ m/s}, \quad \omega_0 = 10^3 \text{ y}, \quad N = 10^{16} \text{ atom/cm}^3, \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}, \quad \epsilon_1 = 3.22, \quad d_1 = 1.5\lambda, \quad d_2 = 16\lambda, \quad w = 0.5\lambda, \quad z = 0.2\lambda, \) and \( k = 2\pi/\lambda \). In the previous studies, bright optical soliton is studied in the transmission under Doppler broadening effect. In this study, the dark and bright optical solitons are controlled by reflection and transmission spectrum simultaneously in the mentioned sodium atomic model. The advantage of the selected methods is that we can simultaneously control both dark and bright solitons in reflection and transmission beams by the application of applied coupled fields in the medium. Hence, one can control the bright to dark and dark to bright solitons in reflection and transmission spectrum with variation in the control coupled fields detuning, Rabi frequency, and phases of control fields.

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In Figure 3, the graphs are traced for reflection and transmission pulses in sodium medium against incident angle \( \theta_i \) and a control field \( E_3 \) of Rabi frequency \( G_3/y \). The incidence angle \( \theta_i \) and control field Rabi frequency \( G_3/y \) determine and discuss the reflection and transmission pulse behaviors. As seen in Figure 3(a), the reflection pulse is periodic and exhibits vivid soliton behaviors with variation in Rabi frequency of control field \( E_3 \) of Rabi frequency \( G_3/y \) and incident angle \( \theta_i \). Multiple solitons are controlled at incident angle \( \theta_i = 0 \) with normal of the medium with variation of control field Rabi frequency \( G_3/y \). The amplitude of reflection pulse variation is in the range of \( 0 < E^{(r)}_3 < 1 \). As seen in Figure 3(b), the transmission pulse exhibits strong, stable, undistorted periodic dark and bright soliton behaviors with variations in Rabi frequency \( G_3/y \) and incident angle \( \theta_i \). The transmission pulse has behaviors contrast to that of the
reflection pulse with ith variations in Rabi frequency $G_i/y$ and incident angle $\theta_i$. The transmission pulse amplitude variation is in the range of $0 \leq E_x^{(t)} \leq 1.5$. The results of these solitons are significant for ultra-high-speed optical communications, optical signal processing, and optical switching technologies.

In Figure 4, the plots are mentioned for reflection and transmission pulses through sodium medium vs collective phase of the driving coupled fields $\varphi$ and Rabi frequency $G_2/y$ of control field $E_2$. The reflection and transmission pulses are elliptical dark and bright solitons behaviors and strong variation function of collective phase of control fields $\varphi$ and Rabi frequency $G_2/y$. The amplitude of reflection pulse variation is in the range of $0 \leq E_x^{(r)} \leq 0.125$. The reflection pulse is elliptical dark soliton with variation in $G_2/y$ and $\varphi$ as discussed in Figure 4(a). The transmission pulse has strong stable undistorted elliptical bright soliton behaviors against $G_2/y$ and $\varphi$ and has behaviors contrast to that of reflection pulse as discussed in Figure 4(b). The amplitude transmission pulse varies in the range of amplitude of reflection pulse variation in the range of $0.25 \leq E_x^{(t)} \leq 0.35$ with variation in $G_2/y$ and $\varphi$. The dark and bright solitons in sodium atomic medium have potential application in ultra-fast optics, pulse shaping, stabilization, and radar technologies.

In Figure 5, the graphs are discussed for reflection and transmission pulses for sodium medium vs control field $E_2$ of detuning $\Delta_2/y$ and another control field $E_1$ of Rabi frequency $G_1/y$. The reflection and transmission pulses are single dark, bright, breather soliton behaviors and strong variation function of control field detuning $\Delta_2/y$ and Rabi frequency $G_1/y$. The reflection pulse is singular dark and breather soliton behaviors with variation in control field detuning $\Delta_2/y$ and Rabi frequency $G_1/y$. The amplitude of reflection pulse varies in the range of $0 \leq E_x^{(r)} \leq 0.5$ as mentioned in Figure 5(a). The transmission pulse is strong stable undistorted singular, bright, and breather soliton behaviors with variation in $\Delta_2/y$ and $G_1/y$. The amplitude of transmission pulse varies in the range of $0.5 \leq E_x^{(t)} \leq 2.5$.
as discussed in Figure 5(b). The modified results of reflection and transmission pulses are used for optical fiber communication systems that provide powerful tool for tuning of the optical fiber network architecture and optimization of signal and information transmission.

4 Conclusion

In conclusion, dark and bright solitons in sodium atomic medium are investigated in this manuscript for reflection and transmission pulses. The key finding in this works is simultaneously controlling dark and bright solitons by reflection and transmission spectrums. Dark soliton is investigated for reflection and bright soliton is reported for transmission pulses with variation in position and coupled driving fields parameters. Density matrix formalism is used to calculate dielectric function for the sodium medium. The reflection and transmission coefficients are calculated from the dielectric function of sodium and free space permittivity. The reflection and transmission pulses are related to reflection and transmission coefficients. The transmission pulse is periodic dark and bright solitonic behaviors and reflection pulse is periodic bright solitonic behavior with variation in incident angle and Rabi frequency of the control field. Elliptical dark and bright solitons as well as breather solitons are also modified for reflection and transmission pulses. Reflection pulse amplitude varies in the range of \(0 \leq E_x^{(r)} \leq 1\) and transmission pulse varies in the range of \(0 \leq E_x^{(t)} \leq 1.5\) with variation in incident angle and control field \(E_3\) of Rabi frequency. Further, the amplitude of reflection pulse varies in the range of \(0 \leq E_x^{(r)} \leq 0.5\) and transmission pulse varies in the range of \(0.5 \leq E_x^{(t)} \leq 2.5\) with variation in control field \(E_2\) of detuning \(\Delta_2/\gamma\) and Rabi frequency of control field \(E_1\) of \(G_1/\gamma\). The dark soliton is obtained due to subluminal propagation and bright soliton is obtained due to superluminal propagation of reflection and transmission pulses simultaneously. Further, breather soliton is also controlled for the reflection and transmission pulses. This work is useful for optical communications, optical signal processing, optical switching, ultra-fast optics, pulse shaping and stabilization, noise suppression, optical modulator, optical sensors, ultra-fast signal routing, data processing systems, and radar technologies.

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