Research Article

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Axial and lateral stiffness of spherical self-balancing fiber reinforced rubber pipes under internal pressure

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Abstract: Fiber reinforced rubber pipes are widely used to transport fluid at locations requiring flexible connections in pipeline systems. The spherical self-balancing fiber reinforced rubber pipes with low stiffness are drawing attention because of their vibration suppression performance under high internal pressure. In this paper, a theoretical model is proposed to calculate the axial stiffness and lateral stiffness of spherical self-balancing fiber reinforced rubber pipes. The inhomogeneous anisotropy of the reinforced layer and the nonlinear stress-strain relationship of the reinforced fiber are considered in the model. The accuracy of the model is verified by experimental results. Theoretical calculation finds that both the axial and lateral stiffness are influenced significantly by the key structural parameters of the pipe (the axial length, the circumferential radius at the end, the meridional radius, and the initial winding angle). The stiffness can be reduced remarkably with optimal meridional radius and initial winding angle, without any side effect on the self-balance of the pipe. The investigation methods and results presented in this paper will provide guidance for design of fiber reinforced rubber pipes in the future.

Keywords: fiber reinforced rubber pipe; axial stiffness; lateral stiffness; composite membrane; Timoshenko beam; winding angle

1 Introduction

Fiber reinforced rubber pipes are widely used to transport fluid at locations requiring flexible connection in pipeline system. They protect the pipeline from damage caused by mechanical vibration and shock, proved extremely useful in marine engineering. The pipes are composed of inner rubber layer, reinforced layer and outer rubber layer, as shown in Figure 1. The reinforced layer is fabricated by multi-layer helically wound unidirectional fabric. With the development of vibration isolation technology for marine engineering, the vibration isolation performance of the float raft system has been greatly improved in recent years. Instead, the propagation of vibration along the pipeline system has been increasingly prominent, becoming one of the key challenges for vibration control of ships [1]. The spherical self-balancing rubber pipes with low stiffness are drawing attention because of their good vibration suppression performance under high internal pressure in pipeline systems. In practical applications, rubber pipes are required to be self-balancing under internal pressure. A rubber pipe with poor balance will produce additional deformation and unbalanced force, damaging the pipelines connected upstream and downstream under high internal pressure. Therefore, the stiffness characteristics of spherical fiber reinforced rubber pipes with good balance needs to be carefully investigated.

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Figure 1: The cross-section of the fiber reinforced rubber pipe.
The stiffness of fiber reinforced pipes has been investigated in depth. Jaszak et al. [2] analyzed the stress distribution of rubber pipes under internal pressure with finite element analysis based on the Mooney-Rivlin model of the rubber material, concluding that the influence of elastic properties of the rubber layer on the stress distribution is negligible. Therefore, the stiffness is mainly determined by the geometry of the rubber pipes and the anisotropy of the reinforced layer. The axial stiffness of cylindrical self-balancing rubber pipes was investigated based on the anisotropic shell theory [3, 4]. Zhou et al. [5] calculated the axial stiffness of spherical rubber pipe based on the thin shell theory, but did not consider the anisotropy of the reinforced layer on the stiffness characteristics. Gao et al. [6] established the axial stiffness model of spherical self-balancing rubber pipes based on anisotropic membrane theory, considering the winding angle variance of rubber pipes under high internal pressure. It should be noted that the balance of pipes could be achieved by specific winding angle of the unidirectional fabric. The self-balancing angle of cylindrical pipes is near 55 degrees [7–9], while the self-balancing angle of spherical pipes is determined by the geometry of the pipe [10]. A number of investigations paid attention to the lateral stiffness of fiber reinforced pipes under internal pressure. Chen et al. [11] studied the lateral stiffness of rubber pipes with an equivalent beam model fixed at both ends. Rafiee et al. [12] established theoretical model to estimate the lateral stiffness of fiber reinforced rubber pipes based on the back-of-envelope technique.

As already known from the research above mentioned, the stiffness of fiber reinforced rubber pipes has gained sufficient attention. However, the stiffness of the spherical self-balancing fiber reinforced rubber pipes needs deeper investigations. Firstly, the comprehensive research on both the axial stiffness and the lateral stiffness is deficient. Reducing the axial stiffness alone will cause the rise of the lateral stiffness possibly and vice versa. To improve the vibration suppression performance of the rubber pipes, the axial stiffness and lateral stiffness are supposed to be optimized together. Secondly, most research is limited to cylindrical fiber reinforced pipes. For spherical pipes, the anisotropy distribution of the reinforced layer is more complicated. The spherical fiber reinforced rubber pipes are manufactured with the curing process, as shown in Figure 2. The reinforced layer is fabricated with unidirectional fabric cross winding helically at a specific initial winding angle before the rubber pipes expanded to the preset shape. The winding angle is determined by the meridian shape on the spherical pipes, while the winding angle is homogeneous on the cylindrical pipes. Therefore, the nonlinear distribution of winding angle leads to inhomogeneous anisotropy of the reinforced layer on the spherical pipes. Thirdly, the variance of the elastic modulus of the reinforce fiber is not negligible when the rubber pipe is subject to different internal pressure. Rao et al. [13] and Kumar et al. [14] found that the reinforced fiber has a nonlinear stress-strain relationship in cord/rubber composites. In the stage with small strain, the elastic modulus of the fiber changes with the strain. In the stage with intermediate strain, the elastic modulus tends to be constant. It could bring extra error in the theoretical model without considering the nonlinear stress-strain relationship of the reinforced fiber.

\[ \text{...} \]

**Figure 2:** The curing process of spherical fiber reinforced rubber pipes.

In this paper, the axial stiffness and lateral stiffness of spherical self-balancing fiber reinforced rubber pipes are investigated based on the anisotropic membrane theory and the composite Timoshenko beam theory, respectively. The inhomogeneous anisotropy of the reinforced layer and the nonlinear stress-strain relationship of the fiber are considered in the theoretical model. The key structural parameters of the spherical pipe are the axial length, the circumferential radius at the ends, the meridional radius and the initial winding angle. Influences of these parameters on the distribution of winding angle, the anisotropy of the reinforced layer, the axial stiffness and the lateral stiffness are investigated in detail. The calculated results of the axial stiffness and the lateral stiffness are in good agreement with the experiment results, proving the accuracy of the theoretical model. To improve the vibration suppression performance, the stiffness characteristics are optimized with adjusting the meridional radius and the initial winding angle. The investigation methods and results presented in this paper will provide guidance for design of fiber reinforced rubber pipes in the future.
2 Theoretical analysis

2.1 Material model of the reinforced layer

The geometric model of the spherical pipe is shown in Figure 3. A curvilinear coordinate system \((a, \beta, \gamma)\) and a Cartesian coordinate system \((x, y, z)\) are established. The geometry of the spherical pipe is simply determined by the structure vector \(H = [L, r, Ra, \theta_0]^T\), where \(L, r, Ra\) and \(\theta_0\) represent the axial length, the circumferential radius at the end, the meridional radius, and the initial winding angle, respectively.

![Figure 3: The geometric model of the spherical pipe. (a) The curvilinear coordinate system and the Cartesian coordinate system. (b) The structural parameters of the spherical pipe.](image)

The single-layer cord/rubber lamina can be regarded as orthotropic material [15]. The equivalent elastic constants of the lamina can be obtained with the following equations [16]

\[
\begin{align*}
E_1 &= E_c V_c + E_r (1 - V_c) \\
E_2 &= E_r (1 + 2V_c) / (1 - V_c) \\
\mu_{12} &= \mu_c V_c + \mu_r (1 - V_c) \\
\mu_{21} &= \mu_{12} E_1 / E_2 \\
G_{12} &= G_r + G_c (G_r - G_c) V_c
\end{align*}
\]

where \(E_c\) and \(E_r\) are the Elastic modulus of the cord and rubber, respectively; \(\mu_c\) and \(\mu_r\) are the Poisson’s ratios of the cord and rubber, respectively; \(G_c\) and \(G_r\) is the shear modulus of the cord and rubber, respectively; \(V_c\) is the volume fraction of cord. The stress-strain relations of the \(k\)th lamina in the reinforced layer are written as [17]

\[
\begin{bmatrix}
\sigma_a \\
\sigma_\beta \\
\tau_{\alpha\beta}\end{bmatrix}_k = \begin{bmatrix} Q_{11}^k & Q_{12}^k & Q_{16}^k \\
Q_{12}^k & Q_{22}^k & Q_{26}^k \\
Q_{16}^k & Q_{26}^k & Q_{66}^k\end{bmatrix} \begin{bmatrix}
\varepsilon_a \\
\varepsilon_\beta \\
\tau_{\alpha\beta}\end{bmatrix}_k
\]

where \(Q_{ij}^k\) represent the off-axis stiffness coefficients of the \(k\)th lamina, which are written as

\[
\begin{bmatrix}
Q_{11}^k & Q_{12}^k & Q_{16}^k \\
Q_{12}^k & Q_{22}^k & Q_{26}^k \\
Q_{16}^k & Q_{26}^k & Q_{66}^k\end{bmatrix} = T_k \begin{bmatrix}
Q_{11}^k & Q_{12}^k & 0 \\
Q_{12}^k & Q_{22}^k & 0 \\
0 & 0 & Q_{66}^k\end{bmatrix} T_k^T
\]

where \(Q_{ij}^k\) represent the material constants of the \(k\)th orthotropic lamina. The fiber coordinates of the lamina are written as 1 and 2, where direction 1 is parallel to the fibers and 2 is perpendicular to them. The \(Q_{ij}^k\) are defined as

\[
Q_{11}^k = \frac{E_1^k}{1 - \mu_{12}^k \mu_{21}^k}, \\
Q_{22}^k = \frac{E_2^k}{1 - \mu_{12}^k \mu_{21}^k}, \\
Q_{66}^k = \mu_{12}^k
\]

where \(E_1^k\) and \(E_2^k\) are elasticity modulus of the \(k\)th lamina in the 1 and 2 directions, respectively; \(G_{12}^k\) is shear modulus and \(\mu_{12}^k\) is the major Poisson’s ratio. \(\mu_{21}^k\) is determined by the equation \(\mu_{21}^k E_2^k = \mu_{12}^k E_1^k\). The transformation matrix \(T_k\) is

\[
T = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & -2 \sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & 2 \sin \theta \cos \theta \\
\sin \theta \cos \theta & -\sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\]

where winding angle \(\theta\) is the angle between direction 1 of the fiber and the direction \(\alpha\) of the curvilinear coordinate system, as shown in Figure 3.

![Figure 4: The winding angle of the reinforced layer.](image)

The anisotropy of the reinforced layer are closely related to the local winding angle of the fabric. Although the initial winding angle is uniform, the winding angle of expanded reinforced layer keeps changing at different positions along the meridional direction. For the curing process, the following two assumptions are proposed:

1. The cords of the fabric is not stretched during the curing process;
2. Both ends of the pipe do not rotate relatively to each other during the curing process.
Supposing the initial winding angle is $\theta_0$, the geometric relationship established based on the assumptions above-mentioned are written as

$$\frac{dy}{\cos \theta_0} = \frac{R_0 d\alpha}{\cos \theta} \quad \frac{\tan \theta_0}{r} dz = \frac{R_0 \tan \theta}{R_0 \sin \alpha} d\alpha$$

where $R_\beta$ represents the circumferential radius, as shown in Figure 3. The distribution of the winding angle $\theta$ along the meridional direction derived from equations (6) is

$$\theta = \arcsin \left( \frac{R_\beta \sin \theta_0}{r} \sin \alpha \right)$$

The structure vector $H$ has significant influence on the distribution of winding angle. Figure 5 shows that the winding angle of cord fabric will become more non-uniform along the axial direction when the initial winding angle and axial length increase, and when the circumferential radius at the end and the meridional radius decrease.

The stress-strain relationship is used to obtain the elastic constant along the axial direction of the rubber pipe:

$$\begin{align*}
E_\alpha &= \frac{Q_{11} Q_{22} - Q_{12}^2}{Q_{11}} \\
E_\beta &= \frac{Q_{11} Q_{22} - Q_{12}^2}{Q_{11}} \\
\mu_{\alpha\beta} &= \frac{\Xi_{12} \Xi_{11}}{Q_{11}} \\
G_{\alpha\beta} &= \frac{Q_{66}}{Q_{11}}
\end{align*}$$

The distribution of the elastic constants is shown in Figure 6a. At any position of the pipe, the shear modulus is greater than the meridional and latitudinal elastic modulus. At the center of the joint, the meridional elastic modulus reaches the minimum value, while the latitudinal elastic modulus reaches a maximum value. Figure 6b shows the relative variance of the elastic constants with different winding angle. The off-axis elastic constants reduce to the orthotropic properties when the winding angle equals 0 degree and 90 degrees. The shear modulus reaches the maximum when the winding angle equals 45 degrees. The stiffness matrix of the 1500D/3-type aramid cords is used in the theoretical calculation, written as follows [15].

$$Q = \begin{bmatrix}
17.8E9 & 7.06E6 & 0 \\
7.06E6 & 13.3E6 & 0 \\
0 & 0 & 2.76E6
\end{bmatrix}$$

Figure 6: (a) Distribution of elastic constants with the structure vector $H = [0.1, 0.1, 0.1, 40]$. (b) The elastic constants at variant winding angle.

The rubber pipes are reinforced by multi-layer fabric. Based on the composite Reissner shell theory, the stiffness matrix of the reinforced layer is

$$\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
A & B \\
B & D
\end{bmatrix} \begin{bmatrix}
\varepsilon^0 \\
\kappa
\end{bmatrix}$$

where $N$ and $M$ are the inplane force and bending moment, respectively; $\varepsilon^0$ is the strain in the midsurface of the reinforced layer; $\kappa$ is the curvature and torsion ratio of the
mid-surface. Each sub-matrix of the stiffness matrix is defined as [18]

$$
\begin{align*}
A_{ij} &= \sum_{k=1}^{n} (Q_{ij})_k (z_k - z_{k-1}) \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (Q_{ij})_k (z_k^2 - z_{k-1}^2) \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (Q_{ij})_k (z_k^3 - z_{k-1}^3)
\end{align*}
$$  \hspace{1cm} (11)

2.2 Analysis of the axial stiffness

The in-plane force is defined as

$$
\begin{bmatrix}
N_a \\
N_\beta \\
N_{\alpha \beta}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\begin{bmatrix}
\epsilon_a \\
\epsilon_\beta \\
\epsilon_{\alpha \beta}
\end{bmatrix} \delta + o(\delta^3) \hspace{1cm} (12)
$$

where $\delta$ is the thickness of the reinforced layer. Under the assumption of thin shell theory, the thickness of the reinforced layer is a small quantity, so the terms related to thickness are considered as high-order small quantities and are negligible in equation (12). Without considering the bending moment and shear force, the geometric equation is simplified as follows [17]

$$
\begin{align*}
\epsilon_a &= \frac{1}{A} \frac{\partial u}{\partial \alpha} + \frac{\partial A}{\partial \beta} \frac{\nu}{A B} + k_a w \\
\epsilon_\beta &= \frac{1}{B} \frac{\partial v}{\partial \beta} + \frac{\partial B}{\partial \alpha} \frac{\nu}{A B} + k_\beta w \\
\epsilon_{\alpha \beta} &= \frac{A}{B} \frac{\partial u}{\partial \beta} \frac{\nu}{A} + \frac{B}{A} \frac{\partial v}{\partial \alpha} \frac{\nu}{B}
\end{align*}
$$  \hspace{1cm} (13)

where $A, B$ are Lamé parameters and $k_a, k_\beta$ are principal curvature of the mid-surface. Combining equations (12) and (13), yields

$$
\begin{align*}
\frac{1}{A} \frac{\partial u}{\partial \alpha} + \frac{\partial A}{\partial \beta} \frac{\nu}{A B} + k_a w &= \frac{1}{\delta} \begin{bmatrix}
N_a \\
N_\beta \\
N_{\alpha \beta}
\end{bmatrix} \\
\frac{1}{B} \frac{\partial v}{\partial \beta} + \frac{\partial B}{\partial \alpha} \frac{\nu}{A B} + k_\beta w &= \frac{1}{\delta} \begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\end{align*}
$$  \hspace{1cm} (14)

The equilibrium equations [17] are

$$
\begin{align*}
\frac{\partial}{\partial \alpha} (BN_a) - \frac{\partial B}{\partial \beta} N_\beta + \frac{\partial A}{\partial \beta} N_{\alpha \beta} + \frac{\partial A}{\partial \alpha} N_\beta + \frac{\partial B}{\partial \alpha} N_{\alpha \beta} + \frac{\partial B}{\partial \beta} N_a &= 0 \\
\frac{\partial}{\partial \beta} (AN_\beta) - \frac{\partial A}{\partial \alpha} N_a + \frac{\partial B}{\partial \alpha} N_{\alpha \beta} + \frac{\partial B}{\partial \beta} N_\beta &= 0 \\
k_a N_a + k_\beta N_\beta - q_\gamma &= 0
\end{align*}
$$  \hspace{1cm} (15)

The spherical pipe is an axisymmetric shell of revolution, so the coefficients in equations (14) and (15) are

$$
\begin{align*}
A &= R_a \\
B &= R_\beta \sin \alpha \\
k_a &= \frac{1}{R_a} \\
k_\beta &= \frac{1}{R_\beta} \\
q_a &= 0 \\
q_\gamma &= p
\end{align*}
$$  \hspace{1cm} (16)

where $p$ is internal pressure. It is assumed that all the constraints and external loads are axisymmetric, and then for any point of the pipe, the equilibrium equation is established in the meridional direction as follows

$$
N_a = \frac{1}{2} p R_\beta + \frac{F_y}{2 \pi R_\beta \sin^2 \alpha} \hspace{1cm} (17)
$$

The axial stiffness of the deformation of joint body is determined by the relationship between $F_y$ and the axial deformation $y$. Combining equations (14), (15) and (17), the solutions of internal force and displacement are obtained as follows

$$
\begin{align*}
N_a &= \frac{1}{2} p R_\beta + \frac{F_y}{2 \pi R_\beta \sin^2 \alpha} \\
N_\beta &= p R_\beta - \frac{p R_\beta^2}{2 R_a} \frac{F_y}{2 \pi R_a \sin^2 \alpha} \\
\int_{-\delta}^{\delta} \sin \alpha \left[ \begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix} \sin \alpha \\
A_{11} & A_{12} & A_{22}
\end{bmatrix} \right] \frac{u}{\delta} \\
w &= \frac{R_\beta}{\delta} \left[ \begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix} \sin \alpha \\
A_{11} & A_{12} & A_{22}
\end{bmatrix} - u \cot \alpha
\end{align*}
$$  \hspace{1cm} (18)
where \( C_2 \) is an integral constant determined by boundary conditions. The boundary condition is

\[
\begin{align*}
\text{u} = 0, & \text{ while } \alpha = \frac{\pi}{2} \\
\text{u} = -y \cos \alpha, & \text{ while } \alpha = \frac{\pi}{2} - \phi
\end{align*}
\]  
(19)

Substituting equations (18) into equation (19) yields the force-displacement relationship for the axial deformation:

\[
F_y = 2\pi By - \pi p \int_0^\phi \frac{A_{11}R_{11}^2 - 2(A_{12} - A_{11})R_{11}R_n + (A_{22} - 2A_{12})R_n^2}{(A_{11}A_{22} - A_{12}^2)\sin \alpha} \, d\alpha
\]  
(20)

The influences of structural parameters are then computed on the axial stiffness, as shown in Figure 7. The axial stiffness will increase significantly when the initial winding angle increases, the meridional radius increases, the axial length decreases, and the circumferential radius at the end increases.

Aramid fiber is generally used as the cord material for large-diameter rubber pipes resistant to high pressure. The elastic modulus of aramid fiber has strong nonlinearity even in the low-stress stage, so it is necessary to experimentally obtain the variation of the elastic modulus of aramid cord with the cord stress. The distribution of stress in the reinforced layer is

\[
\sigma_a = \frac{pR_{11}}{2\delta}
\]  
\( \sigma_b = \frac{R_{11}p(2R_n - R_{11})}{2\delta R_n} \)
(21)

With the formula of off-axis stress, the stress along the direction of cords \( \sigma_1 \) is obtained as follows:

\[
\sigma_1 = \sigma_a \cos^2 \theta + \sigma_b \sin^2 \theta
\]  
(22)

where \( \theta \) is the winding angle of cords. The elastic modulus of cords under different stresses can be obtained by processing the measured force-displacement data of cords. Substituting this elastic modulus into the theoretical model of stiffness yields the stiffness characteristics of the rubber pipe under different internal pressures.

### 2.3 Analysis of the lateral stiffness

Based on the Timoshenko beam theory, a model is established to calculate the lateral stiffness of the spherical fiber reinforced pipes. The deflection equation of beam is [19]

\[
\frac{\partial^4 y}{\partial x^4} = -\frac{M}{EI} + \frac{aq}{GA}
\]  
(23)

where \( a \) is the shear coefficient, \( q \) is the load, \( EI \) is the bending stiffness, and \( GA \) is the shear stiffness. The thickness of the tube is thinner than that of the inner diameter, so the shear stress is approximately uniformly distributed in the thickness direction, and the shear coefficient \( a \) equals 1. According to Shadmehri et al. [20], the bending stiffness of a composite pipe is

\[
\langle EI \rangle = \frac{\pi}{4} \sum_{n=1}^{N} (R_{0,n}^4 - R_{1,n}^4)E_n
\]  
(24)

where \( R_{0,n} \) and \( R_{1,n} \) are the outer diameter and inner diameter of the \( n \)-th layer of fiber, respectively; \( E_n \) is the axial elastic modulus of the \( n \)-th layer, and can be calculated as follows:

\[
E_n = \frac{A_{11}A_{12} - A_{12}^2}{A_{22}(R_{0,n} - R_{1,n})}
\]  
(25)

The influences of the initial winding angle and the meridional radius are analyzed on the spatial distributions of bending stiffness and shear stiffness, as shown in Figure 8. When the initial winding angle is 43 degree, the bending stiffness and shear stiffness reach their minimum and maximum values, respectively.
solution of deflection. Based on the definition of central
difference, the difference equation deflection curve is ob-
tained:
\[
\frac{w_{i+1} - 2w_i + w_{i-1}}{h^2} = -\frac{M_i}{\langle EI \rangle_i} - \frac{q_i}{\langle GA \rangle_i}
\]
where \(h\) is the length step of difference. In this paper, the
influences of structural parameters are analyzed on the
lateral stiffness of a rubber pipe with Clamped-Clamped
boundary condition, and the calculated results are shown
in Figure 9. The result reveals that the lateral stiffness will
significantly decrease when the winding angle and axial
length increase and the circumferential radius decreases.

2.4 The self-balance of spherical fiber
reinforced rubber pipes

A rubber pipe used in ship pipeline systems is required
to have good self-balance; otherwise, when the internal
pressure changes, the pipe will produce additional defor-
mation and may damage the connected pipeline or equip-
ment. Self-balance is evaluated by the following indexes:
1) unbalanced displacement, which is defined as the ratio
of length change under the working pressure when both
ends are free; 2) unbalanced force: the reaction force under
the working pressure when both ends are fixed. The meth-
ods are derived for solving the ratio of length change and
reaction force:

(1) When both ends are free, calculate the ratio of length
change under the working pressure. Based on the
geometric relationship, the axial deformation \(y\) of
joint body is calculated as follows:
\[
y = -\frac{\mu}{\sin \alpha} \left. \sin \alpha \right|_{\alpha = \pi/2 - \varphi}
\]
It is combined with equation (20), and let the external
force in the axial direction \(F_y\) equal 0, and then the
ratio of length change can be calculated:
\[
\eta = \frac{2y}{L}
\]
(2) When both ends of the pipe are clamped, the unbal-
anced force is calculated under the working pressure.
In formula (20), let the axial deformation \(y\) be equal
to 0, and the reaction force can be obtained when both ends of the pipe body are fixed:

\[
F_y = -np \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} A_{11} R_0^2 + 2(A_{12} A_{13}) R_0^2 (A_{22} - 2 A_{21}) R_0 R_1^2 d\alpha \\
\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} A_{11} R_0^2 + A_{22} R_0^2 + 2 A_{21} R_0 R_1^2 d\alpha
\]

(29)

The approach of stiffness calculation of self-balancing fiber reinforced rubber pipes is summarized in a flow chart in Figure 10.

![Flow Chart](image)

**Figure 10:** The approach of stiffness calculation of self-balancing fiber reinforced rubber pipes.

### 3 Experiment results

Experiments are conducted to obtain the variation of the elastic modulus of aramid cord with the cord stress in section 3.1, because the elastic modulus of aramid fiber has strong nonlinearity even in the low-stress stage. The axial and lateral stiffness of four types of spherical self-balancing rubber pipes are also measured in section 3.2, and the measured results are compared with the resulted calculated with the proposed model.

There are four types of self-balanced spherical rubber pipes with different specifications but the same rated working pressure of 3.0MPa. The main structural parameters are shown in Table 1, and the model of test device and the test layout are shown in Table 2 and Figure 10, respectively.

![Experiment Setup](image)

**Figure 11:** (a) Experimental systems used to measure the axial and lateral stiffness; (b) tensile test of aramid cords; (c) axial stiffness test; (d) lateral stiffness test.
3.1 The elastic modulus of aramid cord

Table 3 shows the properties of the aramid cords used in the research. Based on the measured data of tensile test, the fitting results of the elastic modulus of cord are obtained with the least square method and are shown in Figure 12. When the cord strain is less than 0.01, the elastic modulus increases with the increase of the strain; when the strain is greater than 0.01, the elastic modulus becomes linear.

<table>
<thead>
<tr>
<th>Component</th>
<th>Property</th>
<th>Cord</th>
<th>Material</th>
<th>Aramid</th>
<th>Construction</th>
<th>1140D/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diameter</td>
<td>E₁</td>
<td>0.8 mm</td>
<td>21 GPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rubber</td>
<td>Elastic modulus</td>
<td>6 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cord/Rubber Layer</td>
<td>Thickness</td>
<td>1.1 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 Discussion

The experimental result in Figure 13 suggests that the lateral stiffness is larger than the axial stiffness of the tested rubber pipes. In order to reduce the lateral stiffness without causing a significant increase in the axial stiffness, the meridional radius and initial cord-winding angle are varied simultaneous in the theoretical while the self-balance of the rubber pipe is maintained. The problem is to reduce the maximum stiffness of the rubber pipe (i.e. the larger value in the axial stiffness and lateral stiffness) under the following conditions: 1) the internal pressure is kept at 3.0 MPa;
2) the variation of the axial length of joint body is less than 1%; 3) the unbalance force is less than the external axial force required to produce 1% change in the axial length. The constraints of optimization are summarized as

\[
\begin{align*}
\Phi &= \text{minimize}(k_{\text{axial}} \& k_{\text{lateral}}) \\
p &= 3.0 \text{MPa} \\
\eta(R_a, \theta_0) &< 0.01 \\
\Delta F(R_a, \theta_0) &< \eta L k_{\text{axial}} 
\end{align*}
\]

where \( \Phi \) is the objective function of the optimization, \( k_{\text{axial}} \) represents the axial stiffness and \( k_{\text{lateral}} \) represents the lateral stiffness.

Table 4 shows the results of structural-parameter optimization, Table 5 compares the results obtained before and after stiffness optimization, and Figure 14 shows the calculated variations of axial and lateral stiffness with different meridional radius. The data in Table 4 reveals that the optimization of structural parameters does not affect the self-balance of the original rubber pipes. The results in Figure 14 suggest that the optimal meridional radius and initial winding angle can be obtained through simultaneous adjustment of both parameters. The result in Table 5 suggests that the lateral stiffness can be reduced by 19.5% with the optimal structural parameters.

![Figure 14: a) Calculated stiffness vs meridional radius of DN50; b) calculated stiffness vs meridional radius of DN100; c) calculated stiffness vs meridional radius of DN125; d) calculated stiffness vs meridional radius of DN150.](image)

5 Conclusion

The theoretical model is proposed to calculate the axial stiffness and the lateral stiffness of spherical self-balancing fiber reinforced rubber pipes. The accuracy of the theoretical model is verified by the experiment of axial stiffness and lateral stiffness. The theoretical calculation finds that both the axial and lateral stiffness are influenced significantly by the key structural parameters of the pipe (the axial length, the circumferential radius at the end, the meridional radius, and the initial winding angle). The stiffness of the spherical self-balancing fiber reinforced rubber pipes has a remarkable decrease with optimal meridional radius and initial winding angle without any side effect on the self-balance. The investigation methods and results presented in this paper will provide guidance for design of fiber reinforced rubber pipes in the future.

References


