Research Article

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Reflection phenomena of waves through rotating elastic medium with micro-temperature effect

<https://doi.org/10.1515/rams-2020-0036>

Received Feb 26, 2019; accepted Jul 08, 2019

Abstract: In this article, we analyzed the effect of variable thermal conductivity on reflected elastic waves. The waves are propagating through a thermoelastic medium rotating with some angular frequency. The concept of micro-temperature is also been considered, in which microelements of the medium contain a high temperature. A heat conduction phenomenon is encountered by dual phase-lag heat conduction model. P (or SV)-type wave is incident on the medium with some specific angle of incidence. After reflection from the surface incident, P-wave is converted into quasi longitudinal and quasi transverse waves and propagates back into the medium. Helmholtz's potential function along with the harmonic wave solution is used to obtain the solution of the model. Analytically, we calculated the amplitude ratios and attenuation factor for each reflected wave against the angle of incidence. The obtained results are also represented graphically for different values of rotational frequency and variable thermal conductivity for a particular material.

Keywords: reflection phenomena, variable thermal conductivity, rotational frequency, Helmholtz's potential functions, amplitude ratios

1 Introduction

The study of seismic waves and their reflection phenomena from different kinds of boundaries is of countless significance in geophysics. The work on seismic waves contains mostly the analysis on the propagation of plane waves, when they reflect, refract and diffract from gaps existing inside the earth. Great work has been done by investigators regarding wave propagation and reflection and refraction in thermo-elastic waves. Sinha and Sinha [1], Sinha and Elsibai [2], Sinha and Elsibai [3], and Sharma [4] studied the reflection of thermo-elastic waves.

In thermo-elastic media, the wave propagation is of great significance in various fields for example earthquake engineering, soil dynamics, aeronautics, astronautics, nuclear reactors, high energy element accelerator, etc. Several investigators have worked on wave propagation in isotropic thermo-elasticity. Deresiewicz [5] and Beevers and Bree [6] also studied the reflection problems. Grot [7] has introduced the theory of thermodynamics for material which is elastic with microstructure whose elements, with micro-deformation possess micro-temperatures. Grot [7] extended the theory of thermodynamics with microstructure by supposing that each microelement has different temperature for defining this phenomenon the concept of micro-temperature is introduced. Well-posedness of the microtemperature is being studied by Quantanilla [8] and Chirita *et al.* [9]. Steeb and Singh [10] investigated the time harmonic waves in thermoelastic materials with micro temperatures Singh and Yadav [11] have studied reflection plane waves in rotating isotropic medium. Othman and Song [12] studied the reflection of magneto-thermoelastic waves from a rotating elastic half space. Ezzat *et al.* [13], Youssef [14, 15] and Aouadi [16] studied different thermoelastic problems and considered properties of variable material in reference to different generalized theories.

Recently, some new involvement in a dual-phase-lag model has been discussed [17–20]. The stability of DPL model was encountered by Chirita *et al.* [20–22] and Quantanilla and Racke [23]. Some researchers have investigated the problems on elastic deformation by using a dual-

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phase-lag model as, Prasad *et al.* [24], Othman *et al.* [25], Singh [26], Mondal *et al.* [27] and Singh *et al.* [28].

In this paper, we have studied the reflection of plane wave through the stress free surface of the thermo-elastic medium. The medium considered is isotropic and rotating with angular frequency $\vec{\Omega} = (0, \Omega, 0)$. It is found that four waves are generated by reflection of incident waves from the surface of the medium. Theoretical results obtained are represented graphically for the particular material to manifest the effects of rotation and variable thermal conductivity.

2 Problem formulation and basic equations

We have considered a generalized homogeneous isotropic thermoelastic medium with micro temperature. The system considered is without body force and external heat source. The rectangular coordinate system is adopted to represent the problem. Geometrically medium is a half-space medium with a z-axis pointing vertically. Governing equations with description are represented as,

Equation of motion for medium rotating with angular frequency $\vec{\Omega}$,

$$\sigma_{ij,j} = \rho \left[\ddot{u} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega} \times \dot{\vec{u}} \right]_i. \quad (1)$$

The first moment of energy responsible for development of equation is,

$$\Xi \dot{\varepsilon}_i = q_{ji,j} + q_i - Q_i + \Xi M_i \quad (2)$$

The energy equation with the modified Fourier Law is represented as,

$$-q_{i,i} = \rho \theta_0 \dot{S} \quad \text{and} \quad -K_{ij} (\theta_{,j} + \tau_T \dot{\theta}_{,j}) = q + \tau_q \dot{q}_i. \quad (3)$$

The constitutive equations are,

$$\begin{cases} \sigma_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij} - \beta \theta \delta_{ij}, \\ q_{ij} = -k_4 w_{r,r} \delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i}, \\ Q_i = (K - k_3) \theta_{,i} + (k_1 - k_2) w_i, \\ q_i = K \theta_{,i} + k_1 w_i, \\ \Xi \varepsilon_i = -b w_i, \end{cases} \quad (4)$$

By applying the constitutive relations in equation (2), we get the following equation of micro-temperature,

$$k_6 \nabla^2 \vec{w} + (k_4 + k_5) \vec{\nabla} (\vec{\nabla} \cdot \vec{w}) - k_3 \vec{\nabla} \theta - k_2 \vec{w} - b \vec{w} = 0. \quad (5)$$

Equation (3) along with constitutive relations gives parabolic form of dual phase lag heat conduction model with influence of micro-temperature

$$\begin{aligned} & \left(1 + \tau_T \frac{\partial}{\partial t} \right) K \nabla^2 \theta + k_1 (\vec{\nabla} \cdot \vec{w}) \\ & = \left(1 + \tau_q \frac{\partial}{\partial t} \right) (\rho C_E \dot{\theta} + \beta \theta_{,o} \dot{e}_{kk}). \end{aligned} \quad (6)$$

We suppose that all functions are differentiable and continuous in the defined domain. For most of the materials, thermal properties vary with an increase in temperature θ and these temperature increased relations are linear in the range of the temperature θ . Following are the relations for material parameters that are thermal and linear [13]

$$K = K(\theta), \quad (7)$$

$$\rho C_E = \frac{K}{k}, \quad (8)$$

where k is the diffusivity. With effects of variable thermal conductivity heat conduction equation becomes,

$$\begin{aligned} & \left(1 + \tau_T \frac{\partial}{\partial t} \right) \vec{\nabla} \cdot (K(\theta) \vec{\nabla} \theta) + k_1 (\vec{\nabla} \cdot \vec{w}) \\ & = \left(1 + \tau_q \frac{\partial}{\partial t} \right) (\rho C_E \dot{\theta} + \beta \theta_{,o} \dot{e}_{kk}). \end{aligned} \quad (9)$$

We will use the mapping [14],

$$\phi = \frac{1}{K_0} \int_0^\theta K(\theta') d\theta'. \quad (10)$$

By using the fundamental law of calculus and differentiating with respect to special and temporal parameter we get the following relations,

$$K_0 \vec{\nabla} \cdot (\vec{\nabla} \phi) = \vec{\nabla} \cdot (K(\theta) \vec{\nabla} \theta) \quad \text{and} \quad K_0 \dot{\phi} = K(\theta) \dot{\theta}. \quad (11)$$

By using the above transformation and approximation for linearity as $K(\theta) \approx K(\theta_0)$ [13], which is constant depending on the reference temperature θ_0 . Governing system of equation can be modified as

$$\begin{aligned} & \left(1 + \tau_T \frac{\partial}{\partial t} \right) \vec{\nabla} \cdot (\vec{\nabla} \phi) + \frac{k_1}{K_0} (\vec{\nabla} \cdot \vec{w}) \\ & = \left(1 + \tau_q \frac{\partial}{\partial t} \right) \left(\frac{\rho C_E}{K_0} \dot{\phi} + \frac{\beta \theta_{,o}}{K_0} \dot{e}_{kk} \right), \end{aligned} \quad (12)$$

$$\begin{aligned} & \mu \nabla^2 \vec{u} + (\lambda + \mu) \vec{\nabla} (\vec{\nabla} \cdot \vec{u}) - \beta' \vec{\nabla} \theta \\ & = \rho \left[\ddot{u} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega} \times \dot{\vec{u}} \right], \end{aligned} \quad (13)$$

$$k_6 \nabla^2 \vec{w} + (k_4 + k_5) \vec{\nabla} (\vec{\nabla} \cdot \vec{w}) - k_3' \vec{\nabla} \phi - k_2 \vec{w} - b \vec{w} \quad (14)$$

= 0,

where,

$$\frac{\beta K_0}{K(\theta)} \approx \frac{\beta K_0}{T_0} = \beta', \quad k_3' = \frac{k_3 K_0}{K(\theta_0)} = \frac{k_3 K_0}{\theta_0}.$$

Modified form of constitutive relation is represented as,

$$\sigma_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij} - \frac{\beta K_0}{K(\theta)} \delta_{ij} \phi.$$

Following are the non-dimensional variables

$$x'_i = \frac{x_i}{l_0}, \quad t' = \frac{c_1 t}{l_0}, \quad u'_i = \frac{u_i}{l_0}, \quad \theta' = \frac{\theta}{\theta_0}, \quad (15)$$

$$w'_i = w_i l_0, \quad \tau'_T = \frac{c_1}{l_0} \tau_T, \quad \tau'_q = \frac{c_1}{l_0} \tau_q, \quad \nabla^2 = \frac{1}{l_0^2} \nabla'^2,$$

$$\phi' = \frac{\phi}{\theta_0}, \quad \Omega = \frac{c_1}{l_0} \Omega'.$$

Where, l_0 is standard length and c_1 is the standard velocity given by $c_1 = \sqrt{\left(\frac{\lambda+2\mu}{\rho}\right)}$. After non dimensionalization the governing equations can be represented as, (primes are dropped for simplicity),

$$\alpha_0 \nabla^2 \vec{u} + \alpha_1 \vec{\nabla} \left(\vec{\nabla} \cdot \vec{u} \right) - H \vec{\nabla} \phi \quad (16)$$

$$= \left[\ddot{u} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega} \times \dot{u} \right]_i$$

$$\beta_0 \nabla^2 \vec{w} + \beta_1 \vec{\nabla} \left(\vec{\nabla} \cdot \vec{w} \right) - \beta_2 \vec{\nabla} \phi - \beta_3 \vec{w} - \beta_4 \dot{\vec{w}} = 0, \quad (17)$$

$$\left(1 + \tau_T \frac{\partial}{\partial t} \right) \alpha_2 \nabla^2 \phi + \alpha_3 \left(\vec{\nabla} \cdot \vec{w} \right) \quad (18)$$

$$= \left(1 + \tau_q \frac{\partial}{\partial t} \right) \left(\dot{\phi} K'' + \alpha_4 \vec{\nabla} \cdot \dot{\vec{u}} \right)$$

where,

$$\alpha_0 = \frac{\mu}{\rho c_1^2}, \quad \alpha_1 = \frac{(\lambda + \mu)}{\rho c_1^2}, \quad \alpha_2 = \frac{K_0}{l_0 \rho C_E c_1}, \quad (19)$$

$$\alpha_3 = \frac{k_1}{l_0 \rho \theta_0 C_E c_1}, \quad \alpha_4 = \frac{\beta}{\rho C_E}, \quad H = \frac{\beta' \theta_0}{\rho c_1^2}, \quad K'' = \frac{K_0}{K(\theta)},$$

$$\beta_0 = \frac{k_6}{\rho l_0^3 c_1^3}, \quad \beta_1 = \frac{(k_4 + k_5)}{\rho l_0^3 c_1^3}, \quad \beta_2 = \frac{k_3'}{\rho l_0 c_1^3},$$

$$\beta_3 = \frac{k_2}{\rho l_0 c_1^3}, \quad \beta_4 = \frac{b}{\rho l_0^2 c_1^2}.$$

According to Helmholtz decomposition principle, any vector can be decompose into two components an irrotational vector field with scalar potential and a solenoidal vector field with vector potential. The complexity of the problem could be reduced by introducing the appropriate set of potential functions. Displacement and micro-temperature

functions could be converted in terms of potential function by following expression [10],

$$\vec{u} = \text{grad} R + \text{curl} \psi.$$

Can be represented as,

$$u_1 = \left(\frac{\partial R}{\partial x} + \frac{\partial \psi}{\partial z} \right), \quad (20)$$

where, R and v are scalar potentials functions, and vector potential is representing by ψ . Applying equation (20) in non-dimensional form of governing equations, and after some algebraic calculations we get the following relations,

$$[\alpha \nabla^2 - \frac{\partial^2}{\partial t^2} + \Omega^2] R - H \phi + 2\Omega \dot{\psi} = 0, \quad (21)$$

$$[\alpha_0 \nabla^2 - \frac{\partial^2}{\partial t^2} + \Omega^2] \psi - 2\Omega \dot{R} = 0, \quad (22)$$

$$\sigma \nabla^2 v - \beta_2 \phi - \beta_3 v - \beta_4 \dot{v} = 0, \quad (23)$$

$$\alpha_2 \left(\nabla^2 \phi + \tau_T \nabla^2 \dot{\phi} \right) + \alpha_3 \nabla^2 v - \dot{\phi} K'' \left(1 + \tau_q \frac{\partial}{\partial t} \right) \quad (24)$$

$$- \left(1 + \tau_q \frac{\partial}{\partial t} \right) \left(\alpha_4 \nabla^2 \dot{R} \right) = 0.$$

We consider a plane wave (P or SV) incident on the semiconductor nanostructure half space medium. The wave makes an angle θ_0 with an axis normal to the surface at origin. Corresponding to each incident wave, we get quasi longitudinal waves (quasi longitudinal displacement (P_q), quasi transverse displacement (SV_q), quasi thermal wave (T_q)) and quasi micro-temperature wave (MC_q)).

3 Solution of the problem

Solution for each variable is taken in the form,

$$[R, \psi, \phi, v] \quad (25)$$

$$= [R^*, \psi^*, \phi^*, v^*] \exp[i\xi(x \sin \theta + z \cos \theta)] + i\omega t],$$

where, ω is the angular frequency $i = \sqrt{-1}$, a is the wave number and R^*, ϕ^*, v^*, ψ^* represents the amplitude of each wave. By using equation (25) in equations (21)–(24) we get,

$$(-\xi^2 \alpha + A_1) R^* - H \phi^* - A_2 \psi^* = 0, \quad (26)$$

$$(-\xi^2 \alpha_0 + A_3) \psi^* + A_4 R^* = 0, \quad (27)$$

$$(-\xi^2 \sigma + A_5) v^* - \beta_2 \varphi^* = 0, \quad (28)$$

$$(\xi^2 a_2 \pi_1 + A_6) \varphi^* - a_3 \xi^2 v^* - R^* \xi^2 \alpha_4 \pi_2 = 0, \quad (29)$$

where

$$\begin{aligned} A_1 &= \omega^2 + \Omega^2, & A_2 &= -2i\Omega\omega, & A_3 &= \omega^2 + \Omega^2, \\ A_4 &= -2i\Omega\omega, & A_5 &= -\beta_3 - i\omega\beta_4, & A_6 &= \omega^2 K'' \tau_q - i\omega K'', \\ \pi_1 &= -1 - \tau_T i\omega, & \pi_2 &= \omega^2 \tau_q - i\omega. \end{aligned}$$

The determinant of the coefficients must vanish to have a non trivial solution

$$A\xi^8 - B\xi^6 + C\xi^4 - D\xi^2 + E = 0, \quad (30)$$

where,

$$A = \alpha\alpha_0\alpha_2\sigma\pi_1,$$

$$\begin{aligned} B &= -\alpha\sigma A_6\alpha_0 - \alpha\alpha_0\alpha_3\beta_2 + \alpha\sigma A_3\alpha_2\pi_1 + \sigma A_1\alpha_0\alpha_2\pi_1 \\ &\quad + \alpha A_5\alpha_0\alpha_2\pi_1 - H\sigma\alpha_0\alpha_4\pi_2, \\ C &= -\alpha\sigma A_3 A_6 - \sigma\alpha_0 A_1 A_6 - \alpha\alpha_0 A_5 A_6 - \alpha A_3 \alpha_3 \beta_2 \\ &\quad - A_1 \alpha_0 \alpha_3 \beta_2 + \sigma A_1 A_3 \alpha_2 \pi_1 + \sigma A_2 A_4 \alpha_2 \pi_1 + \alpha A_3 A_5 \alpha_2 \pi_1 \\ &\quad + A_1 A_5 \alpha_0 \alpha_2 \pi_1 - H\sigma\pi_2 A_3 \alpha_4 - H A_5 \alpha_0 \alpha_4 \pi_2, \\ D &= -\sigma A_1 A_3 A_6 - \sigma A_2 A_4 A_6 - \alpha A_3 A_5 A_6 \\ &\quad - A_1 A_5 A_6 \alpha_0 - A_1 A_3 \alpha_3 \beta_2 - A_2 A_4 \alpha_3 \beta_2 \\ &\quad + A_1 A_3 A_5 \alpha_2 \pi_1 + A_2 A_4 A_5 \alpha_2 \pi_1 - H A_3 A_5 \alpha_4 \pi_2, \\ E &= -A_1 A_3 A_5 A_6 - A_2 A_4 A_5 A_6, \end{aligned}$$

where $\varepsilon = -\varepsilon_2 + i\omega\varepsilon_3$,

The roots of the equation (30) has four values of $\xi_j = \rho c_j^2$, representing complex phase velocities of P_q , MC_q , T_q and SV_q . The real part of the root of equation (30) represents the velocity of the medium. Geometry of the problem is represented in following figure,

Figure 1: Geometry of the problem

Figure 1, indicate that for incident P or SV wave at the interface will generate reflected P_q , MC_q , T_q and SV_q waves in the half space $z > 0$. The functions R^* , ψ^* , φ^* , v^* will take the following forms:

$$R^* = A_0 \exp[i\xi_0(x \sin \theta + z \cos \theta) + i\omega t] \quad (31)$$

$$+ \sum_{r=1}^4 A_r \exp[i\xi_r(x \sin \theta_r - z \cos \theta_r) + i\omega t],$$

$$\psi^* = \eta_0 A_0 \exp[i\xi_0(x \sin \theta + z \cos \theta) + i\omega t] \quad (32)$$

$$+ \sum_{r=1}^4 \eta_r A_r \exp[i\xi_r(x \sin \theta_r - z \cos \theta_r) + i\omega t],$$

$$\varphi^* = A_0 \zeta_0 \exp[i\xi_1(x \sin \theta + z \cos \theta) + i\omega t] \quad (33)$$

$$+ \sum_{r=1}^4 A_r \zeta_r \exp[i\xi_r(x \sin \theta_r - z \cos \theta_r) + i\omega t],$$

$$v^* = A_0 \Pi_0 \exp[i\xi_0(x \sin \theta + z \cos \theta) + i\omega t] \quad (34)$$

$$+ \sum_{r=1}^4 A_r \Pi_r \exp[i\xi_r(x \sin \theta_r - z \cos \theta_r) + i\omega t],$$

where,

$$\eta_r = -\frac{A_4}{(-\xi_r^2 \alpha_0 + A_3)}, \quad (35)$$

$$\zeta_r = \frac{\xi_r^4 \alpha \alpha_0 - \xi_r^2 (\alpha A_3 + \alpha_0 A_1) + A_1 A_3 + A_2 A_4}{H(-\xi_r^2 \alpha_0 + A_3)},$$

$$r = 0, 1, \dots, 4$$

The corresponding reflection coefficient ratio is represented by $\frac{A_r}{A_0}$, $r = 1, 2, \dots, 4$. Snell's law gives a relation between angle θ , θ_r ($r = 1, 2, \dots, 4$) and the wave number as,

$$\begin{aligned} \xi_0 \sin \theta &= \xi_1 \sin \theta_1 = \xi_2 \sin \theta_2 = \xi_3 \sin \theta_3 \\ &= \xi_4 \sin \theta_4. \end{aligned} \quad (36)$$

4 Boundary conditions

Boundary conditions are given as

1. Mechanical boundary condition

$$\sigma_{zz}(x, 0, t) = 0. \quad (37)$$

2. Thermal condition,

$$\theta(x, 0, t) = \theta_0. \quad (38)$$

Above boundary condition leads to the following algebraic equations:

$$\sum_{j=1}^4 a_{ij} Z_j = b_i, \quad Z_j = \frac{A_j}{A_0}, \quad i, j = 1, 2, \dots, 4, \quad (39)$$

where

$$\begin{aligned} a_{1j} &= \xi_j^2 \lambda \{-\sin^2 \theta_j - \eta_j \sin \theta_j \cos \theta_j\} \\ &\quad + (\lambda + 2\mu) \xi_j^2 \{-\cos^2 \theta_j + \cos \theta_j \sin \theta_j \eta_j\} - \beta \theta_0 \zeta_j, \\ a_{2j} &= -2\mu \xi_j^2 \{\cos \theta_j \sin \theta_j\} + \xi_j^2 \eta_j \{\sin^2 \theta_j - \cos^2 \theta_j\}, \\ a_{3j} &= \Pi_j \xi_j^2 \{k_4 \sin^2 \theta_j + \cos^2 \theta_j (k_4 + k_5 + k_6)\}, \\ a_{4j} &= \zeta_j, \\ b_1 &= \xi_0^2 \lambda \{\sin^2 \theta_0 + \eta_0 \sin \theta_0 \cos \theta_0\} \\ &\quad + (\lambda + 2\mu) \xi_0^2 \{-\cos^2 \theta_0 + \cos \theta_0 \sin \theta_0 \eta_0\} + \beta \theta_0 \zeta_0, \\ b_2 &= 2\mu \xi_0^2 \{\cos \theta_0 \sin \theta_0\} - \xi_0^2 \eta_0 \{\sin^2 \theta_0 - \cos^2 \theta_0\}, \\ b_3 &= -\Pi_0 \xi_0^2 \{k_4 \sin^2 \theta_0 + \cos^2 \theta_0 (k_4 + k_5 + k_6)\}, \\ b_4 &= \frac{f^* \exp \{i \xi(x \sin \theta) + i \omega t\}^{-2}}{A_0} - \zeta_0. \end{aligned}$$

5 Numerical Results and Discussions

The evaluated theoretical results are computed numerically by using the relevant parameters for the case of magnesium crystal. The relevant physical values of elastic constants and micro-temperatures are [10]

$$\begin{aligned} \rho &= 1.74 \times 10^3 \text{ kg m}^{-3}, \quad \lambda = 9.4 \times 10^{10} \text{ Nm}^{-2}, \\ \mu &= 4.0 \times 10^{10} \text{ Nm}^{-2}, \quad \beta = 7.779 \times 10^{-8} \text{ N}, \\ C_E &= 1.04 \times 10^3 \text{ Nm K g}^{-1} \text{ K}^{-1}, \quad \theta_0 = 0.298, \\ b &= 0.15 \times 10^9 \text{ N}. \end{aligned}$$

The micro-temperature parameters are,

$$\begin{aligned} k_1 &= 0.0035 \text{ N s}^{-1}, \quad k_2 = 0.045 \text{ N s}^{-1}, \\ k_3 &= 0.055 \text{ N K}^{-1} \text{ s}^{-1}, \quad k_4 = 0.064 \text{ N s}^{-1} \text{ m}^2, \\ k_5 &= 0.075 \text{ N s}^{-1} \text{ m}^2, \quad k_6 = 0.096 \text{ N s}^{-1} \text{ m}^2. \end{aligned}$$

The computations were carried out for $x = 1$, $t = 0.01$ sec, $t_\tau = 0.015$ sec and $t_q = 0.02$ sec. The amplitude ratio of each wave propagating through the medium after reflecting through the surface are represented graphically for different rotational frequency and variable thermal conductivity of the medium.

Figure 2, is representing the absolute value of amplitude ratio $|Z_1|$ for different values of angle of incidence,

Figure 2: Amplitude ratio of first wave against incident angle

Figure 3: Amplitude ratio of second wave against incident angle

where the variable thermal conductive parameter is set to fix at $K_1 = -0.01$ and the medium is rotating with different angular frequencies. It is observed that the rotational effect reduces the amplitude ratio of wave propagating through the medium. The amplitude of the reflected wave increases exponentially for $\theta \leq 45$ and there is no effect of rotation for $\theta \geq 60$.

Amplitude ratio of second wave against angle of incidence and for different values of rotational frequency of medium is represented in Figure 3. It can be seen clearly, that the behavior of amplitude for the second wave is same as that of the first wave but moving with different absolute values.

Figure 4, represents the graphical analysis of $|Z_3|$ for different values of angle of incidence and rotational frequency of the medium. Rotation is having increasing effects on amplitudes ratio of reflected wave for $\theta > 4.5$. Response of rotation on amplitude ratio for initial values $0 < \theta \leq 5$ of angle of incidence is different. Rotational effect results in low amplitude ratio for the current wave for $\theta < 4.5$, this is also represented on larger scale. The curves

Figure 4: The amplitude ratio of the third wave against incident angle

are in red, blue and green colors for $\Omega = 0, 50$ and 100 respectively.

Figure 5: Amplitude ratio of fourth wave against incident angle

Curves for amplitude ratio of fourth wave against angle of incidence for incident wave are represented in Figure 5. It is clear graphically that the behavior of amplitude curves is same as in Figure 4 indicating the same effects of rotational frequency on the amplitudes ratios of reflected wave. But for initial values of angle of incidence nature of curves are different.

Effect of variable thermal conductivity on amplitude of first reflected wave against angle of incidence is represented in Figure 6. The negative parameter K_1 is directly related with thermal conductivity of the medium i.e., thermal conductivity is maximum for $K_1 = -0.01$ and minimum for $K_1 = -0.8$. It can be seen clearly that in presence of rotational effect, variable thermal conductivity is having increasing effects on amplitude ratio of the reflected wave propagating through the medium.

In Figure 7 effects of variable thermal conductivity on amplitude ratios for second wave is discussed while the

Figure 6: The amplitude ratio of first wave against incident angle for different variable thermal conductivity

Figure 7: Amplitude ratio of second wave against incident angle for different variable thermal conductivity

medium is set on rotation with constant angular frequency Ω . The response of the curves representing the amplitude ratio for second wave is exactly same as that of first wave but with different intensity.

Figure 8: Amplitude ratio of third wave against incident angle for different variable thermal conductivity

Figure 9: Amplitude ratio of fourth wave against incident angle for different variable thermal conductivity

Figure 10: Velocity and attenuation factor for different values of ω and rotational frequency

From Figure 8, it can also be seen that variable thermal conductivity is directly proportional with the amplitude ratio of wave propagating through the medium. The responses of curves represented in Figure 9 are exactly same as that in Figure 8. The curves in both the figures converge to zero as angle reaches its maximum value, indicating that for higher value of angle of incidence the lower will be the amplitude of reflected waves.

Figure 10 and Figure 11, are representing the curves for velocity and attenuation factor for first and second wave against frequency of incident wave injected into the medium. The analysis is done for different intensity of ro-

Figure 11: Velocity and attenuation factor for different values of ω and rotational frequency

Figure 12: Velocity and attenuation factor for different values of ω and rotational frequency

Figure 13: Velocity and attenuation factor for different values of ω and rotational frequency

tational frequency of the medium about x_2 -axis. It can be seen from figures that velocity and attenuation factor of these waves remain unaffected by rotational frequency of the medium. Velocity of these waves is directly proportional to the frequency of incident wave propagating through the medium.

Figure 12 and Figure 13 are representing the velocity profile and attenuation factor profile against frequency of the wave propagating into the medium, rotational effect of medium is also been considered in these figures. It can be seen that the velocity of both the waves behave alike against the considered parameters. Velocity of Quasi-transverse waves is directly proportional to frequency of incident wave and inversely proportional to the rotational frequency of the medium. If the body is set in rotation about y-axis then the absolute value of decreasing intensity of waves increases by increasing the rotational frequency of the medium for quasi-transverse waves.

6 Conclusion

In this paper, the reflection phenomenon through thermoelastic medium in presence of micro-temperature effect is studied. We apply the dual-phase lag heat conduction

model to study the thermal wave propagation through rotating solid. The coefficients, velocities and attenuation factor of reflected waves are also computed and presented graphically. Following are some major outcomes of the work

1. Rotational effect is having decreasing effect on reflected quasi longitudinal waves and having increasing effects on transverse waves for propagating against the depth of the medium.
2. Amplitude ratio of first two waves is directly proportional to thermal conductivity of the medium while the other two waves are inversely proportional to the thermal conductivity for propagating against the vertical component of distance.
3. For zero angular frequency ω of the incident wave there do not exist any reflected waves so the velocity of the reflected waves is also zero.
4. In the case of longitudinal waves the velocity of the medium remains unaffected by different values of rotational frequency, but the velocities of the quasi transverse wave decreases efficiently by increasing the rotational frequency of the medium for different values of angular frequency.
5. Attenuation factor of the quasi longitudinal waves remains unaffected by the different values of rotational frequency, while for the quasi transverse waves, attenuation factor increases by increasing the rotational frequency of the medium.

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