Research Article

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Vibration characteristics analysis of composite floating rafts for marine structure based on modal superposition theory

<https://doi.org/10.1515/rams-2021-0043>

received May 22, 2021; accepted June 16, 2021

Abstract: Composite materials have become a research hotspot in the field of vibration and noise reduction for their high strength, high damping, and other outstanding mechanical properties in recent years. In this paper, the effect of laminated materials on the dynamic performance of floating rafts is investigated based on modal superposition theory using the finite element method. The detailed derivation of the modal superposition theory was made, and taking T700 fiber-reinforced composite material as an example the damping effect of the floating raft structure in three cases was discussed: whether the composite material is laid or not, different layup angles, and different layup positions. The research shows that laying composite materials can improve damping effect of the floating raft and the changes in both the laying angle and the laying position will affect its dynamic performance. Moreover, the damping performance of the structure is inversely correlated with its stiffness within a certain range.

Keywords: composite floating rafts, vibration analysis, modal superposition theory, underwater vehicle

1 Introduction

The vibration and mechanical noise generated by the operation of the marine structure's main engine equipment is the main source of the marine structure's vibration and noise. It is of great significance to effectively suppress the vibration and noise for the comfort and quiet of the ship [1–3]. In fact, since the vibration and noise generated by operating equipment cannot be eliminated completely, vibration isolation technology [4–7] is widely used in the field of vibration protection. The vibration isolation is to use the vibration isolation device to isolate the vibration excitation source and the isolated equipment. The dynamic characteristics of the coupling can be weakened and the energy transfer between the two will be reduced.

Under the continuous research of experts and scholars and the continuous attempts of engineers, the floating raft vibration isolation system [8–10] was created to solve the defects of single-layer vibration isolation system with low stiffness and double-layer vibration isolation system with a large mass. The structure in which multiple vibration excitation source devices are fitted to a common mass which is connected to the installation base through vibration isolators is called the floating raft. Floating raft vibration isolation can reduce both the radiation noise caused by mechanical equipment and the impact of external vibration on the equipment. Researchers have conducted extensive study for the vibration characteristics and vibration isolation performance of the floating raft vibration isolation system. Li et al. [11] analyzed the dynamic performance of a floating raft vibration isolation system composed of constrained damping beams using the analytical method and the power flow method and proved that the floating raft vibration isolation system can provide higher vibration isolation. Lei et al. [12] adopted the inverse resonance method and joint simulation technique to optimize the position of the particle dampers on the basis of the double-layer floating raft vibration isolation system with

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particle damping technology, which greatly improved the damping effect. Niu *et al.* [13] proposed an active floating raft isolation system, in which active actuators were inserted between machines and the intermediate mass as well as the intermediate mass and the foundation. In order to improve the vibration isolation performance of the conventional floating raft system, Sun *et al.* [14] introduced power absorbers into the floating raft structure and verified their vibration isolation effect by numerical simulation.

The traditional floating raft is a steel structure and the low damping of the steel material structure limits the structural design, and hence, its vibration damping effect cannot be further improved. At the same time, steel floating rafts have significant resonance peaks at specific frequencies, which is a high risk in practical applications. In contrast, composite materials have high specific strength, high specific modulus, high damping, and high designability. Therefore, the composite material can achieve better damping effect when applied to the floating raft, which has good application prospects. Experts and scholars have carried out a lot of research work on the structural composition of composite materials [15–18]. In the study of vibration characteristics of composite structures, Zhang *et al.* [19] showed that the rational design of composite structures can improve their dynamic performance. Salim *et al.* [20] analyzed the thermal buckling and free vibration of composite-laminated cylindrical shells controlled by shape memory alloy wires. Amoozgar *et al.* [21] investigated the effectiveness of a resonance avoidance concept for composite rotor blades characterized by tensile-torsional elastic coupling. Zuo *et al.* [22] studied the static and free vibration of laminates using wavelet finite element method and higher-order plate theory. Kamarian *et al.* [23] performed natural frequency analysis on nonuniform nanocomposite beams with surface-bonded piezoelectric layers and concluded that the use of thick piezoelectric layers does not necessarily increase the intrinsic frequency of the structure. Mahi *et al.* [24] proposed a new theory of hyperbolic shear deformation using a set of boundary characteristic orthogonal polynomials associated with the Ritz method to accurately calculate the free-vibration frequency of composite plates. Madhav and Jawaid [25] carried out a comparative theoretical experimental study of composite synthetic caisson bases. Li *et al.* [26,27] used the Jacobi-Ritz method to analyze the free vibration of composite laminated structure and compared the results with finite element calculations to verify the accuracy and reliability of the method. Pang *et al.* [28] calculated the free vibration of composite laminated shells by Rayleigh-Ritz method based on multi-segmentation strategy and first-order shear deformation theory. In addition, there are some experts and scholars who focus on

studying the physical and chemical properties [29–32] of composites with different compositions.

Zhang and Selim [33] combined HSDT with one of the element-free IML-RITZ methods for the first time to study the free vibration behavior of carbon nanotube reinforced functionally graded thick laminated composite plates. The resulting effective material properties of the CNT-reinforced composite are estimated by a detailed and straightforward Mori–Tanaka approach. The numerical results have been compared with the literature showing excellent agreement.

Due to the large and complex structure of the marine structures, it is difficult to achieve the solution by analytical method. It is the numerical calculation method that is an effective means to solve this problem. Finite element method and statistical energy method are the common numerical calculation methods for analyzing the vibration characteristics of marine structures. Since finite element method requires dividing a large number of cells, and the dispersion error caused by dividing cells becomes larger as the frequency increases, cell-based methods are usually limited to the low-frequency range. Therefore, using finite element method to numerically calculate the low-frequency vibration is reasonable and feasible. Bodaghi *et al.* [34] investigated the modeling and vibration control of rectangular plates with integrated polycrystalline NiTi shape memory alloy under dynamic loading. Alhijazi *et al.* [35] explained the theory and the method of modeling and simulation for natural fiber composites. Dehghan and Braradaran [36] performed a vibration characterization of a composite laminate with a certain thickness by using the hybrid finite element method. Lee and Hwang [37] studied nonlinear transient behaviors of carbon nanotube/fiber/polymer composite spherical shells by finite element method. Zhang *et al.* [38] studied the dynamic mechanical response of cross-ply composite laminates under transverse low-velocity impact based on a finite element model of continuum damage mechanics. Thai *et al.* [39] proposed a novel finite element formulation for static and vibration analysis of laminated plates using a new higher-order shear deformation theory. Li *et al.* [40–42] solved the free vibration of composite structure under different boundary conditions by semi-analytical method. Youzera *et al.* [43] studied the damped and nonlinear forced vibrations of a three-layer symmetrical laminated beam and derived its frequency response curve.

Especially, Pang *et al.* [44] used a semi-analytic method to study the free vibration of a hyperboloid rotating shell under arbitrary boundary conditions. In this manuscript, the doubly curved shells of revolution are divided into their segments in the meridional

direction, and the theoretical model for vibration analysis is formulated by applying Flügge's thin shell theory. And then the natural frequencies of the doubly curved shells are obtained by using the Rayleigh–Ritz method. The results compared with those of the numerical method prove the good validity of the method.

In summary, the main differences between this paper and the existing literature can be attributed to the following points. First of all, the paper innovatively uses nickel alloy as the inner mat and T700 carbon fiber-reinforced material, and the resulting laminated composite material is used in the floating raft structure. Second, in order to reduce the complexity of the calculation and improve the rapidity and effectiveness of the analysis, this paper adopts the modal superposition method to calculate the dynamic characteristics of the composite floating raft. Finally, the paper compares and analyzes the differences between the steel and composite floating rafts in terms of intrinsic frequency and dynamic response and suggests possible relevant applications for this composite floating raft. The process is shown in graphical abstract.

2 Theoretical formulations

The modal superposition method is the classical theoretical method for finite element calculations, which can solve most of the dynamics problems [45,46]. The modal superposition method transforms the equations of motion in the physical coordinate system into a series of single-degree-of-freedom equations in the modal coordinate system [47] by performing a modal coordinate transformation of the equations of motion in the physical coordinate system and decoupling the equations of motion using information such as modal frequencies and vibration patterns and then obtains the structural response in the original physical coordinate system by solving the modal coordinate response and combining them. The analysis of the dynamics of the marine structure's floating raft system (Figure 1) using the modal superposition method is a fast and effective means. And the process is derived as discussed in the following.

2.1 Modal coordinate transformation

The floating raft structure is discretized into a finite number of units, and the vibration differential equation of each unit system is listed as follows:

Figure 1: Schematic diagram of floating raft structure.

$$M\ddot{x} + C\dot{x} + Kx = f, \quad (1)$$

where M is the mass matrix, C is the damping matrix, K is the stiffness matrix, x is the displacement vector, and f is the external force vector. The mass matrix, damping matrix, and stiffness matrix are all real symmetric matrices.

The transformation relation between physical and modal coordinates is defined as follows:

$$x = \Phi q, \quad (2)$$

where x denotes the displacement vector in the physical coordinate system, Φ denotes the transformation matrix, q denotes the displacement vector in the modal coordinate system, and the equation is substituted into the above equation to obtain the vibration differential equation in the modal coordinate system as follows:

$$M\Phi\ddot{q} + C\Phi\dot{q} + K\Phi q = f. \quad (3)$$

In order to achieve the decoupling of the equation, both sides of the equation (3) equal sign are multiplied by Φ^T .

$$\Phi^T M \Phi \ddot{q} + \Phi^T C \Phi \dot{q} + \Phi^T K \Phi q = \Phi^T f. \quad (4)$$

Thus, the decoupled forms of the vibration differential equation are obtained.

$$\Phi^T M \Phi = M_p, \quad (5)$$

$$\Phi^T C \Phi = C_p, \quad (6)$$

$$\Phi^T K \Phi = K_p, \quad (7)$$

where M_p , C_p , and K_p are diagonal matrices, and for example, the form of the mass array is as follows:

$$M_p = \begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{pmatrix}. \quad (8)$$

At this point equation (4) can be rewritten as follows:

$$M_p \ddot{q} + C_p \dot{q} + K_p q = f_p, \quad (9)$$

where $f_p = \Phi^T f$. Equation (9) is a system of differential equations of order n . Each coordinate is independent of each other and can be solved independently.

2.2 Solution of coordinate transformation matrix

To solve the coordinate transformation matrix Φ , the transformation operation is performed on equation (5) and substituted into equation (7).

$$\Phi^T = M_P \Phi^{-1} M^{-1}, \quad (10)$$

$$K\Phi = M\Phi M_P^{-1} K_P. \quad (11)$$

Since M_P and K_P are diagonal matrices, we can combine them into one diagonal matrix.

$$\Lambda = M_P^{-1} K_P = \begin{pmatrix} \frac{k_1}{m_1} & 0 & \cdots & 0 \\ 0 & \frac{k_2}{m_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{k_n}{m_n} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}. \quad (12)$$

Separate the coordinate transformation matrix Φ by columns.

$$\Phi = [\phi_1, \phi_2, \cdots, \phi_n]. \quad (13)$$

Each element of Φ separated by column is a column vector, and equation (13) is substituted into equation (11) according to the block matrix multiplication method.

$$[K\phi_1, K\phi_2, \cdots, K\phi_n] = [\lambda_1 M\phi_1, \lambda_2 M\phi_2, \cdots, \lambda_n M\phi_n]. \quad (14)$$

The characteristic equations are as follows:

$$|K - \lambda_n M| = 0. \quad (15)$$

The polynomial is solved and the eigenvalues λ_r and the corresponding eigenvectors ϕ_r are obtained. And they are sorted in the order from small to large. The natural frequency ω_i and the corresponding main mode ϕ_i of the floating raft can be obtained by the equation as follows:

$$\omega_i = \sqrt{\lambda_i}. \quad (16)$$

2.3 Master mass normalization processing

Based on the previous discussion, the equation (17) is easy to conclude.

$$\begin{aligned} M_P = \Phi^T M \Phi &= \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_n^T \end{bmatrix} M [\phi_1, \phi_2, \cdots, \phi_n] \\ &= \begin{pmatrix} \phi_1^T M \phi_1 & \phi_1^T M \phi_2 & \cdots & \phi_1^T M \phi_n \\ \phi_2^T M \phi_1 & \phi_2^T M \phi_2 & \cdots & \phi_2^T M \phi_n \\ \vdots & \vdots & \ddots & \vdots \\ \phi_n^T M \phi_1 & \phi_n^T M \phi_2 & \cdots & \phi_n^T M \phi_n \end{pmatrix} \\ &= \begin{pmatrix} \phi_1^T M \phi_1 & 0 & \cdots & 0 \\ 0 & \phi_2^T M \phi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_n^T M \phi_n \end{pmatrix} \\ &= \begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{pmatrix}. \end{aligned} \quad (17)$$

It is clear that $\phi_i^T M \phi_j = 0$ holds when $\lambda_i \neq \lambda_j$. Combining with equation (14), we get $\phi_i^T K \phi_j = 0$ holds, so for any two vibrational modes, there is orthogonality for both the mass matrices M and the stiffness matrix K .

To facilitate the solution of the vibration differential equation as in equation (9), it is customary to let the coefficient of the first term be 1. In order to distinguish, letting the Φ here be Φ_N .

$$\begin{aligned} M_P = \Phi_N^T M \Phi_N &= \begin{bmatrix} \phi_{N1}^T \\ \phi_{N2}^T \\ \vdots \\ \phi_{Nn}^T \end{bmatrix} M [\phi_{N1}, \phi_{N2}, \cdots, \phi_{Nn}] \\ &= \begin{pmatrix} \phi_{N1}^T M \phi_{N1} & 0 & \cdots & 0 \\ 0 & \phi_{N2}^T M \phi_{N2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_{Nn}^T M \phi_{Nn} \end{pmatrix} = E, \\ \phi_{Nr}^T M \phi_{Nr} &= 1. \end{aligned} \quad (18)$$

Introduce the coefficient matrix μ_r , such that $\phi_{Nr} = \mu_r \phi_r$ holds.

$$\mu_r^2 m_r = 1, \quad (20)$$

$$\Phi_N = [\phi_{N1}, \phi_{N2}, \cdots, \phi_{Nn}] = [\mu_1 \phi_1, \mu_2 \phi_2, \cdots, \mu_n \phi_n]. \quad (21)$$

Combining equations (7) and (12),

$$K_P = \Phi_N^T K \Phi_N = \Phi_N^T M \Phi_N \Lambda = M_P \Lambda = \Lambda. \quad (22)$$

Substitution of $x = \Phi_N q_N$ into equation (1) enables a variation of the original vibration differential equation.

$$\Phi_N^T M \Phi_N \ddot{q}_N + \Phi_N^T C \Phi_N \dot{q}_N + \Phi_N^T K \Phi_N q_N = \Phi_N^T f. \quad (23)$$

Taking the equations (5)–(7) and (22) into the equation above,

$$\ddot{q}_N + C_P \dot{q}_N + \Lambda q_N = f_{NP}. \quad (24)$$

where q_N denotes regular coordinates and f_{NP} denotes regular excitation force.

2.4 Solution of the decoupling equation

When each coefficient matrix of the vibration equation can be diagonalized, its component form can be written as follows:

$$\ddot{q}_i + \frac{c_i}{m_i} \dot{q}_i + \frac{k_i}{m_i} q_i = \frac{f_i}{m_i}, \quad (25)$$

where $\frac{k_i}{m_i} = \omega_{ni}^2$. Let $\frac{c_i}{m_i} = 2\xi_i \omega_{ni}$ and then $\xi_i = \frac{c_i}{2\omega_{ni} m_i} = \frac{c_i}{2\sqrt{m_i k_i}}$. Equation (25) can be written in the following form:

$$\ddot{q}_{Ni} + 2\xi_i \omega_{ni} \dot{q}_{Ni} + \omega_{ni}^2 q_{Ni} = f_{Ni}. \quad (26)$$

The solution of this equation is that

$$q_{Ni}(t) = e^{-\xi_i \omega_{ni} t} \left[q_{Ni}(0) \cos \omega_{di} t + \frac{\dot{q}_{Ni}(0) + \xi_i \omega_{ni} q_{Ni}(0)}{\omega_{di}} \sin \omega_{di} t + \frac{1}{\omega_{di}} \int_0^t f_{Ni}(\tau) e^{-\xi_i \omega_{ni} (t-\tau)} \sin \omega_{di} (t-\tau) d\tau \right], \quad (27)$$

Figure 2: Finite element model of composite laminate.

where $\omega_{di} = \sqrt{1 - \xi_i^2} \omega_{ni}$. In order to obtain the vibration response of the floating raft in the physical coordinate system, a certain deformation of the transformation relation is required.

$$M_N^{-1} \Phi_N^T M x = M_N^{-1} \Phi_N^T M \Phi_N q_N. \quad (28)$$

Because of $M_N^{-1} = E$ and $\Phi_N^T M \Phi_N = M_N$, the above equation can be deformed to $q_N = \Phi_N^T x$. The system response of the floating raft in the physical coordinate system can be derived by equation $x = \Phi_N q_N$.

3 Numerical results and discussion

3.1 Method validation

Take HM-S Graphite/Epoxy square composite laminate in ref. [48] as an example. Its side length is 406 mm, thickness is 4.06 mm, and layup mode $[0^\circ]$, using simple support boundary condition, is simulated with S4R four-node unit. The material parameters are shown in ref. [48], and the established finite element model is shown in Figure 2. Vibration modal analysis was performed on this composite laminate and the results were compared with the literature [49,50] as shown in Table 1 comparison of natural frequency of composite laminate.

From Table 1, it can be seen that the first 10th-order natural frequencies of composite laminates are in good agreement with the results of finite element calculations in the literature, and the maximum error does not exceed 0.6%. In addition, the maximum deviation of the calculated results from the classical layer ensemble damping theory (CLDT) and the discrete layer damping theory (DLDT) considering the interlayer shear resistance does

Table 1: Comparison of natural frequency of composite laminate

n	Natural frequency (Hz)			
	Results	Ref. [48]	CLDT [49]	DLDT [50]
1	131.84	131.88	129.2	128.8
2	162.13	162.21	161.9	161.5
3	237.55	237.66	240.8	124.2
4	361.62	361.85	368.5	367.3
5	504.27	504.26	496.8	490.8
6	521.81	521.36	516.8	510.6
7	530.53	531.07	541.2	538.8
8	563.50	562.35	562.8	556.3
9	642.58	640.6	647.8	640.3
10	741.53	742.72	780.4	772.3

not exceed 5%, which is within the acceptable error range. Therefore, it is effective and feasible to calculate the vibration characteristics of composite laminated structures by this method.

3.2 Vibration characteristics analysis of floating raft structure

According to the above theoretical research, this section takes the ship cabin section and floating raft vibration isolation system as the research object. Taking single equipment operation as an example, the influence of laying of composite materials on vibration and noise reduction effect of the floating raft structure is discussed with the help of ABAQUS.

As shown in Figure 3, the cabin is 6 m long and 7.2 m wide, with a 1.3 m high and 1 m wide pedestal set symmetrically on both sides of the cabin, and a 5.4 m long, 4 m wide, and 0.34 m high floating raft vibration isolation device installed on the pedestal, which are discretized with S4R and S3R grids for a total of 58,017 elements. Vibration isolators with vertical stiffness $k = 500 \text{ N}\cdot\text{mm}^{-1}$ are used to connect the floating raft to the pedestal. Simple-supported boundary conditions are used for the cabin section, and a concentrated force ($F = 1 \text{ N}$) is applied

at the center of the floating raft to assess the vibration acceleration of the lower ten points of the floating raft.

Where not stated, cabin sections are made of marine steel material with $\rho = 7,850 \text{ kg} \cdot \text{m}^{-3}$, $E = 2.1 \times 10^{11} \text{ Pa}$, $\mu = 0.3$. Floating raft structure is the main object. The materials of it are divided into steel and composite materials, of which thicknesses are all 0.015 m. The compositional form of the composite material is discussed in the following. The nickel alloy material [51] with material parameters $\rho = 8,240 \text{ kg} \cdot \text{m}^{-3}$, $E = 2.0 \times 10^{11} \text{ Pa}$, $\mu = 0.285$ is used as the lining to ensure the overall strength. The T700-12K carbon fiber material is bonded to both sides of the nickel alloy material in an ideal way. The parameters of the composite are shown in the Table 2.

The modal analysis of the floating raft is first performed. As we know, there is a need to balance computational accuracy and computational efficiency when performing structural dynamic finite element analysis, and the same is true for calculations based on the modal superposition method. As the modal order of the structure increases, most of the higher-order modes are local modes. Since the lower-order principal modes contribute the majority to the structural response, the higher-order local modes do not have a significant effect on the response. So, the calculation of the modal order need not be too high, and it is necessary to perform a modal truncation of the calculation. In order to improve the

Figure 3: Computational model: (a) geometric models, (b) concentrated force, and (c) boundary condition.

Table 2: The parameters of composite material

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	μ_{12}	ρ (kg·m ⁻³)
281	13.8	9.7	4.8	0.25	1678

Table 3: Modal analysis results of composite floating rafts and steel floating rafts

n	Natural frequency (Hz)	
	Composite material	Steel
1st-order torsional mode	22.009	21.789
1st-order bending mode	43.722	42.715
1st-order bending and torsion combined mode	60.561	58.432
2nd-order bending mode	96.184	103.14

computational efficiency, several calculations were performed for this example, and it was concluded that only the first 120 orders of modalities need to be calculated for

this floating raft structure to meet the need of modal analysis and the computational accuracy requirements.

The work of mechanical equipment first affects the floating raft structure, and its inherent frequency is of great importance to the equipment and the floating raft itself. The single floating raft structure is discretized into 17,988 cells and the Lanczos solver is used to calculate the first 150 orders of modalities of the structure. Comparing the modalities of composite floating raft and steel floating raft structures, the composite floating raft is made of 0.009 m thick nickel alloy plate with 6 layers of unit thickness of 0.001 m of T700-12K carbon fiber composite material in a [0°] layup combination. What can we learn from Table 3 and Figure 4 is that the natural frequency of the floating raft structure is related to the material used. The 1st-order torsional intrinsic frequency of the composite floating raft is 22.009 Hz and the 1st-order torsional intrinsic frequency of the steel floating raft is 21.789 Hz. Similarly, in the 1st-order bending, 1st-order bending-torsion combination, and 2nd-order bending mode, there is a law that the natural frequency of the composite floating raft structure is greater than the natural frequency of the steel floating raft. So, from an

Figure 4: The modal of composite floating raft: (a) 1st-order torsional mode, (b) 1st-order bending mode, (c) 1st-order bending and torsion combined mode, and (d) 2nd-order bending mode.

Figure 5: Comparison of mean square acceleration response of composite and steel floating rafts.

Figure 6: Comparison of acceleration response at the maximum response point of composite and steel floating rafts.

engineering point of view, the natural frequency of the composite material can be designed to avoid the working frequency of the machinery and equipment, thus avoiding resonance, which can cause damage to equipment and structures or even extreme situations.

Figure 5 shows the presence or absence of the degree of influence of composite layup on the vibration and noise reduction performance of the floating raft. The structural mechanical noise caused by the vibration of the power unit is the main component of the underwater low-frequency radiated noise, and the structural noise is formed by the pulsation of air pressure due to the vibration of solid structures. The existence of floating rafts is precisely to reduce the low-frequency vibration generated by the operation of mechanical equipment, so the analysis should focus on the vibration response in the low-frequency region. It can be derived from Figure 5

that, in the low-frequency region, the trends of the vibration acceleration curves of the composite floating raft and the steel floating raft matched well, and the peak frequencies shifted back somewhat. In addition, it can be found that the composite floating raft has a significant reduction at the peak of the steel floating raft, especially in the frequency range of 50–70 Hz and 90–120 Hz, and the vibration damping performance is greatly improved. At the same frequency, the maximum peak value of composite floating raft can be reduced by more than 30% compared to steel floating raft. Beside, the results of vibration acceleration comparison of maximum response points on steel floating rafts before and after laying composite materials are shown in Figure 6. In the range of 10–90 Hz, the vibration damping effect of composite floating raft is worse than steel floating raft, but in the frequency range of 90–140 Hz, the vibration response of the composite

Figure 7: Diagram of pavement: (a) step view and (b) stacked view.

floating raft is smaller than that of the steel floating raft at most frequency points, and the highest peak of the response is also reduced. In general, the damping effect of the composite floating raft at the point of maximum response is satisfactory.

Figure 8: Influence of layup angle on the vibration damping performance.

Considering that each ply of the composite is orthogonally anisotropic, the difference in ply angle will lead to the change of composite properties [52,53]. As we know, 0° is the fiber direction, the highest strength of this layering direction, 90° is perpendicular to the fiber direction, the lowest strength. Here the composite layup diagram shown in Figure 7 is drawn with [0°/60°/90°] layup as an example.

Given the good designability of the laminated structure, the effect of composite layup angle on structural properties is explored. This section considers the floating raft structure of T700-12K carbon fiber composite with nickel alloy plate at three angles of [30°/30°/30°], [60°/60°/60°], and [90°/90°/90°]. And to ensure the strength, the laminates are composed of 0.009 m nickel alloy plate and 6 layers of carbon fiber reinforcement with a thickness of 0.001 m in a single layer. The results are calculated and compared with the mean square vibration acceleration response of the floating raft structure with [0°] pavement for analysis.

Figure 9: Influence of laying position on vibration damping performance.

It can be learnt from Figure 8 that the vibration acceleration curves of the composite floating raft at the three layup angles match well with the trend of the vibration acceleration curves of the composite floating raft with $[0^\circ]$ layup; beside, the damping effect of the floating raft structure at different laying angles is also different. Among the three laying methods, the best laying effect is in the form of $[90^\circ/90^\circ/90^\circ]$. According to the theory described above, the material strength is highest in the $[0^\circ]$ pavement direction and lowest in the $[90^\circ]$ pavement direction. Therefore, theoretically, the material strengths of the three types of pavements $[30^\circ/30^\circ/30^\circ]$, $[60^\circ/60^\circ/60^\circ]$, and $[90^\circ/90^\circ/90^\circ]$ are weakened in turn, and their damping effects are also enhanced in turn. The calculation results do show that $[90^\circ/90^\circ/90^\circ]$ paving method has the best vibration damping performance. So in this structure, the conclusion that the smaller the stiffness the better the damping performance [54] is valid.

Floating raft vibration isolation system can be divided into two parts from the structural composition: floor and faceplate. The panel in direct contact with the equipment and the pedestal is the faceplate, and the supporting role between the two layers of panels becomes the floor. In order to analyze the influence of composite material laying position on the vibration damping performance of floating raft, it is proposed to discuss the three cases of laying only the faceplate, laying only the floor, and laying both the faceplate and floor with composite material. The composite material is composed of a single layer nickel alloy plate and six layers of carbon fiber reinforced material, and the layup angle is $[45^\circ/90^\circ/45^\circ]$. The thickness of the single layer nickel alloy plate is 0.009 m, and the thickness of the single layer carbon fiber reinforced material is 0.001 m. Response analysis was performed on the floating raft, and the mean square vibration acceleration results from the three cases were compared with the result of the steel floating raft. From Figure 9, it can be found that all three laying cases can reduce the peak acceleration of the steel floating raft structure. Further, in the frequency range of 10–60 Hz, there is not much difference between them. In the frequency range of 70–100 Hz, the damping effect of floating rafts with only floor structure is better than that of composite materials with only faceplates. And only the floating raft with faceplate-laying composite resonates near 118 Hz, amplifying the vibration response. In a comprehensive view, it is still the best vibration damping performance of the floating raft with composite materials on the floor and faceplate.

Based on the above discussion, it can be concluded that the use of composite materials for floating raft structures can enhance the vibration damping effect of floating

rafts. Compared to steel floating rafts, composite floating rafts have less structural mass and higher natural frequency, which can significantly reduce the vibration peak of steel floating rafts; also, changing the laying angle of the composite material can change the vibration damping performance of the composite floating raft. At the same time, the different locations of the composite materials will also affect the overall vibration damping effect of the floating raft, in which the best vibration damping performance is achieved when the floors and faceplates are all covered with composite materials.

4 Conclusion

In this paper, based on the modal superposition theory, structural dynamics analysis is carried out for the composite floating raft structure by the software. First, the current research status of floating raft structures, composite materials, and finite element methods is briefly introduced. And then the modal superposition theory is derived in detail. Finally, in the low-frequency range, the influence of the composite material on the floating raft structure is analyzed. The conclusions are as follows:

- (1) The use of composite materials can increase the natural frequency of the floating raft, reduce the resonance peak of the floating raft, and also enhance the vibration damping performance to a certain extent.
- (2) Changing the layering angle has a certain effect on the damping effect of the floating raft, and the smaller the structural stiffness, the better the damping effect.
- (3) The structure will have different vibration damping effects due to the different areas of composite materials laying, and the best vibration damping effect is achieved when the composite materials are laid on the ribs and panels of the floating raft.
- (4) The composite floating raft can be used for efficient vibration damping of ships, underwater vehicle, marine engineering, and even aviation equipment.
- (5) The problems such as the influence of the number of composite layers, the type of fibers and matrix, and the influence of composite joining mode on the vibration damping performance of the floating raft structure still need further study.

Acknowledgements: This research was supported by China National Intellectual Property Administration and Harbin Engineering University.

Funding information: This study was funded by National Natural Science Foundation of China (U2006229), National key Research and Development program (2016YFC0303406), Key Research, Development program of Shandong Province (2019JZZY010125, 2020CXGC010701, 2020CXGC010702), Southern Marine Science and Engineering Guangdong Laboratory (Zhuhai) (311021013).

Author contributions: Five authors have made irreplaceable work on this paper. Jisi Huang provided theoretical support and research ideas. Yang Tang analyzed the data and complete the corresponding chapter. Haichao Li modified and improved the manuscript. Fuzhen Pang provided necessary guidance. Yuxuan Qin performed the simulation.

Conflict of interest: Authors state no conflict of interest.

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