Research Article

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Dynamics of a rotating hollow FGM beam in the temperature field

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Abstract: Dynamic responses and vibration characteristics of a rotating functionally graded material (FGM) beam with a hollow circular cross-section in the temperature field are investigated in this paper. The material properties of the FGM beam are assumed to be temperature-dependent and vary along the thickness direction of the beam. By considering the rigid-flexible coupling effect, the geometrically nonlinear dynamic equations of a hub–FGM beam system are derived by employing the assumed modes method and Lagrange's equations. With the high-order coupling dynamic model, the effect of temperature variations under two different laws of motion is discussed, and the free vibration of the system is studied based on the first-order approximate coupling model. This research can provide ideas for the design of space thermal protection mechanisms.

Keywords: dynamics, rotating FGM beam, hollow circular cross-section, temperature field

1 Introduction

Flexible beam structures are widely used in the field of aerospace. These components often work in an extremely high- or low-temperature environment for a long time. The components made of the isotropic materials and traditional composites gradually cannot meet the requirements of actual working conditions, and so advanced materials such as functionally graded materials (FGM) gradually replace them. With both good mechanical strength and thermal insulation properties, FGMs have

been widely used in aerospace and other fields, and research on mechanical behaviors of FGM has drawn the attention of many scholars.

Hui [1] studied the effects of shear loads on the vibration and buckling of a typical antisymmetric cross-laminated cylindrical thin plate under combined loads. Aminbaghai et al. [2] investigated the effect of torsional warping of the FGM beam on elastic-static behavior, and they used the transfer matrix method to obtain the finite element equations. Paul and Das [3] studied the vibration behavior of prestressed FGM beams and discussed the effect of different FGMs on frequencies. Li et al. [4], Li and Zhang [5] and Dong et al. [6] studied the dynamics of rotating FGM beams based on the rigid-flexible coupling dynamic theory and discussed the frequency veering and mode interactions of the flexible beam. Oh and Yoo [7] proposed a dynamic model, which can be used to analyze the frequencies of rotating FG blades. Yang and He [8] combined the re-modified couple stress theory and the refined zigzag theory to model the functionally graded (FG) sandwich microplates. In their investigation, the vibration and buckling analysis of two types of FG microplates are discussed. Lee and Hwang [9] studied the geometrical nonlinear transient behavior of carbon nanotube/fiber/polymer composite (CNTFPC) spherical shells containing a central cutout, and the results indicated that an appropriate CNT ratio and curvature are important for improving the nonlinear dynamic properties. Parida and Mohanty [10] used the finite element method to develop the dynamic equations of a skew FG plate based on the high order shear deformation theory (HOSDT). Shen et al. [11] studied the dynamic properties of piezoelectric coupled laminated fiber-reinforced cylindrical shells considering the transverse shear effect. The results show that this method is more effective than the finite element method. Zhou et al. [12] developed the three-dimensional dynamic model of the rotating FG cantilever beam based on the Timoshenko beam theory. In their work, the nonlinear coupling deformation term, which describes the stiffening effect of the rotating cantilever FG beam, is considered. Shahmohammadi et al. [13] investigated the free vibration of the traditional, sandwich and laminated

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shells containing the FG material by the isogeometric B3-spline finite strip method (IG-SFSM). Dynamics of the flexible beam structure that are made of FGM are often affected by many factors, such as its shape, the type of graded materials, the external forces, etc. The thermal effect of temperature is a relatively common factor. In recent years, more and more researchers have focused on the thermodynamics of FGM beams. Attia *et al.* [14] investigated the vibration of the temperature-dependent FG plate by employing various four-variable refined plate theories. Bouchafa *et al.* [15] presented a new refined hyperbolic shear deformation theory (RHSDT) to analyze the thermoelastic bending problem of the FG sandwich plates. The influence of temperature on the stability and vibration characteristics of the prestressed sandwich beam covered with FG sheets is investigated by Chen *et al.* [16]. Zaman *et al.* [17] investigated the bending characteristics of a curved FGM piezoelectric cantilever actuator in the electric and temperature fields. The simulations indicated that the thermal load has a significant influence on the electroelastic field of the curved actuator. Based on the classical small deflection plate theory, Xing *et al.* [18] derived the governing equations of the FG plates with the temperature field. Based on Reddy's high-order shear deformation theory, Bisheh *et al.* [19] studied the influence of parameters such as temperature change and FG distribution pattern on the natural frequency of large-amplitude vibration of FG-GRC laminated plates.

Dehrouyeh-Semnani [20] compared the thermal vibration of FGM beams on simplified boundary conditions with that on original boundary conditions and investigated two different temperature types. Jiang *et al.* [21] studied the vibration behavior of composite beams in a thermal environment, but they only discussed the flap-wise vibration and neglected the chord-wise vibration. Varmazyari and Shokrollahi [22] studied the elastic-plastic deformation of rotating FG cylinders used in the strain gradient theory. Based on the first-order shear deformation theory, Kashkoli *et al.* [23] investigated the time-dependent thermoelastic creep problem of the FG thick-walled cylinder and gave the theoretical solution for this problem. Combining Donnell's shell theory, von Kármán nonlinearity terms, the circumferential condition in an average sense, and three-state solution form of deflection, and applying the Galerkin procedure, Nam *et al.* [24] studied the nonlinear deflection torsional buckling problem of the FG carbon nanotube orthogonally reinforced composite cylindrical shells. Li *et al.* [25] investigated the effect of muggy environment on natural frequencies and critical speed of composite thin-walled beams. Based on the higher-order shear deformation beam theory, Shabanlou *et al.* [26]

studied the vibration characteristics of FGM beams in a thermal environment. Ghadiri and Shafiei [27] studied the dynamics of Timoshenko microbeams made of FGM under four different temperature distributions in a thermal environment. Asadi and Beheshti [28] gave a comprehensive parameter analysis on the dynamics of FG composite beams in a thermal environment. Azadi [29] investigated the dynamic analysis of FG beams in a thermal environment. Azimi *et al.* [30] studied the thermomechanical vibrations of rotating axially FG beams. Ghadiri and Jafari [31] proposed an analytical method to study the vibrations of FG beams with a tip mass in a thermal environment. Bich *et al.* [32] analyzed nonlinear vibration and dynamic buckling of FG annular shells in thermal environments. Li *et al.* [33] studied the influence of temperature on vibrations and buckling behaviors of composite beams, and it was assumed that the temperature varies along the thickness at a constant value. Shahrjerdi and Yavari [34] investigated the free vibration analysis of FG nanocomposite beams under a thermal environment. Khosravi *et al.* [35] studied the influence of temperature on the vibration behavior of rotating composite beams based on the Timoshenko beam theory, where the beam was reinforced by employing carbon nanotubes; so it may cause instability and is more sensitive to temperature.

In the above studies, the cross-sections of the FGM beams are usually rectangle, tapered, or trapezoid, while the study on the FGM beam with circular cross-section is quite rare. There are many studies on the nonstatic circular shell and circular disks that rotate around their longitudinal axis. Peng and Li [36] analyzed the thermoelastic problem of a rotating FG hollow circular disk and proposed a new analytical method, which can be used to study steady thermal stresses. Dai and Dai [37] used a semi-analytical approach to investigate the displacement and stress fields of the rotating FGM hollow disk in temperature fields. Huang *et al.* [38] studied the free vibration of a rotating axially FG beam rotating around its longitudinal axis and discussed the effect of axially distributed FGMs on frequencies, critical speeds, and mode shapes. It should be noted that the rigid-flexible coupled dynamics of rotating FG beams with a hollow circular cross-section in the temperature field has not been reported in the open literature.

In this paper, the dynamic responses and free vibrations of rotating FG beams with a hollow circular cross-section in the temperature field are studied. The material properties are assumed to be temperature-dependent and vary along the thickness direction of the beam. By employing the assumed modes method and Lagrange's equations, the governing equations of motion of an FG beam with

hollow circular cross-section attached to a rotating rigid hub are derived in Section 2. The validation of the present dynamic model is shown in Section 3.1. The dynamic responses of the rotating beam driven by an external torque are shown in Section 3.2. The dynamic responses of the beam with the prescribed law of motion are shown in Section 3.3. Free vibrations of the beam rotating at constant angular velocities are discussed briefly in Section 3.4. Some conclusions based on the simulation results are given in Section 4.

2 Dynamic model of the system

Figure 1 shows the schematic of the hub-beam system in which the flexible beam is attached to the rigid body and the displacement field of an arbitrary point on the beam axis. An inertial coordinate system $OXYZ$ and a floating coordinate system $oxyz$ are defined in Figure 1, respectively. θ is the rotating angle of the hub.

Figure 2 shows the geometry of the FG beam with a hollow circular cross-section. The length and density of the beam are L and $\rho(r)$, respectively; the modulus of elasticity is $E(r)$; the outer radius of the beam is r_b ; and the inner radius is r_a .

Figure 1: Schematic of the hub-beam system and the displacement field of an arbitrary point on the beam axis.

The material properties are assumed to vary along the thickness direction of the FG beam with a power-law distribution:

$$P(r) = (P_c - P_m) \left(\frac{r - r_a}{r_b - r_a} \right)^N + P_m, \quad (1)$$

where $P(r)$ can be replaced by $\rho(r)$, $E(r)$, the coefficient of heat conduction $K(r)$, and the coefficient of thermal expansion $\alpha_T(r)$; P_c and P_m are the material parameter of ceramics and metal, respectively.

It is assumed that the temperature varies along the thickness direction of the beam, and the one-dimensional form of the steady-state heat conduction is

$$-\frac{1}{r} \frac{d}{dr} \left[rK(r) \frac{dT(r)}{dr} \right] = 0. \quad (2)$$

The thermal Dirichlet boundary conditions are written as

$$T(r_a) = T_m, \quad T(r_b) = T_c, \quad (3)$$

thus,

$$T(r) = T_m + \frac{T_c - T_m}{C} \int_{r_a}^r \frac{1}{rK(r)} dr, \quad (4)$$

where

$$C = \int_{r_a}^{r_b} \frac{1}{rK(r)} dr. \quad (5)$$

The dependency of Young's modulus and the expansion coefficient varies with the temperature T as follows:

$$P = P_0(P_{-1}/T + 1 + P_1T + P_2T^2 + P_3T^3), \quad (6)$$

where P_0 , P_{-1} , P_1 , P_2 , and P_3 are relevant coefficients of temperature.

According to Figure 1, the position vector $\mathbf{r}_p$ can be expressed as

Figure 2: Geometry of the FG beam with a hollow circular cross-section.

$$\mathbf{r}_p = (a + x + u_x)\mathbf{i} + (y + u_y)\mathbf{j} + (z + u_z)\mathbf{k}, \quad (7)$$

where

$$u_x = w_1 + w_{cy} + w_{cz} + w_{dy} + w_{dz}, u_y = w_2, u_z = w_3, \quad (8)$$

in which w_1 is the axial displacement, $w_{cy} = -(1/2)\int_0^x (\partial w_2/\partial \xi)^2 d\xi$ and $w_{cz} = -(1/2)\int_0^x (\partial w_3/\partial \xi)^2 d\xi$ are the axial shrinkage of the beam caused by the transverse displacement and flapwise displacement, respectively, and $w_{dy} = -yw'_2$ and $w_{dz} = -zw'_3$ are the axial displacements caused by the rotation of the cross-section.

According to equation (7), the velocity vector can be obtained as

$$\dot{\mathbf{r}}_p = [\dot{u}_x - (y + u_y)\dot{\theta}]\mathbf{i} + [(a + x + u_x)\dot{\theta} + \dot{u}_y]\mathbf{j} + \dot{u}_z\mathbf{k}, \quad (9)$$

thus, the kinetic energy of the system can be written as

$$E_{\text{kinetic}} = \frac{1}{2}J_{\text{oh}}\dot{\theta}^2 + \frac{1}{2}\int_V \{\rho(r)[[\dot{u}_x - (y + u_y)\dot{\theta}]^2 + [(a + x + u_x)\dot{\theta} + \dot{u}_y]^2 + \dot{u}_z^2]\}dV. \quad (10)$$

Neglecting the energy caused by shear deformation, the potential energy of the system can be expressed as

$$E_{\text{potential}} = \frac{1}{2}\int_V (\sigma_x \varepsilon_x) dV = \frac{1}{2}\int_V [E(r, T)\varepsilon_x^2] dV. \quad (11)$$

The normal strain of any point can be written as

$$\varepsilon_x = \frac{\partial w_1}{\partial x} - y \frac{\partial^2 w_2}{\partial x^2} - z \frac{\partial^2 w_3}{\partial x^2}. \quad (12)$$

According to Figure 2, $y = r \sin \alpha$ and $z = r \cos \alpha$. Thus, the potential energy can be written as

$$\begin{aligned} E_{\text{potential}} &= \frac{1}{2}\int_V \left[E(r, T) \left(\frac{\partial w_1}{\partial x} - y \frac{\partial^2 w_2}{\partial x^2} - z \frac{\partial^2 w_3}{\partial x^2} \right)^2 \right] dV \\ &= \frac{1}{2}\int_0^L \int_0^{2\pi} \int_{r_a}^{r_b} \left[r E(r, T) \left(\frac{\partial w_1}{\partial x} - r \sin \alpha \frac{\partial^2 w_2}{\partial x^2} - r \cos \alpha \frac{\partial^2 w_3}{\partial x^2} \right)^2 \right] dr d\alpha dx = \pi \int_0^L \int_{r_a}^{r_b} \left[r E(r, T) \left(\frac{\partial w_1}{\partial x} \right)^2 \right] dr dx \\ &\quad + \frac{\pi}{2} \int_0^L \int_{r_a}^{r_b} \left[r^3 E(r, T) \left(\frac{\partial^2 w_2}{\partial x^2} \right)^2 \right] dr dx + \frac{\pi}{2} \int_0^L \int_{r_a}^{r_b} \left[r^3 E(r, T) \left(\frac{\partial^2 w_3}{\partial x^2} \right)^2 \right] dr dx. \end{aligned} \quad (13)$$

Employing the assumed modes method to approximate variables, the axial deformation w_1 , transverse deformation w_2 , and the flapwise deformation w_3 can be expressed, respectively, as follows:

$$\begin{cases} w_1 = \Phi_x(x)\mathbf{q}_1(t) \\ w_2 = \Phi_y(x)\mathbf{q}_2(t) \\ w_3 = \Phi_z(x)\mathbf{q}_3(t), \end{cases} \quad (14)$$

where $\Phi_x(x)$, $\Phi_y(x)$, and $\Phi_z(x)$ are modal function vectors related to longitudinal vibration, transverse bending vibration, and flapwise bending vibration of the FGM beams, respectively. Thus, we can obtain

$$w_{cy} = -\frac{1}{2}\mathbf{q}_2^T \mathbf{H}_1(x) \mathbf{q}_2, \quad (15)$$

$$w_{cz} = -\frac{1}{2}\mathbf{q}_3^T \mathbf{H}_2(x) \mathbf{q}_3, \quad (16)$$

where $\mathbf{H}_1(x)$ and $\mathbf{H}_2(x)$ are coupled shape functions and can be expressed as

$$\mathbf{H}_1(x) = \int_0^x \Phi_y'^T(\xi) \Phi_y'(\xi) d\xi, \quad (17)$$

$$\mathbf{H}_2(x) = \int_0^x \Phi_z'^T(\xi) \Phi_z'(\xi) d\xi. \quad (18)$$

Then, we can obtain

$$\begin{cases} u_x = \Phi_x \mathbf{q}_1 - \frac{1}{2}\mathbf{q}_2^T \mathbf{H}_1 \mathbf{q}_2 - \frac{1}{2}\mathbf{q}_3^T \mathbf{H}_2 \mathbf{q}_3 \\ \quad - r \sin \alpha \Phi_y' \dot{\mathbf{q}}_2 - r \cos \alpha \Phi_z' \dot{\mathbf{q}}_3 \\ u_y = \Phi_y \mathbf{q}_2 \\ u_z = \Phi_z \mathbf{q}_3. \end{cases} \quad (19)$$

Taking the first time derivative of components in equation (19), yields

$$\begin{cases} \dot{u}_x = \Phi_x \dot{\mathbf{q}}_1 - \mathbf{q}_2^T \mathbf{H}_1 \dot{\mathbf{q}}_2 - \mathbf{q}_3^T \mathbf{H}_2 \dot{\mathbf{q}}_3 \\ \quad - r \sin \alpha \Phi_y' \dot{\mathbf{q}}_2 - r \cos \alpha \Phi_z' \dot{\mathbf{q}}_3 \\ \dot{u}_y = \Phi_y \dot{\mathbf{q}}_2 \\ \dot{u}_z = \Phi_z \dot{\mathbf{q}}_3. \end{cases} \quad (20)$$

Let $\mathbf{q} = [\theta \ \mathbf{q}_1^T \ \mathbf{q}_2^T \ \mathbf{q}_3^T]^T$ be the generalized coordinate vector, then the virtual work by temperature can be written as

$$\delta W_T = \int_V [E(r, T)(-\alpha_T(r, T)\Delta T)\delta \varepsilon_x]dV = \mathbf{Q}_T^T \delta \mathbf{q}, \quad (21) \quad \mathbf{M}_{33} = \mathbf{M}_2 + \mathbf{M}_4 + 2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{H}_1 \mathbf{q}_2 \cdot \mathbf{q}_2^T \mathbf{H}_1)]drdx, \quad (30)$$

where $\Delta T = T(r) - T_0$, T_0 is the reference temperature, and

$$\mathbf{Q}_T^T = [0 \quad \mathbf{Q}_{T1}^T \quad \mathbf{Q}_{T2}^T \quad \mathbf{Q}_{T3}^T]^T, \quad (22)$$

$$\mathbf{Q}_{T1} = - \int_V [E(r, T)\alpha_T(r, T)(T(r) - T_0)\Phi_x'^T]dV, \quad (23)$$

$$\mathbf{Q}_{T2} = - \int_V [E(r, T)\alpha_T(r, T)(T(r) - T_0)\Phi_y'^T]dV, \quad (24)$$

$$\mathbf{Q}_{T3} = - \int_V [E(r, T)\alpha_T(r, T)(T(r) - T_0)\Phi_z'^T]dV. \quad (25)$$

By employing Lagrange's equations of the second kind,

$$\frac{d}{dt} \left(\frac{\partial E_{\text{kinetic}}}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial E_{\text{kinetic}}}{\partial \mathbf{q}} = - \frac{\partial E_{\text{potential}}}{\partial \mathbf{q}} + \mathbf{Q}_\tau + \mathbf{Q}_T. \quad (26)$$

The rigid-flexible coupling dynamic equations of the system can be obtained as

$$\begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} & \mathbf{M}_{14} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{M}_{23} & \mathbf{M}_{24} \\ \mathbf{M}_{31} & \mathbf{M}_{32} & \mathbf{M}_{33} & \mathbf{M}_{34} \\ \mathbf{M}_{41} & \mathbf{M}_{42} & \mathbf{M}_{43} & \mathbf{M}_{44} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\mathbf{q}}_1^T \\ \ddot{\mathbf{q}}_2^T \\ \ddot{\mathbf{q}}_3^T \end{bmatrix} = \begin{bmatrix} Q_\theta \\ \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \mathbf{Q}_3 \end{bmatrix} + \begin{bmatrix} \tau \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{Q}_{T1} \\ \mathbf{Q}_{T2} \\ \mathbf{Q}_{T3} \end{bmatrix}, \quad (27)$$

where

$$\begin{aligned} \mathbf{M}_{11} = & J_{oh} + J_{cb} + 2\mathbf{S}_1 \mathbf{q}_1 + \mathbf{q}_1^T \mathbf{M}_1 \mathbf{q}_1 + \mathbf{q}_2^T (\mathbf{M}_2 + \mathbf{M}_4 - \mathbf{C}_1) \mathbf{q}_2 + \mathbf{q}_3^T (\mathbf{M}_5 - \mathbf{C}_2) \mathbf{q}_3 \\ & + \frac{\pi}{2} \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{q}_2^T \mathbf{H}_1 \mathbf{q}_2 \cdot \mathbf{q}_2^T \mathbf{H}_1 \mathbf{q}_2 + \mathbf{q}_3^T \mathbf{H}_2 \mathbf{q}_3 \cdot \mathbf{q}_3^T \mathbf{H}_2 \mathbf{q}_3)]drdx \\ & - 2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{q}_1^T \Phi_x^T \cdot \mathbf{q}_2^T \mathbf{H}_1 \mathbf{q}_2 + \mathbf{q}_1^T \Phi_x^T \cdot \mathbf{q}_3^T \mathbf{H}_2 \mathbf{q}_3)]drdx + \pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{q}_2^T \mathbf{H}_1 \mathbf{q}_2 \cdot \mathbf{q}_3^T \mathbf{H}_2 \mathbf{q}_3)]drdx, \end{aligned} \quad (28)$$

$$\mathbf{M}_{22} = \mathbf{M}_1, \quad (29)$$

$$\mathbf{M}_{44} = \mathbf{M}_3 + \mathbf{M}_5 + 2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{H}_2 \mathbf{q}_3 \cdot \mathbf{q}_3^T \mathbf{H}_2)]drdx \quad (31)$$

$$\mathbf{M}_{12} = \mathbf{M}_{21}^T = -\mathbf{q}_2^T \mathbf{M}_6^T, \quad (32)$$

$$\begin{aligned} \mathbf{M}_{13} = \mathbf{M}_{31}^T = & \mathbf{S}_2 + \mathbf{q}_1^T \mathbf{M}_6 + 2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{q}_2^T \Phi_y^T \cdot \mathbf{q}_2^T \mathbf{H}_1)]drdx \\ & - \pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{q}_2^T \mathbf{H}_1 \mathbf{q}_2 \cdot \Phi_y + \mathbf{q}_3^T \mathbf{H}_2 \mathbf{q}_3 \cdot \Phi_y)]drdx, \end{aligned} \quad (33)$$

$$\mathbf{M}_{14} = \mathbf{M}_{41}^T = 2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{q}_2^T \Phi_y^T \cdot \mathbf{q}_3^T \mathbf{H}_2)]drdx, \quad (34)$$

$$\mathbf{M}_{23} = \mathbf{M}_{32}^T = -2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\Phi_x^T \cdot \mathbf{q}_2^T \mathbf{H}_1)]drdx, \quad (35)$$

$$\mathbf{M}_{24} = \mathbf{M}_{42}^T = -2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\Phi_x^T \cdot \mathbf{q}_3^T \mathbf{H}_2)]drdx, \quad (36)$$

$$\mathbf{M}_{34} = \mathbf{M}_{43}^T = 2\pi \int_0^L \int_{r_a}^{r_b} [\rho(r)(\mathbf{H}_1 \mathbf{q}_2 \cdot \mathbf{q}_3^T \mathbf{H}_2)]drdx, \quad (37)$$

$$\begin{aligned}
Q_\theta = & -2\dot{\theta}[\mathbf{S}_1\dot{\mathbf{q}}_1 + \mathbf{q}_1^T\mathbf{M}_1\dot{\mathbf{q}}_1 + \mathbf{q}_2^T(\mathbf{M}_2 + \mathbf{M}_4 - \mathbf{C}_1)\dot{\mathbf{q}}_2 + \mathbf{q}_3^T(\mathbf{M}_5 - \mathbf{C}_2)\dot{\mathbf{q}}_3] \\
& + 2\pi\dot{\theta} \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{q}_2^T\mathbf{H}_1\mathbf{q}_2 \cdot \Phi_x\dot{\mathbf{q}}_1 + \mathbf{q}_3^T\mathbf{H}_2\mathbf{q}_3 \cdot \Phi_x\dot{\mathbf{q}}_1)]drdx \\
& + 4\pi\dot{\theta} \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{q}_1^T\Phi_x^T \cdot \mathbf{q}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \mathbf{q}_1^T\Phi_x^T \cdot \mathbf{q}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx \\
& - 2\pi\dot{\theta} \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{q}_2^T\mathbf{H}_1\mathbf{q}_2 \cdot \mathbf{q}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \mathbf{q}_3^T\mathbf{H}_2\mathbf{q}_3 \cdot \mathbf{q}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx \\
& - 2\pi\dot{\theta} \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{q}_3^T\mathbf{H}_2\mathbf{q}_3 \cdot \mathbf{q}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \mathbf{q}_2^T\mathbf{H}_1\mathbf{q}_2 \cdot \mathbf{q}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx \\
& - 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{q}_2^T\Phi_y^T \cdot \dot{\mathbf{q}}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \mathbf{q}_2^T\Phi_y^T \cdot \dot{\mathbf{q}}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx,
\end{aligned} \tag{38}$$

$$\begin{aligned}
\mathbf{Q}_1 = & -\mathbf{K}_1\mathbf{q}_1 + \dot{\theta}^2\{\mathbf{S}_1^T + \mathbf{M}_1\mathbf{q}_1 - \pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\Phi_x^T \cdot \mathbf{q}_2^T\mathbf{H}_1\mathbf{q}_2 + \Phi_x^T \cdot \mathbf{q}_3^T\mathbf{H}_2\mathbf{q}_3)]drdx\} \\
& + 2\dot{\theta}\mathbf{M}_6\dot{\mathbf{q}}_2 + 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\Phi_x^T \cdot \dot{\mathbf{q}}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \Phi_x^T \cdot \dot{\mathbf{q}}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx,
\end{aligned} \tag{39}$$

$$\begin{aligned}
\mathbf{Q}_2 = & -\mathbf{K}_2\mathbf{q}_2 + \dot{\theta}^2\{(\mathbf{M}_2 + \mathbf{M}_4 - \mathbf{C}_1)\mathbf{q}_2 \\
& + \pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{H}_1\mathbf{q}_2 \cdot \mathbf{q}_2^T\mathbf{H}_1\mathbf{q}_2 - 2\mathbf{H}_1\mathbf{q}_2 \cdot \Phi_x\mathbf{q}_1 + \mathbf{H}_1\mathbf{q}_2 \cdot \mathbf{q}_3^T\mathbf{H}_2\mathbf{q}_3)]drdx\} - 2\dot{\theta}\mathbf{M}_6^T\dot{\mathbf{q}}_1 \\
& + 4\pi\dot{\theta} \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\Phi_y^T \cdot \mathbf{q}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \Phi_y^T \cdot \mathbf{q}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3 - \mathbf{H}_1\mathbf{q}_2 \cdot \Phi_y\dot{\mathbf{q}}_2)]drdx \\
& - 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{H}_1\mathbf{q}_2 \cdot \dot{\mathbf{q}}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \mathbf{H}_1\mathbf{q}_2 \cdot \dot{\mathbf{q}}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx,
\end{aligned} \tag{40}$$

$$\begin{aligned}
\mathbf{Q}_3 = & -\mathbf{K}_3\mathbf{q}_3 + \dot{\theta}^2\{(\mathbf{M}_5 - \mathbf{C}_2)\mathbf{q}_3 \\
& + \pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{H}_2\mathbf{q}_3 \cdot \mathbf{q}_3^T\mathbf{H}_2\mathbf{q}_3 - 2\mathbf{H}_2\mathbf{q}_3 \cdot \Phi_x\mathbf{q}_1 + \mathbf{H}_2\mathbf{q}_3 \cdot \mathbf{q}_2^T\mathbf{H}_1\mathbf{q}_2)]drdx\} \\
& - 4\pi\dot{\theta} \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{H}_2\mathbf{q}_3 \cdot \Phi_y\dot{\mathbf{q}}_2)]drdx \\
& - 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(\mathbf{H}_2\mathbf{q}_3 \cdot \dot{\mathbf{q}}_2^T\mathbf{H}_1\dot{\mathbf{q}}_2 + \mathbf{H}_2\mathbf{q}_3 \cdot \dot{\mathbf{q}}_3^T\mathbf{H}_2\dot{\mathbf{q}}_3)]drdx,
\end{aligned} \tag{41}$$

in which the constant matrixes are expressed as

$$J_{cb} = \pi \int_0^L \int_{r_a}^{r_b} \{\rho(r)[2r(a+x)^2 + r^3]\} dr dx, \quad (42)$$

$$\mathbf{S}_1 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(a+x)\Phi_x] dr dx, \quad (43)$$

$$\mathbf{S}_2 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(a+x)\Phi_y] dr dx, \quad (44)$$

$$\mathbf{C}_1 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(a+x)\mathbf{H}_1] dr dx, \quad (45)$$

$$\mathbf{C}_2 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)(a+x)\mathbf{H}_2] dr dx, \quad (46)$$

$$\mathbf{M}_1 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)\Phi_x^T \Phi_x] dr dx, \quad (47)$$

$$\mathbf{M}_2 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)\Phi_y^T \Phi_y] dr dx, \quad (48)$$

$$\mathbf{M}_3 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)\Phi_z^T \Phi_z] dr dx, \quad (49)$$

$$\mathbf{M}_4 = \pi \int_0^L \int_{r_a}^{r_b} [r^3 \rho(r) \Phi_y'^T \Phi_y'] dr dx, \quad (50)$$

$$\mathbf{M}_5 = \pi \int_0^L \int_{r_a}^{r_b} [r^3 \rho(r) \Phi_z'^T \Phi_z'] dr dx, \quad (51)$$

$$\mathbf{M}_6 = 2\pi \int_0^L \int_{r_a}^{r_b} [r\rho(r)\Phi_x^T \Phi_y] dr dx, \quad (52)$$

$$\mathbf{K}_1 = 2\pi \int_0^L \int_{r_a}^{r_b} [rE(r, T) \Phi_x'^T \Phi_x'] dr dx, \quad (53)$$

$$\mathbf{K}_2 = \pi \int_0^L \int_{r_a}^{r_b} [rE(r, T) \Phi_y''^T \Phi_y''] dr dx, \quad (54)$$

$$\mathbf{K}_3 = \pi \int_0^L \int_{r_a}^{r_b} [rE(r, T) \Phi_z''^T \Phi_z''] dr dx. \quad (55)$$

Equation (27) provides a three-dimensional HOC dynamic model of a rotating FGM beam with a hollow circular cross-section. Based on the HOC dynamic model, the complex rigid-flexible coupling dynamics of the system can be studied. It can be seen that some of the terms in equations (28, 30, 31, 33–41) are either single underlined or double underlined, or both. These underlined terms are generated by the second-order coupling terms w_{cy} and w_{cz} . If the double underlines are ignored, the HOC dynamic model can be reduced to the traditional FOAC dynamic model.

3 Dynamics and vibration characteristics

3.1 Comparison studies

In order to ensure the validity of the present model, natural frequencies of a nonrotating beam are compared with the results obtained by employing ABAQUS software. Table 1 shows the first four frequencies of a nonrotating homogeneous beam in different temperature fields. It can be seen that the results obtained by the present method are well consistent with those obtained by employing ABAQUS. Table 3 shows the first six frequencies of a nonrotating FGM beam ($N = 1$). In ABAQUS software, the

Table 1: The first four frequencies of a nonrotating homogeneous beam in different temperature fields

Mode	$T = 100 \text{ K}$			$T = 300 \text{ K}$			$T = 500 \text{ K}$		
	Present	ABAQUS	$\Delta R(\%)^*$	Present	ABAQUS	$\Delta R(\%)$	Present	ABAQUS	$\Delta R(\%)$
1	1.4951	1.4951	0.0000	1.4584	1.4584	0.0000	1.4295	1.4295	0.0000
2	9.3693	9.3687	0.0064	9.1395	9.1389	0.0066	8.9587	8.9581	0.0067
3	26.234	26.230	0.0152	25.590	25.586	0.0156	25.084	25.080	0.0159
4	51.405	51.390	0.0292	50.145	50.130	0.0299	49.152	49.138	0.0285

* $\Delta R = (\text{present} - \text{ABAQUS})/\text{present} \times 100\%$.

Table 2: Temperature-dependent coefficients for ceramic and metal

Materials	Properties	P_0	P_{-1}	P_1	P_2	P_3
SUS304	E_m (Pa)	201.04×10^9	0	3.08×10^{-4}	-6.53×10^{-7}	0
	$\alpha_{T,m}$ ($1 \cdot K^{-1}$)	1.23×10^{-5}	0	8.09×10^{-4}	0	0
	ρ_m ($kg \cdot m^{-3}$)	8,166	0	0	0	0
Si_3N_4	E_c (Pa)	348.43×10^9	0	-3.1×10^{-4}	2.16×10^{-7}	-8.95×10^{-11}
	$\alpha_{T,c}$ ($1 \cdot K^{-1}$)	5.87×10^{-6}	0	-9.1×10^{-4}	0	0
	ρ_c ($kg \cdot m^{-3}$)	2,370	0	0	0	0

Table 3: The first six frequencies of a nonrotating FGM beam ($N = 1$)

	1st	2nd	3rd	4th	5th	6th
Present	0.97017	6.0799	17.0235	33.3580	55.1405	82.3656
ABAQUS	0.97026	6.0800	17.022	33.349	55.115	82.306
$\Delta R(\%)$	-0.0093	-0.0016	0.0088	0.0270	0.0462	0.0724

material properties of FGM cannot be set directly with a continuous variation law, so multiple layers of the FGM beam along the radial direction are shown in Figure 3. Table 2 shows the temperature-dependent coefficients of ceramic and metal which are used in this paper. As seen in Table 3, the results calculated by the present method also agree well with those calculated by ABAQUS. Therefore, the accuracy of the present FGM beam model is guaranteed.

3.2 Dynamics of the rotating beam driven by an external torque

Neglecting the radius of the hub by choosing $a = 0$ m, the geometric parameters of the FGM beam are $L = 5$ m,

Figure 3: Multiple layers of an FGM beam along the thickness in ABAQUS.

$r_a = 0.005$ m and $r_b = 0.01$ m. It is assumed that the FGM beam is made up of the ceramic Si_3N_4 and the metal SUS304 with thermal conductivity “Ye et al.” is not cited as an author of ref. 8. Please indicate any changes that are required here. and $K_m = 16.32$ W · mk⁻¹, respectively.

An external torque is applied on the hub to drive the FGM beam rotating around the hub and is defined as

$$\tau(t) = \begin{cases} \tau_0 \sin\left(\frac{2\pi t}{t_s}\right) & 0 \leq t \leq t_s \\ 0 & t \geq t_s, \end{cases} \quad (56)$$

where $t_s = 2$ s is the cycle time.

In this paper, the dynamics of the FGM beam in the flapwise direction are neglected because the rotation has little effect on the responses of the beam. Figures 4 and 5 show comparisons of the responses of the system from the FOAC model and the HOC model when $\tau_0 = 10$ N · m, where the effect of temperature is neglected. The value of the gradient index is $N = 1$. The results show that the rotating angle and angular velocity of the rigid hub, and the tip transverse deformations and velocities of the FGM beam from the two models are basically consistent, except for the tip axial deformations and the tip axial velocities. As shown in Figure 4(a), the rigid hub rotating angle increases to 0.13 rad when $t = 2$ s, and then stops and swings. In Figure 4(b), the rotating angular velocity increases first and then decreases; later it also stops and swings. Figure 5(a) shows that the tip maximum transverse deformation of the FGM beam is about 0.035 m. As shown in Figure 5(c) and (d), the axial deformations and velocities of the beam from the two

Figure 4: Dynamic responses of the hub-beam system ($\tau_0 = 10 \text{ N}\cdot\text{m}$): (a) the rotating angle of the hub and (b) the angular velocity of the hub.

Figure 5: Dynamic responses of the FG beam ($\tau_0 = 10 \text{ N}\cdot\text{m}$): (a) the tip transverse deformation, (b) the tip transverse velocity, (c) the tip longitudinal deformation, and (d) the tip longitudinal velocity.

Figure 6: Dynamic responses of the FG beam ($\tau_0 = 50 \text{ N}\cdot\text{m}$): (a) the tip transverse deformation and (b) the angular velocity.

models are not matched. It can be found that the results calculated by the HOC model are more stable than those by the FOAC model without the high-order small quantities associated with w_{cy} . The high-order small quantities have little effect on the transverse deformation, and due to the axial deformation of the flexible beam being a small amount, the effect of the high-order small quantities on the axial deformation is highlighted. It can also be seen that during the cycle of motion, the axial deformation reaches the maximum at $t = 1 \text{ s}$, but it is very small compared to the transverse deformation.

Figure 6 shows the comparisons of the tip transverse deformations and rotating angular velocities from the HOC model and the FOAC model. The tip maximum transverse deformation can reach approximately 0.175 m , the angular velocity of the rigid hub increases first and then decreases, and the maximum value is about $0.65 \text{ rad}\cdot\text{s}^{-1}$. After the driving torque is removed, the beam stops rotating and swings in the final position. When the FOAC model is used to solve the problem, it is found that sharp divergence occurs after 0.33 s , and both the tip deformation of the FGM beam and the angular velocity of the rigid body increase to infinity, which is inconsistent with the actual situation. For the FGM beam with a low rotating speed, the FOAC model can meet the precise requirement and has better computational efficiency than the HOC model. However, for the FGM beam with relatively high rotating speed, the HOC model should be used to replace the FOAC model due to the advantage in accuracy.

To study the effect of thermal effect on the responses of the FGM beam, three different temperature differences between inner and outer surfaces of the beam are selected

as follows: (1) $T_c = 0 \text{ K}$, $T_m = 0 \text{ K}$; (2) $T_c = 100 \text{ K}$, $T_m = 0 \text{ K}$; and (3) $T_c = 200 \text{ K}$, $T_m = 0 \text{ K}$. The reference temperature is $T_0 = 0 \text{ K}$.

Figure 7 shows the influence of temperature differences on the dynamics of the system. It can be found that the change in the temperature difference has almost no effect on the rotating angle. As the temperature difference increases, the rotating angular velocity also increases. The tip deformation of the FGM beam decreases slightly when the temperature difference increases. The reason is that the temperature is distributed along the radial direction of the FGM beam and is symmetrical on the whole. Therefore, the thermal loads caused by the temperature are also symmetrical; as a result, the thermal loads cancel each other out, but the axial loads caused by the temperature difference still exist. In addition, the tensile stiffness of the FGM beam also becomes smaller, and as the temperature difference increases, the axial deformation of the beam also increases. It can be found that both the transverse and axial velocities of the FGM beam increase with the increase of the temperature difference, and the influence on the axial velocity is more obvious. It can also be found that when the driving torque is removed, both the transverse deformation and velocity of the FGM beam oscillate at the equilibrium position.

Then, three different cases of constant temperature field are selected as follows: (1) $T_c = T_m = 0 \text{ K}$; (2) $T_c = T_m = 100 \text{ K}$; and (3) $T_c = T_m = 200 \text{ K}$. As shown in Figure 8(a), it can be found that the transverse deformation of the FGM beam becomes smaller with the increase of the ambient temperature, and the degree of deformation is smaller than that in Figure 7(c). Figure 8(c) and (d) shows the changes in the axial deformation and velocity of the

Figure 7: Influence of the temperature difference on the dynamics of the system: (a) rotating angle, (b) the angular velocity, (c) the transverse deformation, (d) the transverse velocity, (e) the axial deformation, and (f) the axial velocity.

Figure 8: Influence of temperature variation on the dynamic response of the FG beam: (a) the tip transverse deformation, (b) the tip transverse velocity, (c) the tip axial deformation, and (d) the tip axial velocity.

FGM beam with the ambient temperature, respectively. It can be observed that both the deformation and velocity increase with temperature. Compared with Figure 7(e) and (f), the degree of deformation and velocity have also increased. The reason is that as the temperature increases, the energy of the FGM beam increases and the vibration accelerates during rotation. Because of the symmetrical structure, the transverse deformation of the FGM beam is less affected by temperature than the axial deformation.

3.3 Dynamics of system with prescribed law of motion

The parameters of the FGM beam are still the same as those in Section 3.2, the radius of the rigid body is still

zero, and the large overall motion law of the system is assumed to be

$$\dot{\theta} = \begin{cases} \omega_0 t/t_s - \omega_0 \sin(2\pi t/t_s)/2\pi, & 0 \leq t \leq t_s \\ \omega_0, & t \geq t_s. \end{cases} \quad (57)$$

Figure 9 shows the effect of the functionally gradient indices on the tip deformation of the FGM beam. It can be seen from Figure 9 that as the gradient index increases, the tip transverse and axial deformation of the FGM beam are both slower to reach the maximum amplitude, and both the maximum amplitudes also increase. When the rotating angular velocity becomes uniform, the FGM beam swings at the equilibrium position and the amplitude of the swing also increases. The reason is that as the gradient index increases, the ceramic component proportion of the FGM beam decreases while the metal component proportion increases. As a result, the FGM beam becomes more flexible and is easier to deform.

Figure 9: Influence of the gradient index on the tip deformations of the FG beam: (a) the tip transverse deformation and (b) the tip axial deformation.

Figure 10: The tip dynamic responses of the FGM beam under different temperature differences: (a) the transverse deformation and (b) the transverse velocity.

As in the previous section, we set three different temperature differences. The temperature on the metal side is always set to be 0 K. The temperature differences are set as $T_{cm} = 0$ K, 100 K, 200 K. Figure 10 shows the effect of temperature differences on the transverse deformation and velocity of the FGM beam. It can be found that the transverse deformation decreases with the increase of the temperature difference, while the transverse velocity increases inapparently. The larger the temperature difference, the more significant the reduction of deformation. Figure 11 shows the phase diagram of the transverse

deformation and velocity of the FGM beam under different temperature differences. It can be observed that the vibration times increase rapidly with the increase of the temperature difference. Figure 12 shows the effect of temperature differences on the axial deformation and velocity of the FGM beam. As shown in Figure 12, it can be found that the effect of the temperature difference on the tip axial deformation and velocity of the FGM beam is obvious, as the temperature difference increases, the axial deformation, and velocity both increase. Figure 13 shows the phase diagram of the axial deformation of the

Figure 11: Phase diagram of the transverse deformation of the FGM beam under different temperature differences: (a) $T_{cm} = 0\text{ K}$, (b) $T_{cm} = 100\text{ K}$, and (c) $T_{cm} = 200\text{ K}$.

Figure 12: The tip dynamic responses of the FGM beam under different temperature differences: (a) the axial deformation and (b) the axial velocity.

Figure 13: Phase diagram of the axial deformation of the FGM beam under different temperature differences: (a) $T_{cm} = 0$ K, (b) $T_{cm} = 100$ K, and (c) $T_{cm} = 200$ K.

Figure 14: Influence of temperature variation on the dynamic responses of the FGM beam: (a) tip transverse deformation and (b) tip transverse velocity.

FGM beam under different temperature differences. As shown in the figure, the vibration times increases and the vibration becomes more pronounced as the temperature difference increases.

Figure 14 shows the tip transverse deformation and velocity of the FGM beam in three different temperature fields: $T_c = T_m = 0$ K, 100 K, 200 K. It can be found that as the ambient temperature increases, the tip transverse deformation of the beam decrease, and the velocity increases, but the increase of the velocity is not obvious.

3.4 Numerical simulation procedure

In this section, the free vibration of the FGM beam in the temperature field will be analyzed. The rotating speed is

set as $\dot{\theta} = 100 \text{ rad} \cdot \text{s}^{-1}$. Figure 15 shows variations of the first three frequencies of the FGM beam with the ambient temperature. It can be observed that as the temperature increases, the frequencies become smaller. The reason is that as the temperature increases, the stiffness of the beam decreases. Figure 16 shows the first four normalized bending and tensile mode shapes of the FGM beam when $T = 300$ K and $\dot{\theta} = 100 \text{ rad} \cdot \text{s}^{-1}$.

Figure 17 shows the changes in the first three frequencies of the rotating FGM beam under different temperature differences between the inner and outer surfaces. And the inner temperature is $T_{cm} = 300$ K, the rotating speed is $\dot{\theta} = 200 \text{ rad} \cdot \text{s}^{-1}$. It can be found that the natural frequencies decrease slightly with the increase of the temperature difference.

Figure 15: Effects of temperature on the first three damped frequencies of the FG beam when $\dot{\theta} = 100 \text{ rad} \cdot \text{s}^{-1}$: (a) first damped frequency, (b) second damped frequency, and (c) third damped frequency.

Figure 16: The first four mode shapes of the FG beam when $T = 300$ K and $\dot{\theta} = 100$ rad $\cdot$ s $^{-1}$: (a) first modal bending mode, (b) first modal stretching mode, (c) second modal bending mode, (d) second modal stretching mode, (e) third modal bending mode, (f) third modal stretching mode, (g) fourth modal bending mode, and (h) fourth modal stretching mode.

Figure 17: Influence of temperature difference on the first three damped frequencies of the beam when $\dot{\theta} = 200 \text{ rad} \cdot \text{s}^{-1}$ and $T_{cm} = 300 \text{ K}$: (a) first damped frequency, (b) second damped frequency, and (c) third damped frequency.

4 Discussion

The dynamic modeling and vibration analysis of the rotating FGM beam with a hollow circular cross-section in the temperature field are investigated in this paper. The influences of the temperature difference between the inner and outer surfaces of the beam, the ambient temperature, and the functional gradient index on the dynamic responses of the rotating FGM beam under two laws of motion are discussed. Finally, the free vibration of the rotating FGM beam is discussed. The main conclusions are as follows:

- For the flexible FGM beam system with low rotating speed, the results obtained from the FOAC model are almost the same as those obtained from the HOC model. But for the system with a high rotating speed, the

convergence performance of the numerical results from the HOC model is better than the FOAC model.

- Thermal loads will induce oscillation when the rotating FGM beam is in the temperature field. For rotating FGM beams with a radial temperature gradient, the influences of the temperature on the transverse deformation and velocity of the beam are small, while those on the axial deformation and velocity of the beam are large.
- The variation of the functional gradient index has a significant influence on the dynamic responses of the FGM beam, so it is possible to select the proper functional gradient index for the structure to meet the requirements in engineering applications such as suppression of the unwanted vibration.
- Since only the modulus of elasticity and the coefficient of thermal expansion of the FGM beam are assumed to be

temperature-dependent, it is found that the temperature has little influence on the natural frequencies of the beam based on the present model.

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