Research Article

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Vibration response of functionally graded material sandwich plates with elliptical cutouts and geometric imperfections under the mixed boundary conditions

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Abstract: The present article investigates the effect of elliptical cutouts and geometric imperfections on the vibrational response of functionally graded material (FGM) sandwich plates. Generalised governing equations for the sandwich FGM (SFGM) plate are derived based on non-polynomial higher-order shear deformation theory. Geometric discontinuities have been incorporated as elliptical cutouts in the plates, and the various geometric imperfections are modelled using the generic function. The mathematical modelling has been carried out using the C⁰ continuity isoparametric finite element formulation by considering four-noded elements with seven degrees of freedoms per node. Convergence and validation studies have been performed to demonstrate the efficiency and accuracy of the present methodology. The influence of volume fraction index, geometric imperfections, and elliptical cutouts on the vibrational frequency of SFGM plates have been analysed under the mixed boundary conditions.

Keywords: finite element method, vibrational analysis, functionally graded material, geometric imperfections, elliptical cutouts, mixed boundary conditions

1 Introduction

Functionally graded materials (FGMs) are composite materials created by integrating two materials microscopically, and the properties of the material continuously vary from one surface to another. FGMs have high stiffness to weight ratio, light in weight, high strength, and used for specific purposes. Koizumi [1] conceptualised the concept of FGM in 1984 to prepare thermal barrier materials in Japan. The history of the FGM plates and structures as well as its applications can be found in the report by Jha et al. [2]. FGMs consist of metal and ceramic, which can be tailored according to their intended applications i.e. aerospace, biomedical, chemical science, engineering, defence, nuclear science, and energy. Tomar et al. [3] studied the influence of uncertainties on the processing methods of various composite structures, an overview of several micromechanical models and plate theories. Batra et al. [4] investigated the natural frequencies of thick square plates made of different materials. Gupta and Talha [5] presented a comprehensive review of the structural response of FGM and structures with an emphasis on various fabrication techniques. Thai and Kim [6] presented a comprehensive review of various theories for the modelling and analysis of FGM plates and shells. Extensive investigations on the structural behaviour of FGM plates have been documented in the literature over the past few years by employing various plate theories.

Sandwich functionally graded material (SFGM) plates are made up of three layers in which core layer is bonded with two facing layers. Several studies have been carried out in the last few decades on the structural analysis of SFGM plates. In this context, Zenkour [7] investigated the bending of SFGM plates under the simply supported boundary condition based on the sinusoidal shear deformation plate theory. Adhikari et al. [8] examined the buckling characteristics of porous SFGM plates under a nonuniform in-plane edge load. Burlayenko and Sadowski [9] explored the vibration analysis of SFGM square plates using the 3D finite element method (FEM) and the ABAQUS code. Nguyen et al. [10] investigated the vibration analysis of the SFGM plates using refined higher-order shear

Literature available on the plate/shell with geometric discontinuities such as cutouts are very limited. Lee et al. [14] employed Rayleigh quotient method to investigate the vibrational behaviour of the rectangular isotropic plate. Soleimanian et al. [15] investigated the free vibration analysis of glass fibre reinforced polymer based composite plate with central cut-outs under the free boundary condition. Jain and Kumar [16] explored the post-buckling analysis of the laminate with central cutout based on Mindlin’s plate theory using Newton–Raphson Method. Aydin Komur et al. [17] investigated the buckling analysis of the laminated composite plate with circular and elliptical holes based on the FEM. Apart from this, the SFGM plates have been investigated using various conventional and unconventional boundary conditions. In this context, Gupta et al. [18] explored the vibration analysis of the FGM plate subjected to different boundary constraints using HSDT. Bhandari and Purohit [19] investigated the thermo-mechanical behaviour of FGM plates subjected to various boundaries using the FS DT and FE formulation. Talha and Singh [20] explored the bending analysis of FGM plates under various boundary conditions based on the HSDT using FEM.

In addition to this, several reports are available dealing with the influence of various geometric imperfections on the structural behaviour of FGM plates. In this framework, Kitipornchai et al. [21] explored the vibration analysis of imperfect laminated rectangular plates based on Reddy’s HSDT. Cetkovic [22] investigated the thermal analysis of layer wise laminated composite plate structure using FEM. Gupta and Talha [23] studied the influence of geometric imperfection on non-dimensional frequency parameter (NDFP) of FGM plates based on non-polynomial HSDT. Singh and Gupta [24] investigated the effect of the circular cutouts and various imperfections on the free vibration response of the SFGM plates based on the Lagrangian principle using HSDT. The vibration behaviour of SFGM plates with geometric discontinuities and microstructural defects was investigated using the logarithmic shear–strain function.

Several studies have focused on the influence of the elastic foundations under the thermal environment on the structural behaviour of SFGM plates. In this context, Liu et al. [25] investigated the deflection response of exponentially porous FGM nanoplate under the visco-elastic foundation based on TSDT using Hamilton’s principle. Van Vinh et al. [26] studied the static bending and buckling analysis of bi-directional porous FGM plates based on improved FSDT using mixed FEM and Hamilton’s principle. Djilali et al. [27] investigated the non-linear cylindrical bending analysis of FGM plate reinforced by single walled carbon nanotubes (SWCNTs) under the thermal environment based on simple integral HSDT. Van Vinh and Tounsi [28] studied the free vibration analysis of the FGM doubly curved nano-shells using non-local FSDT with variable nonlocal parameters. Tahir et al. [29] investigated the influence of the three-variable visco-elastic foundation on wave propagation in the FGM sandwich plates based on simple quasi-3D plate theory using Hamilton’s principle. Hebali et al. [30] analysed the bending and dynamic behaviours of advanced composite plates under the visco-Pasternak foundations using a simple shear deformation integral plate model and Hamilton’s principle. Bouafia et al. [31] investigated the flexural and free vibrational response of FGM plate subjected to elastic foundation based on higher order quasi-3D formulation using Hamilton’s principle. Zaitoun et al. [32] studied the buckling response of FGM sandwich plate subjected to viscoelastic foundation under the hygrothermal conditions using HSDT. Mudhaffar et al. [33] investigated the linear and nonlinear effect of temperature and moisture concentration on the bending analysis of advanced FGM plate with viscoelastic foundation under the hygro-thermo-mechanical load based on simple HSDT using the power law distribution. Kouider et al. [34] analysed the static and free vibration behaviour of sandwich plates based on quasi-3D higher-order theories using Hamilton’ principle. Merazka et al. [35] studied the hygro-thermo-mechanical bending responses of FGM plate under the Winkler–Pasternak elastic foundation using HSDT. Hachemi et al. [36] studied the bending of FGM plates under the transverse load for simply supported boundary conditions based on HSDT using Navier-type analytical solution. Bakoura et al. [37] investigated the mechanical buckling analysis of FGM plates with simply-supported boundary condition based on the HSDT using simple power law distribution.

Tomar and Talha [13] studied the effect of material uncertainties on vibration and bending analysis of FGM...
skewed sandwich plates using Reddy’s HSDT. Tomar and Talha [38] investigated the flexural and vibration behaviour of imperfection sensitive higher order FGM skew sandwich plates in thermal environment based on the Reddy’s HSDT using power law distribution. Huang et al. [39] investigated the vibration analysis of FGM rectangular plate with circular cutouts under various boundary conditions based on the three-dimensional elasticity theory using Lagrangian energy principle and Rayleigh–Ritz method.

In the view of afore-discussed literature survey, it can be observed that very few reports are available on the vibration response of geometrically imperfect SFGM structures with elliptical cutouts, although these reports are limited to the single layer FGM plates. The novelty of the present work lies in considering geometric discontinuities (elliptical cutouts) and geometric imperfections while exploring the vibration response of SFGM plates. In addition to this, the influence of the geometric imperfections on the vibration behaviour of SFGM plates with elliptical cutouts under various conventional and unconventional (mixed) boundary conditions have not been documented yet in the literature. Very limited studies are available for the vibrational analysis of SFGM plates with elliptical cutouts under the mixed boundary conditions, according to the author’s knowledge. Therefore, the aim of the present article is to investigate the effect of geometric imperfections and elliptical cutouts on the vibration response of SFGM plates under mixed boundary conditions. The effect of volume fraction index, geometric imperfections, and elliptical cutouts on NDFP of SFGM plates under the mixed boundary conditions have been investigated. The findings of this work will help in designing the engineering structures in extreme working environmental conditions.

In Section 2, the mathematical formulation of the vibration response of SFGM square plates derived based on Lagrangian FEM using the HSDT is discussed. In Section 3, convergence and validation studies for the vibration analysis of SFGM square plates with elliptical cutouts and geometric imperfections have been discussed. In Section 4, concluding remarks are presented.

2 Mathematical formulation

A SFGM plate with elliptical cutouts comprises three layers, two FGM face sheets and a metal core, as shown in Figure 1.

2.1 Power law distribution

The material properties of SFGM face sheets (top and bottom layers) are assumed to vary across the plate thickness based on the power law distribution. This variation in material properties is governed by the power law distribution, and it is defined as follows:

$$P_e = P_m + (P_e - P_m)V(z),$$

where $P_e$ represents the effective material property, $V(z)$ is the volume fraction of the metallic constituent and is expressed as follows: [40]

$$V^{(b)}(z) = \left(\frac{z - t_0}{t_1 - t_0}\right)^n \quad z \in [t_0, t_1],$$

$$V^{(c)}(z) = 0 \quad z \in [t_1, t_2],$$

$$V^{(d)}(z) = \left(\frac{z - t_2}{t_3 - t_2}\right)^n \quad z \in [t_2, t_3],$$

where $(n)$ represents the volume fraction exponent, which controls the gradation of the material constituents in the SFGM plate.

The vibration analysis was performed on SFGM plate with a metal core, as shown in Figure 2. The thickness of
the three layers is defined by $t_b$, $t_c$, and $t_t$ for the bottom layer, core layer, and top layer, respectively, where the overall thickness of the plate is denoted by “$t$”. Figure 3 depicts SFGM plates with two distinct configurations (1-1-1) and (1-2-1) that have been taken into consideration for the investigation. Plate configurations are altered by changing the thickness of the core where the middle layer is considered a metal core.

The configuration (1-1-1) represents that all the layers have the same thickness, while configuration (1-2-1) represents that the core thickness has twice the face sheets.

### 2.2 Displacement field

In the current study, the FEM formulation based on non-polynomial HSDT has been employed. The trigonometric function $\Phi(z)$ used in the present study was initialized by Sarangan and Singh [41] for laminated composite structures, whereas it has been applied for the first time in the SFGM plate structure. The displacement field can be represented as follows:

$$
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} - \Phi(z) \frac{\partial \theta_x}{\partial x} - z \frac{\partial \varphi_z}{\partial x}, \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} - \Phi(z) \frac{\partial \theta_y}{\partial y} - z \frac{\partial \varphi_z}{\partial y}, \\
\varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \Phi(z) \left( \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right) \\
- &- z \left( \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \right), \\
\gamma_{yz} &= \frac{\partial w}{\partial y} - \Phi(z) \theta_y - \varphi_y, \\
\gamma_{yz} &= \frac{\partial w}{\partial x} - \Phi(z) \theta_x - \varphi_x.
\end{align*}
$$

The above strain–displacement equations can be expressed as follows:

$$
\{\varepsilon\}_{lx=1} = [H]_{lx=11} [d]_{lx=1},
$$

where $\{\varepsilon\}_{lx=1} = \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{xy} \quad \gamma_{yz} \quad \gamma_{yx}\}$,

$$
[H]_{lx=11} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi(z) & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi(z)
\end{bmatrix},
$$

and

$$
[d]_{lx=11} =
\begin{bmatrix}
\tilde{a}_1^1 & \tilde{a}_1^2 & \tilde{a}_1^3 & \tilde{a}_1^4 & \tilde{a}_1^5 & \tilde{a}_1^6 & \tilde{a}_1^7 & \tilde{a}_1^8 & \tilde{a}_1^9 & \tilde{a}_1^{10}
\end{bmatrix},
$$

where

$$
\begin{align*}
\tilde{a}_1^1 &= \frac{\partial u_0}{\partial x}, & \tilde{a}_1^2 &= \frac{\partial v_0}{\partial y}, & \tilde{a}_1^3 &= \frac{\partial \theta_x}{\partial x} - \mu_v, & \tilde{a}_1^4 &= \frac{\partial \varphi_z}{\partial x} - \mu_z,
\tilde{a}_1^5 &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}, & \tilde{a}_1^6 &= \frac{\partial \theta_y}{\partial x} - \mu_v, & \tilde{a}_1^7 &= \frac{\partial \varphi_z}{\partial y} - \mu_z,
\tilde{a}_1^8 &= -\frac{\partial \phi_x}{\partial x}, & \tilde{a}_1^9 &= \frac{\partial \phi_y}{\partial y}, & \tilde{a}_1^{10} &= \frac{\partial \phi_z}{\partial x} - \phi_z, & \tilde{a}_1^{11} &= \frac{\partial \phi_y}{\partial y} - \phi_z.
\end{align*}
$$

### 2.3 Strain–displacement relations

The strain–displacement can be represented as follows:
The $[d_{13}]_{13×1}$ matrix can be further expressed as follows:

$$[d_{13}]_{13×1} = [B]_{13×7}[\{\beta\}]_{7×1}, \quad (6)$$

where $[\{\beta\}]_{1×7} = \{u^0\; v^0\; w^0\; \varphi_x\; \varphi_y\; \theta_x\; \theta_y\}$.

$$[B]_{7×13} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{\partial}{\partial y} & -\frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -\frac{\partial}{\partial x} & 0 & -\frac{\partial}{\partial y} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\partial}{\partial x} & 0 & -\frac{\partial}{\partial y} & 0 & 0 & 0 & -1 \end{bmatrix}.$$ 

2.4 Constitutive relation

Stress and strain vectors based on Hooks law is as shown in the following equations:

$$\begin{align*}
\begin{cases}
\sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy}
\end{cases} & = \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}, \\ \begin{cases}
\tau_{x} \\ \tau_{y}
\end{cases} & = \begin{bmatrix} E(z) & 0 \\ 0 & 2(1+v) \end{bmatrix} \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \end{bmatrix},
\end{cases} \quad (8)$$

$$\begin{cases}
\varepsilon & \text{strain vector} \\ \sigma & \text{stress vector} \\ \varepsilon_{x} & \text{strain in } x \text{ direction} \\ \varepsilon_{y} & \text{strain in } y \text{ direction} \\ \gamma_{xy} & \text{shear strain}
\end{cases}$$

where, $\{\varepsilon\}$ and $\{\sigma\}$ are the strain and stress vectors of the sandwich plate, respectively.

2.5 Finite element implementation

A four-noded isoparametric element with seven-degrees of freedom (DOF) per node was used to discretise the SFGM plate.

Substituting equation (6) in equation (5), we get

$$\{\varepsilon\}_{6×1} = [H]_{6×13}[B]_{13×7}[\{\beta\}]_{7×1}. \quad (7)$$

$$\{N\} = \sum_{i=1}^{4} N_i[\beta_i], \quad x = \sum_{i=1}^{4} N_i x_i, \quad y = \sum_{i=1}^{4} N_i y_i, \quad (10)$$

• where the ith node shape function is denoted by $N_i$ and $[\beta_i]$ represents the ith node displacement vector.

• where $x$ is the generalised geometrical coordinate, $x_i$ is the corresponding coordinate value at ith node, and $y$ is the generalised field variable with $y_i$ as the value of the field variable at ith node.

The element shape functions associated with the master element shown in Figure 4 can be described as follows:

$$N_j = (1 + \xi \xi')(1 + \eta \eta'), \quad (11)$$

where $j = 1, 2, 3, 4$.

Hence,

$$N_1 = \frac{1}{4}(1 - \xi)(1 - \eta) \quad N_2 = \frac{1}{4}(1 - \xi)(1 + \eta),$$

$$N_3 = \frac{1}{4}(1 + \xi)(1 + \eta) \quad N_4 = \frac{1}{4}(1 + \xi)(1 - \eta).$$
2.5.1 Strain energy relations

The kinetic energy (KE) of the plate can be expressed as follows:

\[ \chi_e = \frac{1}{2} \int \{\varepsilon\}^T \{\sigma\} dV, \]

where \( \chi_e \) represents strain energy. \( \{\varepsilon\}_{a1} = [H]_{6 \times 1}[B]_{13 \times 7}\) and \( \{\sigma\}_{a1} = [Q]_{6 \times 6}\),

\[ \chi_e = \frac{1}{2} \int \{\varepsilon\}^T \{Q\} \{\varepsilon\} dV, \]

\[ \chi_e = \frac{1}{2} \int \{\beta\}^T \{B\}^T \{H\} [Q] \{H\} \{\beta\} dV, \]

\[ \chi_e = \frac{1}{2} \int \{\beta\}^T \{K\} \{\beta\} dV, \]  

In equation (13), \( \{D\}_{13 \times 13} = [H]_{13 \times 6}[Q]_{6 \times 6}[H]_{6 \times 13} \),

\[ \chi_e = \frac{1}{2} \int \{\beta\}^T \{K\}_{17 \times 7}\{\beta\} dV, \]  

In equation (14), stiffness matrix, \( \{K\}_{17 \times 7} = \int \{B\}^T \{D\} \{B\} dV \),

For the total “nel” elements

\[ \chi_e = \sum_{z=1}^{nel} \chi_e^z. \]

2.5.2 KE of the FGM sandwich plate

The KE (\( \kappa_e \)) is expressed as shown in equation (16).

\[ \kappa_e = \frac{1}{2} \int \rho\{\beta\}^T\{\dot{\beta}\} dV, \]

where \( \rho \) is the density, \( \{\beta\} \) is the element displacement vector, \( \{\dot{\beta}\} \) is the element velocity vector of the \( j \)th node, which can be further expressed as follows:

\[ \{\dot{\beta}\} = [\ddot{\beta}]_{3 \times 7}\{\dot{\beta}\}_{7 \times 1}, \]

where \([\ddot{\beta}]\) is the function of the thickness coordinates and can be expressed as follows:

\[ [\ddot{\beta}]_{3 \times 7} = \begin{bmatrix} 1 & 0 & 0 & -z & 0 & -\Phi(z) & 0 \\ 0 & 1 & 0 & 0 & -z & 0 & -\Phi(z) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \]

where \( \{\beta\}_{1 \times 7} = \{u^0, v^0, w^0, \varphi_x, \varphi_y, \theta_x, \theta_y\} \).

The element level KE can be obtained by substituting equation (17) in equation (16).

\[ \kappa_e^e = \int \int_A \rho(z)[\ddot{\beta}]^T[\ddot{\beta}] dV, \]

where \( \{\dot{R}\}_{7 \times 7} = \int \int_A \rho(z)[\ddot{\beta}]^T[\ddot{\beta}] dV. \]

2.6 Governing equation

The governing equation of motion for free vibration of SFGM plate can be driven through the variational principle, which is the generalisation of the principle of virtual displacement. The generalised Lagrange equation for a system is given as follows:

\[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0; \quad i = 1, 2, \ldots, \]

where \( \{d_i\} \) and \( \{\dot{d}_i\} \) are the generalised coordinates and generalised velocities, respectively. The governing equation for the free vibration analysis can be written as follows:

\[ [M]\ddot{d} + [K]\dot{d} = 0. \]

\[ [M] = \lambda [\Omega], \quad [K] = \lambda [\Omega]. \]

For all “nel” elements, the total KE of the plate is given as follows:

\[ \kappa = \sum_{c=1}^{nel} \kappa_e^c. \]
3 Numerical analysis

The above formulations have been applied to investigate the vibrational frequencies of a SFGM plate. A generalised program is developed in MATLAB environment for numerical computation. Numerous illustrations have been presented to investigate the influence of various parameters like volume fraction exponent, elliptical cutouts, geometric configurations, and conventional and unconventional boundary conditions on vibrational behaviour SFGM plates. The numerical results, which have been exhibited in graphical and tabular form, can be used as a benchmark for further research. The effects of various important influencing parameters considered in the present work are listed below:

- Influence of the volume fraction exponent
- Influence of the geometric imperfections
- Influence of the elliptical cutouts
- Influence of the SFGM plate configurations
- Influence of the various mixed boundary conditions

4 Results and discussion

4.1 Convergence and validation studies

Example 1. This example demonstrates the convergence of the present solution for the vibration analysis of SFGM square plate with geometric discontinuities. The NDFP \( (\omega = \omega_n a^2/\sqrt{\rho_m E_m}) \) is used in the current study. The current problem is investigated for geometrically imperfect SFGM plates made of \((\text{Al/Al}_2\text{O}_3)\) materials with sine-type, local-type, and global-type materials at the volume fraction index \(n = 10\), side to thickness ratio \(a/h = 10\), and plate configuration (1-1-1).

Figure 5 shows the vibration analysis of a simply supported (SSSS) SFGM plate obtained by changing the mesh size from \((10 \times 10)\) to \((25 \times 25)\). The current solution is found to have good convergence for the vibration analysis of the SFGM square plate. The NDFP and various material properties are shown in Table 1.

Example 2. In the second example, the side-to-thickness ratio \(a/h = 10\), volume fraction indices are \(n = 5\) and \(n = 10\), along with configurations (1-1-1) and (1-2-1) and NDFP and various material properties are shown in Table 1. This example is performed to ascertain the ability of the current model to predict the free vibrational response of the SFGM plate subjected to SSSS boundary conditions. Figure 6 shows the comparison of NDFP acquired from the present theory with those given by Hadji and Avcar [42] based on HSDT, showing that the results are in excellent agreement.

The NDFP obtained from the present HSDT theory based on (1-2-1) sandwich plate configuration is compared with the results given by Hadji and Avcar [42] and Li et al. [43]. Hadji and Avcar used HSDT to analyse the SFGM plate, whereas Li et al. [43] used a three-dimensional (3D) solution. Figure 7 demonstrates the present study, which is in good agreement with the referred results.

Table 1: Various material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m(^3))</th>
<th>NDFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium (Al) (m)</td>
<td>70</td>
<td>0.3</td>
<td>2,702</td>
<td>(\omega_2 = \omega_n \left(\frac{\bar{a}}{h}\right)^2/\sqrt{\rho_m E_m})</td>
</tr>
<tr>
<td>Alumina (\text{Al}_2\text{O}_3) (c)</td>
<td>380</td>
<td>0.3</td>
<td>3,800</td>
<td></td>
</tr>
</tbody>
</table>
4.1.1 Parametric studies

Parametric studies have been conducted to study the influence of elliptical cutouts, geometric imperfections for the vibration analysis of SFGM plate under various boundary conditions. The results are computed for \( n = 0, 0.5, 1, 5, \) and \( 10 \) at \( a/h = 10 \). The conventional and unconventional boundaries are considered for the vibration analysis of the SFGM plate. The obtained results are shown in graphical form in Figures 5–20.

4.1.1.1 Influence of geometric imperfections, elliptical cutouts, plate configurations on the vibration response of SFGM plate with conventional boundary conditions

A. Influence of geometric imperfections on the NDFP of the SFGM plate:

The plate’s vibration response is investigated with various geometrical imperfections and elliptical cutouts. A wide range of initial imperfection modes can be modelled using this expression given by Kitipornchai et al. [21]. The following function (equation (23)) has been used to develop the various imperfection modes, i.e. sine-type, local-type, and global type as shown in Figure 8.

\[
Z_m = \eta t \text{sech} \left( \delta_1 \left( \frac{X}{a - \psi_1} \right) \cos \left( \eta \mu_1 \left( \frac{X}{a - \psi_1} \right) \right) \right) \times \text{sech} \left( \delta_2 \left( \frac{Y}{a - \psi_2} \right) \cos \left( \eta \mu_2 \left( \frac{Y}{a - \psi_2} \right) \right) \right),
\]  

(23)

where \( \eta \) represents the maximum amplitude of the initially deformed geometry. \( \psi_1 \) and \( \psi_2 \) are the constants defining the localisation degree of the imperfection symmetric about \( X_1 = \psi_1 \) and \( X_2 = \psi_2 \). \( \mu_1 \) and \( \mu_2 \) represent the half-wave numbers of the imperfection in \( X_1 \) and \( X_2 \) axes, respectively.

Figure 9, shows the effect of the geometrical imperfections on the vibration response of the SFGM plate without cutout under the SSSS boundary condition using configuration (1-1-1) at volume fraction index \( n \) value 0, 5, and 10. The NDFP values for a perfect plate with an imperfection of the sine-type, local-type, and global-type at \( n = 0 \) is 1.6534, 1.6816, 1.6783, and 1.7753, respectively. It is observed from the results that the global-type imperfection has higher value as compared to other imperfections. Similar trends can be observed for other values of “\( n = 5 \) and 10” considered in the present study. It is concluded that the global-type geometric imperfection has a greater influence on the NDFP than other geometric imperfections, i.e. sine-type and local-type.

B. Influence of volume fraction index and elliptical cutouts on NDFP of SFGM plate with elliptical cutouts:

The vibration analysis of SFGM square plate with elliptical cutouts have been investigated using HSDT under various boundary conditions. Figure 10 illustrates a schematic representation of a square plate with an ellipse cutout. The \( B/A \) ratio is calculated using an ellipse with a minor to major axis distance. The influence of elliptical cutouts on the NDFP of SFGM plates has been studied.
Figure 11 demonstrates the effect of the elliptical cutout on the NDFP of the SFGM plate under the SSSS boundary condition using configuration (1-1-1) and elliptical cutout $B/A$ ratios are considered to be 0.25 and 0.5. The value of NDFP is higher for the perfect SFGM plate at each volume fraction index $n$ as compared to the SFGM plate with elliptical cutout. The NDFP of the perfect plate without cutout is 1.6534 at $n = 0$. The NDFP of the plate with the elliptical cutout is 1.4915 and 1.4688, when the $B/A$ ratio is 0.25 and 0.5, respectively at $n = 0$. It is observed that the NDFP reduces by 9.79% after elliptical cutout at the $B/A$ ratio 0.25. Similar to this, when $B/A$ ratio is 0.5 at $n = 0$, the NDFP decreases by 11.16%. According to the results, the NDFP decreases as the elliptical cutout $B/A$ ratio increases. Similar trends can be observed for other values of “n” considered in the present study.

Figure 12 shows the effect of elliptical cutout and various geometric imperfections (sine-type, local-type, global-type) on the NDFP of the SFGM plate.
and 15 illustrate the effect of elliptical cutouts on the NDFP of SFGM square plate by varying \( B/A \) ratio under the SSSS boundary condition using (1-1-1) configuration.

Figure 10: Schematic representation from the top of the SFGM square plate with elliptical cutout.

**Figure 11:** Effect of elliptical cutouts on the NDFP of SFGM square plate by varying \( B/A \) ratio under the SSSS boundary condition using (1-1-1) configuration.

The NDFP decreases by 9.79% when the elliptical cutout \( B/A \) ratio is 0.25 incorporated in the plate. It is observed that the NDFP decreases as the elliptical cutout size increases. This is due to the fact that mass as well as stiffness decreases as geometric discontinuities (cutout) included in the plate. The NDFP majorly depends on the dominance of either two values. In the present case, the stiffness of the plate decreases more compared to the mass of the plate, consequently the NDFP decreases. When geometric imperfections are included in the SFGM plate, it is found that the NDFP increased by 0.82, 0.70, and 6.50% for sine-type, local type, and global-type of geometric imperfections, respectively, at volume fraction index \( (n) = 0 \). Hence, it can be concluded that global-type geometric imperfection has the highest impact on the NDFP. In contrast, local-type geometric imperfection has the least impact, and a similar trend can be seen for each volume fraction index \( (n) \).

C. **Influence of conventional boundary conditions and plate configuration on the NDFP of SFGM plate with elliptical cutouts:**

In this section, various volume fraction index \( (n) \), geometric imperfections, sandwich plate configurations (1-1-1 and 1-2-1), and conventional boundary conditions (SSSS and CCC) are selected for the vibration analysis of the SFGM plate as shown in the Figure 13.

Figures 14 and 15 illustrate the effect of the geometrical imperfections, SSSS and CCC boundary conditions on NDFP of the SFGM plate with elliptical cutout \( (B/A \) is 0.25) using configuration (1-1-1). It is observed that the NDFP values are less in the case of the SSSS boundary condition as compared to the CCC boundary condition, as more degrees of freedom restricted the plate’s boundaries. In the case of CCC which leads to increases in the stiffness of the plate, consequently, NDFP is more in CCC boundary condition compared to SSSS. Similar variation in the results can be observed for all geometric imperfection models with both the SSSS and CCC boundary conditions for each value of the “\( n \)”.

Figures 16 and 17 illustrate the effect of various imperfections and SSSS and CCC boundary conditions on the vibration behaviour of the SFGM plate with elliptical cutout \( (B/A \) is 0.25) using configuration (1-2-1). It is found that the NDFP decreases as the volume fraction
Figure 13: Representation of conventional boundary conditions (a) SSSS and (b) CCCC.

Figure 14: Effect of geometric imperfections and SSSS boundary condition on the NDFP of the SFGM plate with elliptical cutouts using (1-1-1) plate configuration with $a/h = 10$ and $\eta = 0.3$.

Figure 15: Effect of geometric imperfections and CCCC boundary condition on the NDFP of the SFGM plate with elliptical cutouts using (1-1-1) plate configuration with $a/h = 10$ and $\eta = 0.3$.

Figure 16: Effect of geometric imperfections and SSSS boundary condition on the NDFP of the SFGM plate with elliptical cutouts using (1-2-1) plate configuration with $a/h = 10$ and $\eta = 0.3$.

Figure 17: Effect of geometric imperfections and CCCC boundary condition on the NDFP of the SFGM plate with elliptical cutouts using (1-2-1) plate configuration with $a/h = 10$ and $\eta = 0.3$. 
index \( (n) \) increases for both the conventional boundary conditions (SSSS and CCCC), It is also found that the NDFP values decrease when the plate configurations are changed from (1-1-1) to (1-2-1) due to the thickness of the core is twice in the case of (1-2-1) as compared to (1-1-1) and material reduction is more when elliptical cutouts are present. It is observed that when the plate is geometrically imperfect, the NDFP values increase at specific imperfection model. Local-type and global-type of geometric imperfection have the minimum and maximum influence on the NDFP value, respectively. It can also be concluded that as the metal content increases \( (n > 0) \), the influence of geometric imperfection increases for all the conditions considered herewith.

4.1.1.2 Effect of geometric imperfections and elliptical cutouts on the vibration response of the SFGM plate under the unconventional boundary conditions

In the real-time environment, composite structures with unconventional boundary conditions are very common.
Figure 19: Effect of unconventional boundary conditions on the NDFP of SFGM square plate with elliptical cutout along with various imperfection models using (1-1-1) configuration.
especially in mechanical, civil, and aerospace sectors. Hence, one aspect which has to be specifically addressed is to analyse how discontinuous boundary constraints affect the frequency parameter of the composite structures. In this section, the influence of various mixed boundary conditions on the NDFP of SFGM square plates made up of FGM face sheets and an aluminium core using (Al/Al₂O₃) materials are investigated.

The clamped ratios (Cr; clamped length/total length) of 1/5, 1/4, and 1/2 are considered in this study. Figure 18 shows the schematic illustration of several unconventional boundary conditions for SFGM square plate considered in the present work. Cases 1–4 have the same clamping ratio (Cr = 1/5) at different positions of the DOF restrictions at the plate boundaries, whereas in cases 5 and 6, the “Cr” is considered as 1/4 and 1/2, respectively.

Figure 19 demonstrates the computed NDFP of SFGM square plate with elliptical cutout along with various imperfection models using (1-1-1) configuration for cases 1–6. It is clear from Figure 18 that cases 1–4 have the same clamping ratio Cr, i.e. 1/5. It is observed that the NDFP is different for all the cases in spite of the same Cr. It is evident from the results that the NDFP is maximum in case 1 and minimum in case 6. It is also observed that the NDFP increases as the clamping ratio increases from (Cr = 1/4 to 1/2) in case 5 and 6, respectively.

5 Conclusion

The influence of geometric imperfection, elliptical cutouts, and conventional and unconventional (mixed) boundary conditions on the vibrational responses of SFGM square plates are investigated. Convergence and validation studies using (1-1-1) and (1-2-1) sandwich plate configurations under simply supported boundary condition have been performed, and the findings are validated with the reported literature. Abovementioned influences have a significant impact on the vibrational behaviour of the FGM sandwich plates. The following outcomes can be drawn from the present study.

• It is observed from the results that the NDFP decreases as the volume fraction index (n) increases because the metallic content increases consequently decreasing the stiffness of the plate.
• The NDFP decreases as the elliptical cutout size increases. The NDFP of a perfect plate without cutout is 1.6534, and the plate with elliptical cutout at B/A = 0.5 is 1.4688 at volume fraction index (n = 0). The results clearly show that the NDFP has decreased by 11.16% when the SFGM plate has an elliptical cutout using (1-1-1) configuration. The variation in NDFP is not monotonic when the elliptical cutouts ratio (B/A) varies because the plate’s stiffness and mass are reduced in the presence of cutouts.
• The NDFP values for a perfect plate with an imperfection of the sine-type, local-type, and global-type at n = 0 is 1.6534, 1.6816, 1.6783, and 1.7753, respectively. It has been concluded that the global-type geometric imperfection has the maximum influence on the vibration characteristics of the plates.
The NDFP largely depends on the clamped ratio (Cr) and the position where the boundary constraints are applied. The fully clamped (CCCC) boundary condition has the highest NDFP.

It is observed that the NDFP increases as the clamping ratio increases from (Cr = 1/4 to 1/2). It can be concluded that the NDFP is significantly influenced by the DOF arrested at the boundaries and it also depends on the arrested location.

Conflict of interest: The authors have no conflict of interest to declare.

Data availability statement: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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