Research Article

Jianhui Tian*, Hongrui Zhang, Jinjuan Sun, Jialiang Wu and Guangchu Hu

Dynamic response of functionally graded plate under harmonic load with variable gradient parameters

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Abstract: This article used the strip element method to study the dynamics problems of the functionally graded plate with variable gradient parameters under the harmonic loads. The dynamic model of the functionally graded plate is established by using the strip element method, the rationality and accuracy of the theoretical results are verified by finite element method, and the displacement response under different gradient parameters is also calculated. The results show that under different gradient parameters, the displacement varies harmonically with time, and with the increase of gradient parameters, the fluctuation period of displacement with time increases continuously, and the displacement peak also gradually increases. The displacement along the thickness direction also shows the harmonic form. Through comparison, it is found that the gradient parameters have a greater impact on the dynamic response for the functionally graded plate; with the increase in the gradient parameters, the displacement response also increases, but the displacement response trend slows down.

Keywords: functionally graded plate, strip element method, harmonic load, dynamic response

1 Introduction

Due to the high specific strength and specific stiffness, and easy design, functionally graded materials (FGMs) can work under some extreme environmental conditions and are widely used in aerospace, biomedicine, petrochemical, heavy industry, and other fields [1–3]. With the development of FGMs, the research on the dynamic characteristics of FGMs becomes more and more important. In the process of dynamics research, at present, in the field of traditional FGMs research, linear parameters are generally used, which results in certain limitations in practical applications. Therefore, the dynamics of FGMs with variable gradient parameters is extremely critical and important.

Compared with other composite materials, FGMs have better mechanical properties. The key researches mainly focus on vibration, shock, and dynamic response under different external conditions and different boundary conditions by using different methods, such as theoretical solutions, numerical calculations, or experiments. Shariyat et al. [4] used finite element method and iterative algorithm to study the dynamic stress, displacement distribution, hygrothermal elastic wave propagation, and reflection response of a functionally graded hollow sphere under thermomechanical shock. Hao et al. [5] studied the nonlinear forced vibration and natural frequency of a double-bent flat shell with a rectangular sandwich FGM and verified its effectiveness using numerical simulation methods. Gupta and Talha [6] focused on the structural characteristics of FGMs plates and shells under different boundaries and environmental conditions under thermo-electromechanical loads. Bakhtiari and Kheradpisheh [7] studied the transient response of an inflatable multilayer hollow functionally graded cylinder with interlayer adhesion defects under load. This structure is widely used in aerospace structures. Bozyigit et al. [8] applied the theory of univariate shear deformation to the analysis of the free vibration and harmonic response of the multi-layer frame model.
considering the basic flexibility. Aris and Ahmadi [9] studied the nonlinear vibration and resonance analysis of truncated gradient tapered shells under harmonic excitation. Parandvar and Farid [10] established a nonlinear finite element model of the dynamic response of a FGM plate under thermal, static, and harmonic loads and studied the effect of initial conditions and static pressure on the dynamic response of the system. Najarzadeh et al. [11,12] studied the free vibration of thin plates under arbitrary load using the boundary element method. Parida et al. [13] used the finite element method to perform dynamic analysis on the simply supported beam structure and studied the effect of temperature as a function of dynamic parameters on the mechanical properties. Guo et al. [14] analyzed the modes of simply supported plates with uniform thickness and stepped thickness by using a dynamic shape function. Using a dynamic stiffness method (DSM), Kumar and Jana [15] studied the free vibration characteristics of rectangular FGM thin plates with S-FGM and E-FGM characteristics along the thickness direction, and they studied the free vibration characteristics of functionally gradient rectangular plate using DSM [16].

For functionally graded plates with variable gradient parameters, previous research work has involved the study of the thermal conduction of functionally graded plates with variable gradient parameters under heat source load [17], but the research on the dynamic performance of variable gradient parameters has not been carried out, so based on the strip element method, fixed boundary conditions are applied to study the dynamic response of functionally graded plates with variable gradient parameters. In addition, Karamia et al. [18] studied the influence of various boundary conditions and found that for all boundaries. Dhital et al. [19] used the properties of two materials to smoothly transition from one material to another for reducing thermal stress, residual stress, and stress concentration factor. These studies are very helpful for the development of the research on functionally graded plates with variable gradient parameters. And Liu et al. [20] verified the correctness of the strip element method. However, the study of functionally graded plates with variable gradient parameters under harmonic load has not been mentioned. In this article, the variable gradient parameter model will be established on the basis of the strip element method to calculate the displacement response of the functionally gradient plate under different gradient parameters and verify its rationality and accuracy, this work provides a new way for the variable gradient parameter model.

2 Establishment of dynamic model

2.1 Dynamic theory model

When studying the dynamic performance of a functionally graded plate, as shown in Figure 1, the domain is defined by \( D = (x, z) \), in which \( x \in (-\infty, +\infty), z \in (0, H) \), and \( D_p \in D, D_p \) are composed of boundaries \( B_1, B_2, B_3, \) and \( B_4 \). Since the functionally graded plate is isotropic in the \( x-y \) plane and the \( y \)-direction is infinite, simplifying the problem to the two-dimension relation with \( x \) and \( z \), there are only two displacement components \( u \) and \( v \) in \( x \)-direction and \( z \)-direction, respectively.

Without considering the internal damping of the functionally graded plate, it is assumed that the dynamic control equation of the functionally graded plate in matrix form as

\[
M \ddot{u} - L^T \sigma = Q,
\]

where \( M \) is the mass matrix, \( \dot{u} \) is the displacement vector, \( L^T \) is the differential operator matrix, \( \sigma \) is the stress vector, and \( Q \) is the load vector.

2.2 System equation

The relationship between strain and displacement in matrix form is

\[
\varepsilon = LU,
\]

where \( \varepsilon \) is the vector of the strain, defined as

![Figure 1: Functionally graded plate discrete model.](image)
\( \mathbf{e} = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{bmatrix} \). \hspace{1cm} (3)

The vector form of the displacement is
\( \mathbf{U} = \begin{bmatrix} u \\ v \end{bmatrix} \). \hspace{1cm} (4)

where \( u \) and \( v \) are the \( x \)-direction and \( z \)-direction displacements, respectively.

The differential operator matrix is
\[
\mathbf{L} = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 \\
0 & \frac{\partial}{\partial z} \\
\frac{\partial}{\partial z} & \frac{\partial}{\partial x}
\end{bmatrix}.
\hspace{1cm} (5)
\]

Simplify the differential operator matrix to
\[
\mathbf{L} = \mathbf{L}_x \frac{\partial}{\partial x} + \mathbf{L}_z \frac{\partial}{\partial z},
\hspace{1cm} (6)
\]

where \( \mathbf{L}_x \), \( \mathbf{L}_z \) is a constant matrix
\[
\mathbf{L}_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},
\hspace{1cm} (7)
\]
\[
\mathbf{L}_z = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.
\hspace{1cm} (8)
\]

The relationship between stress and strain in matrix form is
\[
\mathbf{\sigma} = \mathbf{c}\mathbf{e},
\hspace{1cm} (9)
\]

where the vector form of stress is
\[
\mathbf{\sigma} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \tau_{xz} \end{bmatrix}.
\hspace{1cm} (10)
\]

The stiffness coefficient matrix is
\[
\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}.
\hspace{1cm} (11)
\]

Transforming the equation (1) could obtain the motion balance equation in the form of displacement
\[
\mathbf{M}\mathbf{U} - \mathbf{L}'\mathbf{c}\mathbf{U} = \mathbf{Q},
\hspace{1cm} (12)
\]

where
\[
\mathbf{L}'\mathbf{c}\mathbf{L} = \mathbf{D}_{xx} \frac{\partial^2}{\partial x^2} + 2\mathbf{D}_{xz} \frac{\partial^2}{\partial x \partial z} + \mathbf{D}_{zz} \frac{\partial^2}{\partial z^2},
\hspace{1cm} (13)
\]

and \( \mathbf{D}_{xx}, \mathbf{D}_{xz}, \) and \( \mathbf{D}_{zz} \) could be calculated

\[
\mathbf{D}_{xx} = \mathbf{L}_x^\mathbf{T}\mathbf{c}\mathbf{L}_x = \begin{bmatrix} c_{11} & c_{13} \\ c_{13} & c_{33} \end{bmatrix},
\hspace{1cm} (14)
\]
\[
\mathbf{D}_{xz} = \frac{1}{2} [\mathbf{L}_x^\mathbf{T}\mathbf{c}\mathbf{L}_z + \mathbf{L}_z^\mathbf{T}\mathbf{c}\mathbf{L}_x] = \frac{1}{2} \begin{bmatrix} c_{33} + c_{21} \\ c_{21} + c_{33} + c_{12} \end{bmatrix},
\hspace{1cm} (15)
\]
\[
\mathbf{D}_{zz} = \mathbf{L}_z^\mathbf{T}\mathbf{c}\mathbf{L}_z = \begin{bmatrix} c_{33} & c_{33} \\ c_{33} & c_{33} \end{bmatrix}.
\hspace{1cm} (16)
\]

Assuming boundary stress form as
\[
\mathbf{R} = \begin{bmatrix} \mathbf{R}_x \\ \mathbf{R}_z \end{bmatrix},
\hspace{1cm} (17)
\]

when \( x \) is a constant, the stress acting on the \( x \) plane can be expressed as
\[
\mathbf{R}_x = \mathbf{L}_x^\mathbf{T}\mathbf{c}\mathbf{U} = \mathbf{L}_x^\mathbf{T}\mathbf{c}\mathbf{L}_x \frac{\partial \mathbf{U}}{\partial x} + \mathbf{L}_x^\mathbf{T}\mathbf{c}\mathbf{L}_z \frac{\partial \mathbf{U}}{\partial z} + \mathbf{D}_{xx} \frac{\partial \mathbf{U}}{\partial x} + \mathbf{D}_{xz} \frac{\partial \mathbf{U}}{\partial z},
\hspace{1cm} (18)
\]

when \( z \) is a constant, the stress acting on the \( z \)-plane can be expressed as
\[
\mathbf{R}_z = \mathbf{L}_z^\mathbf{T}\mathbf{c}\mathbf{U} = \mathbf{L}_z^\mathbf{T}\mathbf{c}\mathbf{L}_x \frac{\partial \mathbf{U}}{\partial x} + \mathbf{L}_z^\mathbf{T}\mathbf{c}\mathbf{L}_z \frac{\partial \mathbf{U}}{\partial z} + \mathbf{D}_{xz} \frac{\partial \mathbf{U}}{\partial x} + \mathbf{D}_{zz} \frac{\partial \mathbf{U}}{\partial z},
\hspace{1cm} (19)
\]

where
\[
\mathbf{D}'_{xz} = \mathbf{L}_x^\mathbf{T}\mathbf{c}\mathbf{L}_x = \begin{bmatrix} c_{33} & c_{33} \\ c_{33} & c_{33} \end{bmatrix} = [\mathbf{D}_{xz}]^\mathbf{T}.
\hspace{1cm} (20)
\]
\[
\mathbf{D}'_{xz} = \mathbf{L}_z^\mathbf{T}\mathbf{c}\mathbf{L}_x = \begin{bmatrix} c_{33} \\ c_{33} \end{bmatrix} = [\mathbf{D}_{xz}]^\mathbf{T}.
\hspace{1cm} (21)
\]

## 3 Dynamic theory of variable gradient parameter strip element method

### 3.1 Establishment of the theoretical model of variable gradient parameters

In the functionally graded plate, the volume fraction of ZrO₂ is assumed to be
\[
V_c = \left(1 - a \left(1 - \frac{z}{H}\right) + b \left(1 - \frac{z}{H}\right)^p\right),
\hspace{1cm} (22)
\]

where \( a, b, r, \) and \( p \) is the gradient parameters and \( z \) is the coordinate thickness.

In this calculation model, gradient parameters are \( a = 1, b = 0, \) and gradient parameter \( p \) is a variable
parameter from 0 to positive infinity. The volume fraction of ceramics in the model changes with the gradient value. When the gradient parameter \( p \) approaches infinite, the volume fraction of ceramics is close to zero, and when the gradient parameter \( p \) approaches zero, the volume fraction of ceramics is close to 1. Density, Poisson’s ratio and the elastic modulus of the functionally graded plate all depend on the volume fraction of ceramics. Different gradient parameters correspond to the volume fraction of ceramics in the direction of thickness as shown in Figure 2.

The functionally graded plate is composed of ZrO\(_2\) and Ti–6Al–4V, and its material properties are shown in Table 1.

As the volume fraction in the functionally graded plate changes, the density of the model also changes accordingly. The density in the model is assumed to be

\[
\rho(z) = \rho_M(1 - V_C) + \rho_C V_C,
\]

where \( \rho_C \) is the density of ZrO\(_2\), \( \rho_M \) is the density of Ti–6Al–4V.

Similarly, the elastic modulus of the functionally graded plate changes as

\[
E(z) = E_M(1 - V_C) + E_C V_C,
\]

where \( E_C \) is the elastic modulus of ZrO\(_2\) and \( E_M \) is the elastic modulus of Ti–6Al–4V.

The variation of density corresponding to different gradient parameters in the thickness direction is shown in Figure 3. The change law of elastic modulus is consistent with the change law of the mass density.

### 3.2 Variable gradient parameter strip element method theory

Divide the domain \( D_g \) into \( N \) strip elements, assuming that the displacement field of the element is

\[
U(x, z) = N(z)V(x) \exp(-i\omega t),
\]

where \( V(x) \) is the displacement vector, \( N(z) \) is the shape function matrix, \( t \) is the time, and \( \omega \) is the angular frequency.

\[
N(z) = [(1 - 3z + 2z^2)I, \quad 4(z - z^2)I, \quad (-z + 2z^2)I],
\]

\[
V = [V_U^T, \quad V_M^T, \quad V_L^T]^T,
\]

where the \( I \) is the identity matrix, which \( V_U^T, V_M^T, V_L^T \) are the displacements of the upper, middle, and lower nodal lines of the elements.

Assume the form of the applied harmonic external load is

\[
Q = \tilde{Q} \exp(-i\omega t),
\]

where \( \tilde{Q} \) is the amplitude vector.

According to the principle of virtual work, we could obtain

\[
\delta V^T \cdot Q = \delta V^T \cdot R + \int_0^h \delta U^T(M\ddot{U} - \dot{U}cL\ddot{U} - Q) \, dz.
\]
Solve the approximate differential set of a set of elements
\[ \ddot{Q} = A_1 \frac{\partial^2 V}{\partial z^2} + A_0 \frac{\partial V}{\partial z} + A_0 V - \omega^2 M V, \]  
where
\[ A_0 = \frac{1}{3h} \begin{bmatrix} 7D_{xx} & -8D_{xx} & D_{xx} \\ -8D_{xx} & 16D_{xx} & -8D_{xx} \\ D_{xx} & -8D_{xx} & 7D_{xx} \end{bmatrix}, \]  
\[ A_1 = \frac{1}{3} \begin{bmatrix} 3(D_{xz} - D_{zz}) & -4D_{xz} & D_{xz} \\ -4D_{xz} & 0 & -4D_{xz} \\ -D_{xz} & 4D_{xz} & -3(D_{xz} - D_{zz}) \end{bmatrix}, \]  
\[ A_2 = \frac{h}{30} \begin{bmatrix} 4D_{zz} & 2D_{zz} & -D_{zz} \\ 2D_{zz} & 16D_{zz} & 2D_{zz} \\ -D_{zz} & 2D_{zz} & 4D_{zz} \end{bmatrix}, \]  
\[ M = \int_0^h \rho N'(z)N(z) \, dz = \frac{\rho h}{30} \begin{bmatrix} 4I & 2I & -I \\ 2I & 16I & 2I \\ -I & 2I & 4I \end{bmatrix}, \]  
in the mass matrix, \( I \) is a \( 2 \times 2 \) identity matrix.

The obtained element equations are extended to the domain \( D_g \) to obtain the approximate differential equations
\[ \ddot{Q}_g = A_{ig} \frac{\partial^2 V_g}{\partial x^2} + A_{ig} \frac{\partial V_g}{\partial x} + A_{ig} V_g - \omega^2 M_g V_g, \]  
where \( g \) represents the matrix or vector in the whole domain, and the matrix \( A_{ig}, M_g \) and vector \( Q_g, V_g \) are formed by combining the corresponding matrices and vectors of all elements. \( A_{ig} \) and \( M_g \) are \( M \times M \) matrices, where \( M = 4N + 2 \) and \( N \) is the number of elements.

Equation (35) represents a set of second-order differential equations with constant coefficients that can be accurately solved. Assuming the form of displacement and load is
\[ V_g = d_g \exp(ikx), \]  
\[ \ddot{Q}_g = P_g \exp(ikx), \]  
where \( k \) is the wave number.

Substituting equations (36) and (37) into equation (35) could obtain
\[ P_g = [k^2A_{ig} + ikA_{ig} + A_{ig} - \omega^2 M_g]d_g, \]  
where \( P_g \) is the amplitude vector of the external force acting on the boundary node line.

For the given \( P_g \), the characteristic equation could be obtained:
\[ 0 = [k^2A_{ig} + ikA_{ig} + A_{ig} - \omega^2 M_g]d_g - P_g. \]  
For a given \( \omega \), equation (39) can be transformed into a standard characteristic equation:
\[ 0 = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \omega^2 M_g - A_{ig} - iA_{ig} \end{pmatrix} \begin{pmatrix} d_g \end{pmatrix} = \begin{pmatrix} P_g \end{pmatrix}. \]  
Solving equation (41) can get \( 2m(m = 6N - 2(N - 1) = 4N + 2) \) eigenvalues \( k(1...2m) \), if the upper part of the \( j \)th eigenvectors corresponding to \( d_g \) is represented by \( \phi_i \), then there is
\[ \phi^T_j = [\psi_{ij} \, \psi_{2j} \, \psi_{3j}]. \]  
According to the mode superposition method, the displacement of the \( x \)-direction can be written as the superposition of all eigenvectors
\[ V_g = \sum_{j=1}^{2m} C_j \phi_j \exp(ikx) = G(x)C, \]  
where \( C_j \) is the undetermined coefficient, and equation (43) is a basic solution system in the \( D_g \) domain. The constant vector \( C \) can be determined from the boundary \( B_1, B_2 \) to obtain a special solution of the problem domain \( D_p \). There are \( m \) nodes and \( 2m \) boundary conditions on the boundary \( B_1, B_2 \), respectively, resulting in \( 2m \) constants \( C_j \).

According to equation (43), the constant vector \( C \) can be expressed as the displacement vector of the node on the boundary \( B_1, B_2 \)
\[ C = G^{-1}_B V_{bg}, \]  
where
\[ V_{bg} = \begin{bmatrix} V_{B1}^1 \ V_{B2}^1 \ end{bmatrix} \end{bmatrix} \begin{bmatrix} u_1 \ v_1 \ u_2 \ v_2 \ \vdots \ u_m \ v_m \end{bmatrix}, \]  
\[ G_{B1} = \begin{bmatrix} \phi_{11} \phi_{12} \cdots \phi_{1m} & \phi_{21} \phi_{22} \cdots \phi_{2m} & \phi_{31} \phi_{32} \cdots \phi_{3m} \end{bmatrix}, \]  
\[ Z_j^B = \exp(ikz_j^B), \]  
where \( B_1 \) is the left boundary, and \( B_2 \) is the right boundary. In equation (45), \( u_i \) and \( v_i \) represent the displacements in the \( z \)-direction and the \( x \)-direction at the point \( i \) on the boundaries \( B_1 \) and \( B_2 \), respectively. In equation (47), \( z_j^B \)
3.3 Application of boundary conditions

Using equation (18), at any point of x is a constant, the stress vector in one element is

\[ \mathbf{R}_x = \mathbf{D}_x' \frac{\partial \mathbf{U}}{\partial x} + \mathbf{D}_{xx} \frac{\partial \mathbf{U}}{\partial x}, \]

(48)

where

\[ \mathbf{D}_x' = \begin{bmatrix} c_{11} & c_{13} \\ c_{13} & c_{23} \end{bmatrix}, \]

(49)

and

\[ \mathbf{R} = \{ \mathbf{R}_x \}_{x=0} \mathbf{R}_x \left[ \begin{array}{c} \mathbf{R}_x \left[ \begin{array}{c} \mathbf{R}_x \left[ \begin{array}{c} 0 \\ 0 \\ \mathbf{D}_{xx} \end{array} \right] \right] \\ 0 \\ \mathbf{D}_{xx} \end{array} \right] + \mathbf{R}_0 \frac{\partial \mathbf{V}}{\partial x}, \]

(50)

where

\[ \mathbf{R}_1 = \frac{1}{h} \begin{bmatrix} -3\mathbf{D}_x' & 4\mathbf{D}_x' & -\mathbf{D}_x' \\ -\mathbf{D}_x' & 0 & \mathbf{D}_x' \\ \mathbf{D}_x' & -4\mathbf{D}_x' & 3\mathbf{D}_x' \end{bmatrix}, \]

(51)

\[ \mathbf{R}_2 = \begin{bmatrix} \mathbf{D}_{xx} & 0 & 0 \\ 0 & \mathbf{D}_{xx} & 0 \\ 0 & 0 & \mathbf{D}_{xx} \end{bmatrix}. \]

(52)

Assemble matrix \( \mathbf{R}_1, \mathbf{R}_2 \) into the overall unit matrix and get the stress vector of the nodal line as

\[ \mathbf{R}_g = \mathbf{R}_g \mathbf{V}_g + \mathbf{R}_g \frac{\partial \mathbf{V}}{\partial x}. \]

(53)

Simultaneous equations (43)–(43) could obtain

\[ \mathbf{R}_{bg} = \mathbf{K} \mathbf{V}_{bg}, \]

(54)

where

\[ \mathbf{R}_{bg} = \begin{bmatrix} \mathbf{R}_{bg}^l \\ \mathbf{R}_{bg}^r \end{bmatrix}. \]

(55)

\( \mathbf{R}_{bg}^l \) and \( \mathbf{R}_{bg}^r \) respectively represent the outer vectors of the left and right boundaries.

\[ \mathbf{K} = \begin{bmatrix} \mathbf{R}_{bg} & 0 \\ 0 & \mathbf{R}_{bg} \end{bmatrix} + \begin{bmatrix} \mathbf{R}_{bg} \frac{\partial \mathbf{G}_u}{\partial x} \mathbf{G}^{-1}_b \\ \mathbf{R}_{bg} \frac{\partial \mathbf{G}_u}{\partial x} \mathbf{G}^{-1}_b \end{bmatrix}, \]

(56)

where \( \mathbf{K} \) is the stiffness matrix. Equation (54) gives the relationship between stress and displacement at the left and right boundaries. For a given \( \mathbf{R}_{bg}, \mathbf{V}_{bg} \) could be calculated by equation (54), and \( \mathbf{V}_{bg} \) can be calculated from equations (43) and (44). The global displacement in the time domain can be solved by equation (25).

3.4 Application of the dynamic strip method under simple harmonic load

The thickness of the functionally graded plate along the z direction is \( H = 90 \), and it is divided into 10-layer elements. Within the boundary formed by \( B_1, B_2, B_3, \) and \( B_4 \), the length of the functionally graded plate along the x-direction is \( x = H = 90 \). Harmonic load action time \( t = 2 \) s. The surface materials of the first layer and the tenth layer of this material are ZrO\(_2\) and Ti–6Al–4V, respectively, and the properties of the intermediate material change continuously along the thickness direction.

The amplitude of the applied harmonic load \( \mathbf{Q} = 4.9 \times 10^4 \), by the load equation (45), and the loading form of the harmonic load varying with time are shown in Figure 4.

4 Results and discussion

Before the output of the theoretical results, the rationality and accuracy of the theoretical results need to be verified. For the theoretical in this article, further finite element simulation is needed to verify its accuracy and correctness. The study of functionally graded plates with variable gradient parameters shows that when the gradient
parameters tend to infinity, the volume fraction of metal materials in functionally graded plates tends to 1. Therefore, the displacement response of pure metal material under harmonic loading is analyzed by the finite element method, and the results are in agreement with the strip element method (Figure 5).

According to the displacement form of the strip element method, the law of the overall displacement of the functionally graded plate with time at \( z = 0 \) in the lower surface, gradient parameters \( p = 0.5, p = 1, p = 2, p = 5, \) and \( p = 10 \) are calculated as shown in Figure 6 under the harmonic load. It can be obtained that the overall displacement of the functionally gradient plate under the action of harmonic load presents sinusoidal fluctuation with time. Under the different gradient parameters, the displacement is in the form of simple harmonic motion with time, and with the increase of gradient parameters, the fluctuation period of displacement with time increases continuously, and the displacement peak also gradually increases, because the metal composition of the functionally gradient plate increases with the increase of gradient parameters, and the plasticity of metal is greater than that of ceramic.

Figure 7 shows the variation law of displacement with gradient parameters at \( t = 0.2 \) s and \( t = 1.3 \) s. It can be found that the displacement of the functionally graded plate increases with the gradient parameters, but the increasing trend first increases and then decreases.

Figure 8(a)–(c) respectively shows at the gradient parameters \( p_1 = 0.5, p_2 = 1, p_3 = 2, p_4 = 5, p_5 = 10 \) of the functionally graded plate as the displacement changes along the thickness at \( x = 0 \) mm, \( x = 45 \) mm, and \( x = 90 \) mm. It can be obtained that the displacements along the thickness direction at different positions of \( x \) are in harmonic form within the boundary. The peak displacement appears between \( x = 20 \) and \( x = 80 \). The displacement peak value is different under different gradient parameters. As the gradient parameter increases, the displacement peak value gradually increases and the displacement phase also increases.

Figure 9(a)–(c) respectively shows the displacement of the upper surface, middle surface, and lower surface of
the functionally graded plate with gradient parameters at $x = 10$ mm, $x = 45$ mm, and $x = 90$ mm. It can be obtained that the displacement of the upper surface, the middle surface, and the lower surface is related to the gradient parameters and increases with the increase in the gradient parameters. With the increase in the gradient parameters, the volume fraction of metal in the functionally graded plate increases, and the plastic deformation capacity of metals is higher than that of ceramics. Therefore, as the gradient parameters increase, the displacement increases gradually.

Figure 10(a) and (b), respectively, shows the variation of displacement along the $x$-direction on the upper, middle, and lower surfaces of the functionally graded plate at $p = 0.5$ and $p = 2$. It can be seen that the displacement of the functionally graded plate has a faster downward trend along the $x$-direction and gradually tends to zero on the boundary. Among them, the displacement when the displacement $p = 0.5$ is smaller than that when $p = 2$, and the displacement is maximum on the middle surface.
The strip element method is first used to study the dynamics of functionally graded plates with variable gradient parameters under harmonic loads, and useful conclusions are obtained:

1. Under the different gradient parameters, the displacement varies harmonically with time, and with the increase of gradient parameters, the fluctuation period of displacement with time increases continuously, and the displacement peak also gradually increases.

2. Under the fixed boundary conditions, the displacement of the functionally graded plate along the thickness direction shows the harmonic form.

3. By comparing the displacement at different positions for gradient parameters, as the gradient parameters increase, the peak displacement of the functionally graded plate increases, but the trend of displacement changes decreases.

4. The displacement perpendicular to the load direction gradually tends to zero along the $x$-axis, and the displacement peak appears on the middle surface of the functionally graded plate.

Figure 9: The upper, middle, and lower surface displacement changes with gradient parameters with different $x$ positions. (a) $x = 0$, (b) $x = 45$, and (c) $x = 90$. 

5 Conclusion
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References


Figure 10: The upper, middle, and lower surface displacement changes along the x-direction with different parameters. (a) $p = 0.5$ and (b) $p = 2$. 

![](attachment:image1.png)

![](attachment:image2.png)


