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Multi-level factor analysis of bond risk premia

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Abstract:

Earlier studies in the finance literature show that macroeconomic fundamentals can predict excess bond returns. We employ a multi-level factor model to estimate global and sectoral factors separately and show that (i) the real factors possess most important predictive power existing in the panel; (ii) the financial factors might have some predictive power but less than the real factors; (iii) the inflation factors have almost no predictive power and (iv) the excess bond returns have a countercyclical component.

Keywords: common factors, excess bond returns, predictive regression

JEL classification: E0, E4, G10, G12

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1 Introduction

In the finance literature, a large body of research shows that macroeconomic variables have important predictive power for risk premiums. Related studies include Ang and Piazzesi (2003), Piazzesi and Swanson (2006), Ludvigson and Ng (2009), Huang and Shi (2009), Cooper and Priestley (2011), Wright (2011), and Joslin, Priebisch, and Singleton (2014) among many others.

This paper is particularly related to Ludvigson and Ng (2009, 2011), who show that common factors, extracted from a large panel of macroeconomic variables, possess important predictive power for excess bond returns on US government bonds. Earlier to Fama and Bliss (1987) and Ludvigson and Ng (2009) show that the spread between the n -year forward rate and the 1 year yield forecasts n -year excess bond returns. Cochrane and Piazzesi (2005) find that a factor, obtained from forward rates, explains a significant portion of the 1 year ahead variation in excess bond returns. Ludvigson and Ng (2009) claim that the macroeconomic factors have significant predictive information that is independent of the predictive information in Cochrane and Piazzesi (2005) factor. In addition, they show that bond risk premia, computed by using the macroeconomic factors, have a strong countercyclical component so that investors are compensated with higher excess returns during recessions. Ghysels et al. (2014) show that a sizable portion of the predictive power that Ludvigson and Ng (2009) show exists in the revision of the data. They show that the predictability of the macroeconomic factors drops substantially when the real time data is used.

Common factors are regarded as capturing the most important time variation that is common across variables in a panel. Hence, Ludvigson and Ng (2009) result is understood as statistical evidence that macroeconomic fundamentals can forecast excess bond returns. While this is a significant contribution, we further study the informational content of the macroeconomic factors. The questions that we attempt to answer are as follows: (i) is the predictive information in the macroeconomic panel evenly spread out across all variables or concentrated over a certain type of variables?; (ii) if the information is concentrated over a certain type, is it exclusively contained in that group or still existent outside of that group?; (iii) are there any variables in the panel that can be a good proxy for the factors?

The econometric method to be used is a multi-level factor model. In the economics and finance literature, there are a number of applications and theoretical developments of multi-level factor models similar to ours. For example, see Gregory, Head, and Raynauld (1997), Wang (2008), Kose, Otrok, and Whiteman (2003), Dias, Pinheiro, and Rua (2013), Moench, Ng, and Potter (2013), and Ando and Bai (2015, 2016). The multi-level factor model that we use assumes that there are two levels of factors, global and sectoral ones. Global factors can affect all variables in the panel, and sectoral ones can affect only those in one sector. The panel of data we use has the same variables as the one in Jurado, Ludvigson, and Ng (2015) but covers a different time span. It includes diverse time series, ranging from those measuring real activities to those for financial markets and inflation measures. We put the panel data into three sectors, namely, “real”, “finance”, and “price” sectors, and call the sectoral factors as real factors, financial factors, and inflation factors. Global and sectoral factors are uncorrelated. This ensures that the effects of a sectoral factor do not leak into other sectors.

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We use the method proposed by Choi et al. (2014) to estimate the multi-level factors. Then, we investigate which multi-level factors have significant predictive power for excess bond returns by using the best subset selection method. It turns out that the two real factors, the square of the first real factor and one of the two financial factors enter the predictive regression. We call them the main predictors. We show that the main predictors have important predictive power regardless of the presence of Cochrane and Piazzesi's single forward factor in the predictive regression. This result is in line with Ludvigson and Ng (2009). Between the real factors and the financial factor, it is the real factors that have more predictive information. On the other hand, the global factors, the other financial factor and the inflation factors have almost no predictive power.

We construct a single predictor factor based on the main factors. It is the projection image of excess bond returns averaged across maturities 2 and 3 years on the four main predictors. The predictive power of the single predictor factor is almost identical to that of the main predictors. The 12 month moving average of the single predictor factor shows marked countercyclicity while it enters the predictive regression with a positive coefficient. The same is observed for the first real factor, which presumably has the most important predictive power among the main predictors. This result suggests that the excess bond returns have a countercyclical component.

The rest of the paper is organized as follows. In Section 2, we explain the details of our econometric framework including the multi-level factor model. In Section 3, we discuss the data set. In Section 4, we present the main results including the in-sample predictive regressions as well as the out-of-sample predictions. Also, some justification of the multi-level factor model is provided. We conclude in Section 5.

2 Econometric methods

Let $rx_t^{(n)}$ denote the continuously compounded log excess return on an n -year discount bond at time t . (See Section 3 for a more detailed description.) The 1 year ahead predictive regression is given by

$$rx_{t+12}^{(n)} = \beta' Z_t + \varepsilon_{t+12}, \quad (1)$$

where Z_t denotes a vector of predictors. The core of this econometric analysis is to find out variables that significantly enter the above predictive regression and to measure how much of excess bond returns is explained by the set of predictive regressors.

Our candidate predictors are obtained from the multi-level factor model. Suppose that there is a panel of macroeconomic variables, which is classified into M sectors. Let x_{mit} stand for the i^{th} variable in the m^{th} sector at time t . Then, the multi-level factor model assumes that

$$x_{mit} = G_t' \gamma_{mi} + S_{mt}' \psi_{mi} + e_{mit}, \quad (2)$$

for $m = 1, \dots, M$, $i = 1, \dots, N_m$, and $t = 1, \dots, T$. An $s \times 1$ vector G_t stands for a vector of global factors, which affects all variables. An $r_m \times 1$ vector S_{mt} refers to a vector of sectoral factors for sector m , which affects variables in sector m only. e_{mit} is an idiosyncratic error assumed to be uncorrelated across all individuals. Once we obtain the estimates of these factors from (2), we investigate which of these factors, their squares and cubes has predictive power in (1).

In the multi-level factor model, G_t , S_{1t} , ... and S_{Mt} are assumed to be uncorrelated. Thus, the global factor can be correlated with variables in all sectors since

$$E(x_{mit} G_t) = \gamma_{mi}' E(G_t G_t') \text{ for all } m.$$

On the other hand, a sectoral factor can be correlated with variables only in one sector since

$$E(x_{mit} S_{nt}) = \psi_{mi}' E(S_{nt} S_{nt}') \text{ for all } m$$

and

$$E(x_{mit} S_{nt}) = 0 \text{ for all } m \neq n.$$

Also, the assumption of uncorrelatedness of factors plays an important role in the estimation procedure. (See Step 1 below.)

2.1 Estimation of the multi-level factor model

The multi-level factor model has seen a number of applications in the literature. The examples include Gregory, Head, and Raynauld (1997), Kose, Otrok, and Whiteman (2003), and Moench, Ng, and Potter (2013) among others. In most applications, multi-level factors are estimated via Bayesian methods. We estimate the multi-level factor model in (2) via the procedure proposed in Choi et al. (2014), which is in line with the principal component method by Bai (2003).

Rewrite the model in (2) in matrix notation.

$$\begin{aligned} X_{mt} &= \Gamma_m G_t + \Psi_m S_{mt} + e_{mt} \\ &= [\Gamma_m, \Psi_m] \begin{bmatrix} G_t \\ S_{mt} \end{bmatrix} + e_{mt} \\ &= \Theta_m K_{mt} + e_{mt}, \text{ say,} \end{aligned} \quad (3)$$

where

$$X_{mt} = \begin{bmatrix} x_{m1t} \\ \vdots \\ x_{mN_m t} \end{bmatrix}, e_{mt} = \begin{bmatrix} e_{m1t} \\ \vdots \\ e_{mN_m t} \end{bmatrix}, \Gamma_m = \begin{bmatrix} \gamma'_{m1} \\ \vdots \\ \gamma'_{mN_m} \end{bmatrix}, \Psi_m = \begin{bmatrix} \psi'_{m1} \\ \vdots \\ \psi'_{mN_m} \end{bmatrix} \text{ and } \Theta_m = [\Gamma_m, \Psi_m].$$

Stacking X'_{mt} s vertically across t yields that

$$\begin{aligned} X_m &= G\Gamma'_m + S_m\Psi'_m + e_m \\ &= K_m\Theta'_m + e_m, \end{aligned}$$

where

$$X_m = \begin{bmatrix} X'_{m1} \\ \vdots \\ X'_{mT} \end{bmatrix}, G = \begin{bmatrix} G'_1 \\ \vdots \\ G'_T \end{bmatrix}, S_m = \begin{bmatrix} S'_{m1} \\ \vdots \\ S'_{mT} \end{bmatrix}, K_m = [G, S_m] \text{ and } e_m = \begin{bmatrix} e'_{m1} \\ \vdots \\ e'_{mT} \end{bmatrix}.$$

The estimation procedure proposed in Choi et al. (2014) is as follows.

Step 1: Select two sectors, say $m = 1$ and 2 , and obtain an estimate of K_{mt} by the principal component method, say $\hat{K}_{mt}$, which is consistent for some linear combinations of the global and sectoral factors, from the result of Bai (2003). Let $\hat{Q}_{ab} = T^{-1} \sum_{t=1}^T \hat{K}_{at} \hat{K}'_{bt}$ ($a, b = 1, 2$) and $\hat{\lambda}_1 \geq \dots \geq \hat{\lambda}_{s+r_1}$ be the generalized eigenvalues in decreasing order, solving

$$|\hat{Q}_{12} \hat{Q}_{22}^{-1} \hat{Q}_{21} - \hat{\lambda}_j \hat{Q}_{11}| = 0. \quad (4)$$

The vector $\hat{p}_j$ is the eigenvector corresponding to $\hat{\lambda}_j$. Then, the initial estimate of the global factors is given by $\hat{G}_t^{(1)} = [\hat{p}'_1, \dots, \hat{p}'_s] \hat{K}_{1t}$. The superscript (1) is employed to distinguish the current estimate from the final estimate of G_t obtained in *Step 3*. Given that sectoral factors from different sectors are uncorrelated, pre-multiplying the eigenvectors $[\hat{p}'_1, \dots, \hat{p}'_s]$ has an effect to disentangle the global factors from the sectoral factors.

Step 2: Rewrite the model in (3) as

$$\begin{aligned} X_{mt} &= \Gamma_m D^{-1} \hat{G}_t^{(1)} + \Psi_m S_{mt} + e_{mt} - \Gamma_m D^{-1} (\hat{G}_t^{(1)} - D G_t) \\ &= \Gamma_m^* \hat{G}_t^{(1)} + \Psi_m S_{mt} + e_{mt}^{(1)}, \text{ say,} \end{aligned} \quad (5)$$

and, estimate Ψ_m and S_{mt} by the principal component method after projecting out $\hat{G}_t^{(1)}$. These estimates are denoted as $\hat{\Psi}_m^{(1)}$ and $\hat{S}_{mt}^{(1)}$, respectively.

Step 3: Using $\hat{\Psi}_m^{(1)} \hat{S}_{mt}^{(1)}$, rewrite the model in (3) as

$$\begin{aligned} X_{mt} - \hat{\Psi}_m^{(1)} \hat{S}_{mt}^{(1)} &= \Gamma_m G_t + e_{mt} - (\hat{\Psi}_m^{(1)} \hat{S}_{mt}^{(1)} - \Psi_m S_{mt}) \\ &= \Gamma_m G_t + e_{mt}^{(2)}, \text{ say,} \end{aligned} \quad (6)$$

stack them as

$$\begin{bmatrix} X_{1t} - \hat{\Psi}_1^{(1)} \hat{S}_{1t}^{(1)} \\ \vdots \\ X_{Mt} - \hat{\Psi}_M^{(1)} \hat{S}_{Mt}^{(1)} \end{bmatrix} = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_M \end{bmatrix} G_t + \begin{bmatrix} e_{1t}^{(2)} \\ \vdots \\ e_{Mt}^{(2)} \end{bmatrix}, \quad (7)$$

and estimate $\{\Gamma_m\}$ and G_t by the principal component method. These estimates are written as $\{\hat{\Gamma}_m^{(1)}\}$ and $\hat{G}_t^{(2)}$. $\hat{G}_t^{(2)}$ is the final estimate of the global factors.

Step 4: Using $\{\hat{\Gamma}_m^{(1)}\}$ and $\hat{G}_t^{(2)}$, rewrite the model in (3) as

$$\begin{aligned} X_{mt} - \hat{\Gamma}_m^{(1)} \hat{G}_t^{(2)} &= \Psi_m S_{mt} + e_{mt} - (\hat{\Gamma}_m^{(1)} \hat{G}_t^{(2)} - \Gamma_m G_t) \\ &= \Psi_m S_{mt} + e_{mt}^{(3)}, \text{ say,} \end{aligned} \quad (8)$$

and estimate Ψ_m and S_{mt} by the principal component method. Denote these estimates as $\hat{\Psi}_m^{(2)}$ and $\hat{S}_{mt}^{(2)}$, respectively. $\hat{S}_{mt}^{(2)}$ is the final estimate of the sectoral factors. We refer to Choi et al. (2014) for further asymptotic theory such as consistency of these multi-level factor estimates.

One may think of splitting the panel at the sector level to extract sectoral factors. For example, Ang and Piazzesi (2003) extract a real factor from a group of real variables and an inflation factor from a group of price variables. Ludvigson and Ng (2011) also split the panel at the sectoral level and extract common factors separately from each sector. These factors are linear combinations of the global and sectoral factors from the perspective of the multi-level factor model and consequently have correlation with variables in other sectors due to the existence of global factors. Thus, even if one finds that common factors obtained from one specific sector have predictive power, one cannot tell if the source of predictive power is the time variation specific to that sector or the time variation common to all sectors. The estimation method employed in this paper avoids such an issue.

2.2 Selection of predictive regressors

Having estimated the global and sectoral factors, we determine which variable enters Z_t in (1). In a related study, Ludvigson and Ng (2009) estimate factors, without the multi-level structure we use, from a similar panel of macroeconomic variables and show that some of the factors possess significant predictive power. They also consider the squares and cubes of the factors as candidate predictors. A rationale for this is what enters Z_t is some unknown function of each factor which can be well approximated by the third order polynomial. We also set our candidate predictors as the global and sectoral factors themselves as well as their squares and cubes.

Two methods are considered to determine the composition of Z_t in (1). They are the best subset selection and the backward elimination.¹ In principle, the best subset selection finds the combination of predictors that gives the smallest residual sum of squares for each subset size and chooses the best subset size by comparing the model fits penalized by the subset size. To save on computation time, we first find the preliminary best subset by excluding the squares and cubes from candidates. We then take the variables in the preliminary best subset as well as their squares and cubes as new candidates and find the final best subset. For the comparison across subset sizes, we use Bayes Information Criterion (BIC). The backward elimination starts from a model with all candidate predictors and sequentially deletes the predictor with the greatest p-value until the p-values of all predictors are under a certain threshold. The Newey-West robust standard errors which are obtained using 18 lags are used to compute p-values.²

We prefer the best subset selection because the choice of a threshold in the backward elimination is difficult to justify while the final selection of predictors critically relies on the threshold. Furthermore, when the threshold is set at a common choice of 0.05, the resulting predictive regression yields an adjusted R^2 slightly lower than the predictive regression determined by the best subset selection.

3 Data

The log yield data from 1- to 3-year maturity zero coupon U.S. treasury bond, $y_t^{(n)}$, are obtained from the Federal Reserve Economic Data (FRED, DGS1 ~ 3). They are in monthly frequency spanning from 1976:6 to 2011:12. We construct the log price of an n-year discount bond at time t by $p_t^{(n)} = -ny_t^{(n)}$ and the log 1 year holding

period return is computed as $r_{t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}$. Then the continuously compounded log excess return on an n -year discount bond in period $t + 12$ is calculated by $rx_{t+12}^{(n)} = r_{t+12}^{(n)} - y_t^{(1)}$. Thus the log excess return spans from 1977:6 to 2011:12.

The panel of data we use has the same variables as the one in Jurado, Ludvigson, and Ng (2015) but covers a different time span. It has 132 variables spanning from 1977:6-2011:12 ($T = 415$). This set of variables has been used in multiple studies such as Ludvigson and Ng (2009, 2011) among others.

We classify the 132 variables into three sectors, namely, “real”, “finance”, and “price” sectors ($M = 3$). There are 70 variables in the real sector ($N_1 = 70$), 37 variables in the finance sector ($N_2 = 37$), and 25 variables in the price sector ($N_3 = 25$). In the appendix, the complete list of variables is provided. Those in the real sector are 1 ~ 70, those in the finance sector are 71 ~ 107, and those in the price sector are 108 ~ 132. All variables are transformed as described in the list to ensure stationarity and they are subsequently standardized. The variables in the real sector measure real activities. Personal income, industrial production indices, employment and unemployment related hours, and housing related statistics belong to the real sector. The ones in the finance sector include money stocks such as M1, M2, and M3, S&P’s indices, US treasury bill rates, bond yields, bond future spreads, and exchange rates. The price sector includes various price indices. Our classification of variables into three sectors may seem arbitrary. However, this does not cause econometric issues, unless the numbers of global factors and sectoral factors are predetermined. For example, unnecessarily splitting a sector into two sectors would only generate additional global factors instead of sectoral factors. When two sectors are merged, sectoral factors would still remain as sectoral factors of the merged sector. As extreme cases, one can merge all sectors to get the usual factor model with no multi-level structure or split the usual factor model into an arbitrary number of sectors to get a multi-level model with global factors only.³

4 Main results

4.1 In-sample prediction

The total number of factors is estimated as 8 by the information criteria proposed by Bai and Ng (2002), agreeing with other studies using the same data set. We assume that there are 2 global factors and 2 sectoral factors in each sector. As a matter of notation, we denote the global factors by G_1 and G_2 , the real factors by S_{11} and S_{12} , the financial factors by S_{21} and S_{22} , and the inflation factors by S_{31} and S_{32} . We estimate these factors as described earlier and now investigate with what variables in the panel each factor is correlated. Figure 1–Figure 4 report the marginal R^2 values from regressing each multi-level factor on each variable in the panel. Note that the marginal R^2 value is the square of the correlation coefficient and remains the same when the dependent variable and the regressor switch around. Figure 1 shows that both global factors are correlated with the variables in the real and financial sectors. In our definition, any factor that affects multiple sectors is regarded as a global one. Figure 2–Figure 4 respectively correspond to the real, financial, and inflation factors. These sectoral factors have correlation mostly in the sector of their name, as assumed by the model.

Figure 1: Marginal R^2 for global factors.

Notes: The marginal R^2 values are obtained from regressing a global factor, G_1 or G_2 , on each variable in the panel. The list of variables is provided in the appendix. The variables numbered from 1 to 70 are real variables, the ones from 71 to 107 are financial variables, and the ones from 108 to 132 are inflation variables.

Figure 2: Marginal R^2 for real factors.

Notes: The marginal R^2 values are obtained from regressing a real factor, S_{11} or S_{12} , on each variable in the panel. The list of variables is provided in the appendix. The variables numbered from 1 to 70 are real variables, the ones from 71 to 107 are financial variables, and the ones from 108 to 132 are inflation variables.

Figure 3: Marginal R^2 for financial factors.

Notes: The marginal R^2 values are obtained from regressing a financial factor, S_{21} or S_{22} , on each variable in the panel. The list of variables is provided in the appendix. The variables numbered from 1 to 70 are real variables, the ones from 71 to 107 are financial variables, and the ones from 108 to 132 are inflation variables.

Figure 4: Marginal R^2 for inflation factors.

Notes: The marginal R^2 values are obtained from regressing an inflation factor, S_{31} or S_{32} , on each variable in the panel. The list of variables is provided in the appendix. The variables numbered from 1 to 70 are real variables, the ones from 71 to 107 are financial variables, and the ones from 108 to 132 are inflation variables.

The best subset selection procedure is applied to the predictive regression where the predicted variable is the excess return on a 2-year discount bond. The selected variables are the two real factors (S_{11} and S_{12}), the square of the first real factor (S_{11}^2) and the second financial factor (S_{22}). These are the main predictors of ours. To check the robustness of our choice, we apply the best subset selection to subsamples in a recursive manner. The subsample first covers data up to 2005:2 and increases by a month until it includes the entire data. Table 1 shows the frequency that each factor is selected from the recursive exercise. The second and third rows correspond to the case where the predicted variable is the 2-year and 3-year bond excess returns respectively. The frequencies are similar across the two rows. The first real factor (S_{11}) and the second financial factor (S_{22}) are selected in all subsamples. The second global factor (G_2), the second real factor (S_{12}) and the first financial factor (S_{21}) are selected with a frequency less than a half, and the other factors are never selected.⁴ The first real factor (S_{11}) and the second financial factor (S_{22}) are the two most consistently selected predictors.

Table 1: Selection frequencies of predictors by the best subset selection.

	G_1	G_2	S_{11}	S_{12}	S_{21}	S_{22}	S_{31}	S_{32}
$rx_{t+12}^{(2)}$	0.00	0.37	1.00	0.42	0.38	1.00	0.00	0.00
$rx_{t+12}^{(3)}$	0.00	0.21	1.00	0.13	0.00	1.00	0.00	0.00

The composition of a predictor vector of a predictive regression is determined by the best subset selection. The best subset selection is applied recursively from a sample of 1977:6–2005:2 to a sample of 1977:6–2011:12. The first column of the table denotes the predicted variables and the first row denotes candidate predictors. The numbers in the table show the frequencies each candidate predictor is selected.

The predictive regression results using the main predictors are reported in Table 2(A). The upper and lower part of the table respectively correspond to the 2-year and 3-year bond excess returns. CP stands for the single forward rate factor that is shown to have predictive power by Cochrane and Piazzesi (2005). $F4$ is the projection image of excess bond returns averaged across maturities 2 and 3 years on the four main predictors. In other words, this is the linear combination of the main predictors that best explains the average excess bond return. In row (a), only CP is used as a predictor and it is significant at 5% significance level⁵ with the adjusted R^2 being close to 0.2. This confirms the predictive power of CP in our data. In row (c), the main predictors are added and the adjusted R^2 rises to 0.3. In row (b), CP is dropped and the resulting adjusted R^2 is slightly over 0.1, which is roughly the same as the increment in the adjusted R^2 from row (a) to row (c). This suggests that our main predictors possess some predictive power independent of CP . This result is in line with the findings in Ludvigson and Ng (2009) where unrestricted factors are used. In rows (d) and (e), the main predictors are replaced by the single predictor $F4$. Comparing them with rows (b) and (c), there is little change in the adjusted R^2 values. The single predictor $F4$ has as much predictive power as the set of four main predictors. While S_{11} and S_{22} are significant in all specifications, S_{12} and S_{11}^2 have less significance especially when the dependent variable is $rx_{t+12}^{(2)}$. Hence, it can be said that S_{11} and S_{22} are more important predictors than S_{12} and S_{11}^2 , which also matches the implication from Table 1.

Table 2: In-sample predictive regression with the main predictors.

(A) Main predictors		S_{11}	S_{12}	S_{22}	S_{11}^2	CP	$F4$	$\bar{R}^2$
$rx_{t+12}^{(2)}$	(a)					0.46 (4.19)		0.19
	(b)	1.13 (1.98)	−1.00 (−1.37)	−1.20 (−2.35)	−0.42 (−1.68)			0.11
	(c)	1.16 (2.87)	−0.97 (−1.90)	−1.03 (−2.51)	−0.32 (−1.60)	0.46 (4.23)		0.30
	(d)						0.69 (2.77)	0.12
	(e)					0.45 (4.18)	0.68 (3.94)	0.31
$rx_{t+12}^{(3)}$	(a)					0.81 (3.78)		0.18
	(b)	2.15 (2.31)	−2.05 (−1.75)	−2.12 (−2.25)	−0.81 (−2.08)			0.12
	(c)	2.20 (3.40)	−2.01 (−2.58)	−1.81 (−2.41)	−0.63 (−2.02)	0.81 (3.77)		0.30
	(d)						1.31 (3.25)	0.13
	(e)					0.80 (3.73)	1.28 (4.82)	0.30

(B) Real factors vs. financial factor

		S_{11}	S_{12}	S_{22}	S_{11}^2	CP	$\bar{R}^2$
$rx_{t+12}^{(2)}$	(f)	1.10 (1.85)	-1.09 (-1.41)		-0.33 (-1.19)		0.08
	(g)	1.14 (2.66)	-1.06 (-1.92)		-0.24 (-1.11)	0.47 (4.24)	0.28
	(h)			-1.24 (-2.47)			0.04
	(i)			-1.10 (-2.65)		0.45 (4.25)	0.22
$rx_{t+12}^{(3)}$	(f)	2.10 (2.14)	-2.22 (-1.78)		-0.66 (-1.47)		0.09
	(g)	2.16 (3.13)	-2.15 (-2.56)		-0.50 (-1.42)	0.83 (3.78)	0.27
	(h)			-2.20 (-2.39)			0.04
	(i)			-1.94 (-2.56)		0.80 (3.81)	0.21

The variables in the first column are the predicted variables and the ones in the first row are the predictors except for the adjusted R-square, denoted by $\bar{R}^2$. $rx_t^{(n)}$ is the log excess return on an n-year discount bond at time t . S_{11} and S_{12} are the real factors, S_{22} is the second financial factor, CP is Cochrane and Piazzesi (2005) forward rate factor, and $F4$ is the single predictor factor constructed using S_{11} , S_{12} , S_{22} and S_{11}^2 . The numbers in the table are the coefficient estimates of the predictors and those in parentheses are t-values obtained using the Newey and West (1987) robust standard errors which are obtained using 18 lags. Those significant at 5% are marked bold.

In Table 2(B), we compare the predictive power between the real sector and the financial sector. In rows (f) and (g), we exclude the financial factor (S_{22}). The adjusted R^2 values do not drop much. They are slightly lower than 0.1 without CP and slightly lower than 0.3 with CP . In rows (h) and (i), we exclude the real factors (S_{11} , S_{12} and S_{11}^2). Now the adjusted R^2 values drop more substantially. Especially with CP , the adjusted R^2 values are around 0.2, which is only slightly higher than what is achievable with CP only (rows (a)).

To sum up, the main predictors seem to have predictive power irrespective of whether or not CP is controlled. Between the real factors and the financial factor, the real factors appear to have more predictive power. The financial factor might have some predictive power but its predictive power is less clear in the presence of CP .

We now investigate if the other factors not selected as the main predictors have any predictive power. Table 3 reports the predictive regression results with the global factors (G_1 and G_2), the first financial factor (S_{21}) and the inflation factors (S_{31} and S_{32}). The model specification in each row is the same as in Table 2. In row (b), only these factors are used as predictors and the resulting adjusted R^2 is close to 0, suggesting that they have no predictive power. In row (c), CP is added and the adjusted R^2 is close to 0.2, what can be achieved with CP alone. (See Table 2 row (a).) Furthermore, none of these variables is significant. $F5$ is the single predictor constructed in the same way as $F4$, but based on G_1 , G_2 , S_{21} , S_{31} and S_{32} . The results in rows (d) and (e) suggest that $F5$ does not have any meaningful predictive power. Hence, these factors are deemed to have no predictive power.

Table 3: In-sample predictive regression with other predictors.

		G_1	G_2	S_{21}	S_{31}	S_{32}	CP	$F5$	$\bar{R}^2$
$rx_{t+12}^{(2)}$	(b)	0.67 (0.79)	0.68 (0.82)	-0.61 (-1.16)	0.08 (1.43)	0.13 (0.95)			0.02
	(c)	0.28 (0.35)	-0.71 (-1.25)	0.35 (0.74)	0.10 (1.50)	0.24 (1.58)	0.51 (4.12)		0.19
	(d)							0.64 (1.68)	0.03
	(e)						0.48 (3.79)	-0.20 (-0.64)	0.19
$rx_{t+12}^{(3)}$	(b)	0.88 (0.61)	1.16 (0.76)	-1.56 (-1.62)	0.15 (1.45)	0.31 (1.28)			0.03
	(c)	0.21 (0.16)	-1.20 (-1.07)	0.07 (0.09)	0.18 (1.46)	0.50 (1.87)	0.86 (3.60)		0.18
	(d)							1.36 (2.01)	0.03
	(e)						0.82 (3.37)	-0.07 (-0.13)	0.18

The variables in the first column are the predicted variables and the ones in the first row are the predictors except for the adjusted R-square, denoted by $\bar{R}^2$. $rx_t^{(n)}$ is the log excess return on an n-year discount bond at time t . G_1 and G_2 are the global factors, S_{21} is the first financial factor, S_{31} and S_{32} are the inflation factors, CP is Cochrane and Piazzesi (2005) forward rate factor, and $F5$ is the single predictor factor constructed using G_1 , G_2 , S_{21} , S_{31} and S_{32} . The numbers in the table are the coefficient estimates of the predictors and those in parentheses are t-values obtained using the Newey and West (1987) robust standard errors which are obtained using 18 lags. Those significant at 5% are marked bold.

Next, we inspect if these factors can boost the predictive power of the main predictors, although they do not have independent predictive power. We augment the predictive regressions reported in Table 1 with G_1 , G_2 , S_{21} , S_{31} and S_{32} . Table 4 reports the adjusted R^2 values of the augmented regressions. The column denoted by "None" has no added regressors and thus is the same as the adjusted R^2 values in Table 2(A). As we add the global factors, the first financial factor, the inflation factors or all of them, the adjusted R^2 values remain almost unchanged. Using variables other than the main predictors does not contribute to the predictability.

Table 4: In-sample predictive regression with augmented predictors.

		Augmented Regressors				
		None	G_1, G_2	S_{21}	S_{31}, S_{32}	All the others
$rx_{t+12}^{(2)}$	(b)	0.11	0.12	0.12	0.11	0.13
	(c)	0.30	0.31	0.31	0.30	0.31
$rx_{t+12}^{(3)}$	(b)	0.12	0.12	0.14	0.12	0.14
	(c)	0.30	0.30	0.30	0.29	0.30

The predictive regressions whose results are presented in Table 2(A) are augmented with additional variables. The variables in the first column are the predicted variables and the ones in the second row are the additional predictors. The numbers in the table are the adjusted R-square values. G_1, G_2, S_{21}, S_{31} and S_{32} are as explained in the note of Table 3. The numbers in the column denoted by "None" are the same as the adjusted R^2 in Table 2(A).

In the finance literature, a number of studies show that not only measures on real activities such as output gap, employment growth and Chicago Fed National Activity Index (CFNAI) but also measures on inflation are important predictors for excess bond returns. For example, see Ang and Piazzesi (2003), Cooper and Priestley (2011), and Joslin, Priebsch, and Singleton (2014). Our result indicates, however, that the time variation that exists exclusively in inflation measures does not help forecasting bond excess returns, at least for the data set we use. One possible reconciliation is that if an inflation variable with predictive power has a component that is correlated with real variables and if that component is not common among the other inflation variables, then that inflation variable can have some predictive power due to correlation with real variables while that correlation is not captured as a common factor among inflation variables.

4.2 Out-of-sample prediction

In Table 5(A), the 1 year ahead out-of-sample forecast performance of the main predictors is reported. The benchmark model uses only a constant or CP_t in addition to a constant, while the comparison model uses the main predictors in addition to the variables of the benchmark model. The model parameters and factors are recursively estimated in the estimation sample to generate 1 year ahead out-of-sample forecasts. For the forecast sample of 1996:8–2011:12, the initial estimation sample spans from 1976:6 to 1995:8, where the dependent variable spans from 1977:6 to 1995:8 and the predictors span from 1976:6 to 1994:8. The predictors are estimated from the panel spanning from 1976:6 to 1994:8. The forecast for the excess return at 1996:8 is obtained by combining the coefficient estimates with the value of predictors at 1995:8. Then the estimation sample expands by a month to include data from 1976:6 to 1995:9 and the forecast for 1996:9 is obtained in the same manner. This process continues until the forecast for 2011:12 is obtained.

Table 5: Out-of-sample prediction.

(A) main predictors			
Forecast sample	Comparison	Relative MSE	Test statistics
$rx_{t+12}^{(2)}$	1996:8–2011:12	Main predictors vs. const	1.275
	2005:4–2011:12	Main predictors vs. const	1.871
	1996:8–2011:12	Main predictors + CP vs. const + CP	0.922
	2005:4–2011:12	Main predictors + CP vs. const + CP	0.885
$rx_{t+12}^{(3)}$	1996:8–2011:12	Main predictors vs. const	1.069
	2005:4–2011:12	Main predictors vs. const	1.412
	1996:8–2011:12	Main predictors + CP vs. const + CP	0.854
	2005:4–2011:12	Main predictors + CP vs. const + CP	0.770
(B) the first real factor (S_{11})			
Forecast sample	Comparison	Relative MSE	Test statistics
$rx_{t+12}^{(2)}$	1996:8–2011:12	S_{11} vs. const	0.995
	2005:4–2011:12	S_{11} vs. const	1.114
	1996:8–2011:12	S_{11} + CP vs. const + CP	0.918
	2005:4–2011:12	S_{11} + CP vs. const + CP	0.833

$rx_{t+12}^{(3)}$			
1996:8–2011:12	S_{11} vs. const	0.939	29.43*
2005:4–2011:12	S_{11} vs. const	0.976	20.69*
1996:8–2011:12	$S_{11} + CP$ vs. const + CP	0.893	23.10*
2005:4–2011:12	$S_{11} + CP$ vs. const + CP	0.803	19.20*

(C) the second financial factor (S_{22})

Forecast sample	Comparison	Relative MSE	Test statistics
$rx_{t+12}^{(2)}$			
1996:8–2011:12	S_{22} vs. const	1.061	9.43*
2005:4–2011:12	S_{22} vs. const	1.256	−0.80
1996:8–2011:12	$S_{22} + CP$ vs. const + CP	0.980	7.22*
2005:4–2011:12	$S_{22} + CP$ vs. const + CP	1.038	1.33*
$rx_{t+12}^{(3)}$			
1996:8–2011:12	S_{22} vs. const	1.054	8.00*
2005:4–2011:12	S_{22} vs. const	1.270	−1.84
1996:8–2011:12	$S_{22} + CP$ vs. const + CP	0.983	5.96*
2005:4–2011:12	$S_{22} + CP$ vs. const + CP	1.035	0.79*

One year ahead out-of-sample forecast performances are reported. The model parameters and factors are estimated recursively from the estimation sample. The column labelled as “relative MSE” contains the ratio of the mean squared forecast error (MSE) of our model relative to that of the benchmark model. The column labelled as “Test statistic” reports the values of the ENC-NEW test statistic proposed by Clark and McCracken (2001). * denotes rejection of the null of equal predictability at the asymptotic size 5 percent.

The column labelled as “relative MSE” contains the ratio of the mean squared forecast error (MSE) of our model relative to that of the benchmark model. Under the presence of CP , the MSE of our model is smaller than that of the benchmark model for both maturities and for both forecast samples. However, the performance reverses in the absence of CP . The column labelled as “Test statistics” reports the values of the ENC-NEW test statistic proposed by Clark and McCracken (2001). The test rejects the null of equal predictive ability at the asymptotic 5% size in all cases, implying that the main predictors can improve the forecast of the benchmark model.

The superior performance of the main predictors is clear with CP , while the evidence is mixed without CP . This might be due to the fact that the main predictors are selected after CP is controlled. Having mentioned that, we cannot say that the above exercise is strictly out-of-sample, because the entire sample is used to select the main predictors.⁶ One might consider a forecast model which selects the predictors in each recursive estimation sample. Then, what can be obtained is the performance of a model that is made of a set of candidate predictors and a specific choice rule. This is different from what we are interested in, that is, the performance of our predictors. Instead, we consider an out-of-sample prediction exercise where only one of our main predictors is included, and report the results for the two most important predictors, the first real factor (S_{11}) and the second financial factor (S_{22}). This not only avoids the issues arising from the predictor selection but also is meant to compare the out-of-sample forecast ability between the two predictor factors.

Table 5(B) reports the results when only S_{11} is used instead of the main predictors while Table 5(C) reports the results when only S_{22} is used. The out-of-sample predictability is much clearer with S_{11} . With S_{11} , the relative MSE is smaller than one in all cases but one and the ENC-NEW test rejects in all cases. With S_{22} , the relative MSE is smaller than one in only two cases and the ENC-NEW test fails to reject in two cases.

4.3 Countercyclicity of excess bond returns

In Figure 5 and Figure 6, we plot each of S_{11} and $F4$ against the growth of the industrial production (IP) as 12 month moving averages. In the figures, the shaded areas denote the recessions designated by the NBER. As can be seen, both S_{11} and $F4_t$ show clear countercyclicity, with their respective correlation coefficient with the IP growth being -0.82 and -0.55 . Both S_{11} and $F4_t$ also have positive coefficients in the predictive regression across all maturities (See Table 2). This suggest that the excess bond returns have a strong countercyclical component.

Figure 5: Countercyclicality of excess return, first real factor and IP growth.

Notes: S₁₁ stands for the 12 month moving average of the first real factor. IP growth stands for the 12 month moving average of the industrial production growth. The shaded areas denote the recessions designated by the NBER.

Figure 6: Countercyclicality of excess return, single predictor factor and IP growth.

Notes: F₄ stands for the 12 month moving average of the single predictor factor, constructed with the main predictors. IP growth stands for the 12 month moving average of the industrial production growth. The shaded areas denote the recessions designated by the NBER.

4.4 Choice of factor numbers

Our model, when collected across sectors, can be written as

$$\begin{aligned} X &= [X_1, \dots, X_M] \\ &= [G, S_1, \dots, S_M] \begin{pmatrix} \Gamma'_1 & \dots & \Gamma'_M \\ \Psi'_1 & & 0 \\ & \ddots & \\ 0 & & \Psi'_M \end{pmatrix} + [e_m, \dots, e_M] \\ &= F\Lambda' + e, \end{aligned}$$

which is a standard factor model when the zero restrictions in Λ' are ignored.

If the numbers of global and sectoral factors are correctly specified, each unrestricted factor must be perfectly spanned by the multi-level factors, because the unrestricted factors are exact rotations of the multi-level factors, that is, $F\Psi = [G, S_1, \dots, S_M]$ for some invertible matrix Ψ . Naturally, one is tempted to choose the factor numbers by looking at how close to one the R^2 value from the regression of each unrestricted factor estimate on the multi-level factor estimates is.⁷ However, the interpretation of such R^2 values is challenging. Both the unrestricted and multi-level factors are obtained with estimation errors, and the cause of a low R^2 value can be on either side. Also, because each unrestricted factor estimate will be approximated with a varying degree of precision across possible combinations of factor numbers, one has to decide how much weight to put on each of the R^2 values to assess the adequacy of the model specification.

We choose the factor numbers so that the unrestricted factor estimates with predictive power are replicated as much as possible. This is motivated by the aim of our study to uncover the informational contents of Ludvigson and Ng type macro factor predictors. We first estimate eight unrestricted factors from our data set, say $F = [F_1, \dots, F_8]$, via principal component analysis and obtain five predictors by the best subset selection. They are F_1, F_2, F_4, F_8 and F_1^2 . Table 6(A) reports the predictive regression results using these predictors. To choose the factor numbers, we obtain the projection images of the unrestricted factor estimates onto the space spanned by the multi-level factor estimates for each possible choice of factor numbers. Then, we look at how well the predictive regressions with the unrestricted factors in Table 6(A) are replicated when these projection images replace the unrestricted factor estimates. When there are two global factors and two sectoral factors in each of the three sectors, the predictive regressions with the unrestricted factor estimates are replicated remarkably well. Table 6(B) reports the results. As can be seen, the coefficient estimates and R^2 s are very similar across Table 6(A) and (B).

Table 6: In-sample predictive regression.

(A) Unrestricted factor estimates									
		F_1	F_2	F_4	F_8	F_1^2	CP	$F5$	$\bar{R}^2$
$rx_{t+12}^{(2)}$	(b)	1.91 (2.82)	1.22 (1.70)	−0.02 (−0.02)	−0.96 (−1.17)	− 1.40 (−2.55)			0.13
	(c)	1.71 (3.65)	0.92 (2.22)	− 1.63 (−3.66)	− 1.51 (−1.99)	− 1.14 (−2.66)	0.51 (4.50)		0.33
	(d)							0.70 (3.30)	0.14
	(e)						0.43 (4.23)	0.64 (4.19)	0.31
									0.13
$rx_{t+12}^{(3)}$	(b)	3.42 (3.26)	2.54 (2.03)	0.11 (0.08)	−1.95 (−1.28)	− 2.54 (−2.81)			0.13
	(c)	3.08 (4.35)	2.00 (2.73)	− 2.72 (−3.46)	− 2.93 (−2.01)	− 2.08 (−2.92)	0.90 (4.10)		0.32
	(d)							1.30 (4.02)	0.14
	(e)						0.76 (3.81)	1.19 (5.37)	0.30
(B) Generated unrestricted factor estimates									
		$\hat{F}_1$	$\hat{F}_2$	$\hat{F}_4$	$\hat{F}_8$	$\hat{F}_1^2$	CP	$F5$	$\bar{R}^2$
$rx_{t+12}^{(2)}$	(b)	1.89 (2.87)	1.22 (1.73)	−0.03 (−0.04)	− 3.06 (−3.05)	− 1.42 (−2.65)			0.14
	(c)	1.71 (3.73)	0.91 (2.26)	− 1.68 (−4.13)	− 2.19 (−2.62)	− 1.15 (−2.73)	0.50 (4.54)		0.33
	(d)							0.71 (3.56)	0.15
	(e)						0.42 (4.13)	0.63 (4.31)	0.31
									0.14
$rx_{t+12}^{(3)}$	(b)	3.41 (3.33)	2.53 (2.06)	0.08 (0.06)	− 4.73 (−2.60)	− 2.57 (−2.91)			0.14
	(c)	3.09 (4.47)	1.99 (2.76)	− 2.82 (−3.88)	− 3.19 (−2.08)	− 2.10 (−2.98)	0.88 (4.11)		0.32
	(d)							1.29 (4.26)	0.15
	(e)						0.74 (3.71)	1.15 (5.45)	0.29

(A): The variables in the first column are the predicted variables and the ones in the first row are the predictors except for the adjusted R-square, denoted by $\bar{R}^2$. $rx_t^{(n)}$ is the log excess return on an n-year discount bond at time t . Eight unrestricted factors are estimated, and F_1, F_2, F_4, F_8 and the square of F_1 are selected as the predictors by the best subset selection. CP is Cochrane and Piazzesi (2005) forward rate factor, and $F5$ is the single predictor factor constructed using F_1, F_2, F_4, F_8 and F_1^2 . The numbers in the table are the coefficient estimates of the predictors and those in parentheses are t-values obtained using the Newey and West (1987) robust standard errors which are obtained using 18 lags. Those significant at 5% are marked bold.

(B): The variables in the first column are the predicted variables and the ones in the first row are the predictors except for the adjusted R-square, denoted by $\bar{R}^2$. $rx_t^{(n)}$ is the log excess return on an n-year discount bond at time t . $\hat{F}_1, \hat{F}_2, \hat{F}_4$ and $\hat{F}_8$ are the projection images of F_1, F_2, F_4 and F_8 on the multi-level factors, respectively. CP is Cochrane and Piazzesi (2005) forward rate factor, and $F5$ is the single predictor factor constructed using $\hat{F}_1, \hat{F}_2, \hat{F}_4, \hat{F}_8$ and $\hat{F}_1^2$. The numbers in the table are the coefficient estimates of the predictors and those in parentheses are t-values obtained using the Newey and West (1987) robust standard errors which are obtained using 18 lags. Those significant at 5% are marked bold.

One may wonder if there is an one-to-one correspondence between the multi-level factors and the unrestricted factors. Figure 7 shows the marginal R^2 values from regressing an unrestricted predictor on each multi-level factor. F_1 is a linear combination of G_1 and S_{11} where S_{11} has greater importance than G_1 . F_2 and F_4 are a linear combination of various multi-level factors. F_8 is not well represented as a linear combination of multi-level factors, but this has little impact on the predictive regression as evidenced in Table 6(B).

Figure 7: Composition of unrestricted factors in terms of multi-level factors.

Notes: The marginal R^2 values are obtained from regressing an unrestricted factor, F_1, F_2, F_4 or F_8 , on each multi-level factor.

4.5 Proxy macroeconomic variables

We search for macroeconomic variables that can be proxies for the main predictor factors by regressing each of the main predictors on each variable in the panel. The first real factor S_{11} is highly correlated with variables representing employment growth. The log difference of “Employees on Nonfarm Payrolls: Total Private” gives the highest R^2 at 0.71, followed by similar employment variables. For the second real factor S_{12} , housing related variables show high correlation. The log of “Houses Authorized by Building Permits, West” yields the highest

R^2 at 0.52, followed by a number of housing starts variables. The second financial factor S_{22} is highly correlated with exchange rate variables. The log difference of the effective exchange rate (the multilateral exchange rate model index) gives the highest R^2 at 0.59, followed by the exchange rates with UK and Switzerland.

We replace in the predictive regression the main predictor factors with these variables. The regression results are reported in Table 7. The statistical significance of these proxy variables drops slightly, when compared with the main predictor factors in Table 2. This might be a natural consequence because these proxies are contaminated versions of the true factors. However, the adjusted R^2 values are almost as high as the ones observed in Table 2. $P4_t$ is the single predictor factor constructed using these proxy variables.

Table 7: In-sample predictive regression with proxy macroeconomic variables.

		Emp	House	FX	Emp ²	CP	P4	$\bar{R}^2$
$rx_{t+12}^{(2)}$	(b)	-0.29 (-1.06)	-0.49 (-1.60)	0.21 (2.21)	-0.12 (-1.26)			0.10
	(c)	-0.31 (-1.36)	-0.45 (-2.09)	0.29 (2.72)	-0.11 (-1.33)	0.46 (4.24)		0.30
	(d)						0.69 (2.39)	0.11
	(e)					0.46 (4.26)	0.69 (3.26)	0.30
	(b)	-0.56 (-1.16)	-0.96 (-1.96)	0.31 (1.80)	-0.23 (-1.45)			0.11
$rx_{t+12}^{(3)}$	(c)	-0.58 (-1.50)	-0.89 (-2.61)	0.45 (2.35)	-0.21 (-1.56)	0.82 (3.76)		0.29
	(d)						1.31 (2.87)	0.12
	(e)					0.81 (3.80)	1.30 (4.08)	0.30

The variables in the first column are the predicted variables and the ones in the first row are the predictors except for the adjusted R-square, denoted by $\bar{R}^2$. $rx_t^{(n)}$ is the log excess return on an n-year discount bond at time t . The main predictors in Table 2(A) are replaced by proxy variables. Emp stands for "total private employees on nonfarm payrolls" replacing S_{11} . House stands for "houses authorized by building permits" replacing S_{12} . FX stands for "effective exchange rate (multilateral exchange rate model index)" replacing S_{22} . CP is Cochrane and Piazzesi (2005) forward rate factor. P4 is the single predictor factor constructed using Emp, House, FX and Emp². The numbers in the table are the coefficient estimates of the predictors and those in parentheses are t-values obtained using the Newey and West (1987) robust standard errors which are obtained using 18 lags. Those significant at 5% are marked bold.

Our finding is in agreement with Piazzesi and Swanson (2006), who show that excess returns on fed funds futures are well predicted by employment growth. Employment growth is one of the common choices for a macroeconomic predictor found in the literature. Our result justifies this common choice using the multi-level factor model. The macroeconomic variable that resembles the most the multi-level factor with the greatest predictive power is employment growth.

5 Conclusion

We have four conclusions. First, the real factors are the most important predictors for excess bond returns. Second, the financial factors might have some predictive power but not so much as the real factors, as evidenced by both in-sample and out-of-sample predictions. Third, the inflation factors are almost irrelevant. Fourth, the excess bond returns have a strong countercyclical component.

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Appendix

List of macroeconomic variables in the panel data

N	Name	Description	Tran
1	PI	Personal Income	$\Delta \ln$
2	PI less transfers	Personal Income Less Transfer Payments	$\Delta \ln$
3	Real Consumption	Real Consumption	$\Delta \ln$
4	M&T sales	Manufacturing and Trade Sales	$\Delta \ln$

5	Retail sales	Sales of Retail Stores	$\Delta l n$
6	IP: total	Industrial Production Index - Total Index	$\Delta l n$
7	IP: products	Industrial Production Index - Products, Total	$\Delta l n$
8	IP: final prod	Industrial Production Index - Final Products	$\Delta l n$
9	IP: cons gds	Industrial Production Index - Consumer Goods	$\Delta l n$
10	IP: cons dble	Industrial Production Index - Durable Consumer Goods	$\Delta l n$
11	IP: cons nondble	Industrial Production Index - Nondurable Consumer Goods	$\Delta l n$
12	IP: bus eqpt	Industrial Production Index - Business Equipment	$\Delta l n$
13	IP: matls	Industrial Production Index - Materials	$\Delta l n$
14	IP: dble matls	Industrial Production Index - Durable Goods Materials	$\Delta l n$
15	IP: nondble matls	Industrial Production Index - Nondurable Goods Materials	$\Delta l n$
16	IP: mfg	Industrial Production Index - Manufacturing (Sic)	$\Delta l n$
17	IP: res util	Industrial Production Index - Residential Utilities	$\Delta l n$
18	IP: fuels	Industrial Production Index - Fuels	$\Delta l n$
19	NAPM prodn	Napm Production Index (Percent)	$l v$
20	Cap util	Capacity Utilization (Mfg.)	$\Delta l v$
21	Help wanted indx	Index of Help-Wanted Advertising in Newspapers	$\Delta l v$
22	Help wanted/unemp	Employment: Ratio; Help-Wanted Ads:No. Unemployed Clf	$\Delta l v$
23	Emp CPS total	Civilian Labor Force: Employed, Total	$\Delta l n$
24	Emp CPS nonag	Civilian Labor Force: Employed, Nonagric. Industries	$\Delta l n$
25	U: all	Unemployment Rate: All Workers, 16 Years & Over	$\Delta l v$
26	U: mean duration	Unemploy. By Duration: Average (Mean) Duration in Weeks	$\Delta l v$
27	U < 5 wks	Unemploy. By Duration: Persons Unempl. Less than 5 Wks	$\Delta l n$
28	U 5-14 wks	Unemploy. By Duration: Persons Unempl. 5 to 14 Wks	$\Delta l n$
29	U 15+ wks	Unemploy. By Duration: Persons Unempl. 15 Wks +	$\Delta l n$
30	U 15-26 wks	Unemploy. By Duration: Persons Unempl. 15 to 26 Wks	$\Delta l n$
31	U 27+ wks	Unemploy. By Duration: Persons Unempl. 27 Wks +	$\Delta l n$
32	UI claims	Average Weekly Initial Claims, Unemploy. Insurance	$\Delta l n$
33	Emp: total	Employees on Nonfarm Payrolls: Total Private	$\Delta l n$
34	Emp: gds prod	Employees on Nonfarm Payrolls - Goods-Producing	$\Delta l n$
35	Emp: mining	Employees on Nonfarm Payrolls - Mining	$\Delta l n$
36	Emp: const	Employees on Nonfarm Payrolls - Construction	$\Delta l n$
37	Emp: mfg	Employees on Nonfarm Payrolls - Manufacturing	$\Delta l n$
38	Emp: dble gds	Employees on Nonfarm Payrolls - Durable Goods	$\Delta l n$
39	Emp: nondbles	Employees on Nonfarm Payrolls - Nondurable Goods	$\Delta l n$
40	Emp: services	Employees on Nonfarm Payrolls - Service-Providing	$\Delta l n$
41	Emp: TTU	Employees on Nonfarm Payrolls - Trade, Transportation, and Utilities	$\Delta l n$
42	Emp: wholesale	Employees on Nonfarm Payrolls - Wholesale Trade	$\Delta l n$
43	Emp: retail	Employees on Nonfarm Payrolls - Retail Trade	$\Delta l n$
44	Emp: FIRE	Employees on Nonfarm Payrolls - Financial Activities	$\Delta l n$
45	Emp: Govt	Employees on Nonfarm Payrolls - Government	$\Delta l n$
46	Agg wkly hours	Index of Aggregate Weekly Hours	$\Delta l n$
47	Avg hrs	Avg Weekly Hrs of Prod or Nonsup Workers on Private Nonfarm Payrolls - Goods-Producing	$l v$
48	Overtime: mfg	Avg Weekly Hrs of Prod or Nonsup Workers on Private Nonfarm Payrolls - Mfg Overtime Hours	$\Delta l v$
49	Avg hrs: mfg	Average Weekly Hours, Mfg.	$l v$
50	NAPM empl	Napm Employment Index	$l v$
51	Starts: nonfarm	Housing Starts: Nonfarm (1947-58); Total Farm & Nonfarm (1959-)	$l n$
52	Starts: NE	Housing Starts: Northeast	$l n$
53	Starts: MW	Housing Starts: Midwest	$l n$
54	Starts: South	Housing Starts: South	$l n$
55	Starts: West	Housing Starts: West	$l n$
56	BP: total	Housing Authorized: Total New Priv Housing Units	$l n$
57	BP: NE	Houses Authorized By Build. Permits: Northeast	$l n$
58	BP: MW	Houses Authorized By Build. Permits: Midwest	$l n$
59	BP: South	Houses Authorized By Build. Permits: South	$l n$
60	BP: West	Houses Authorized By Build. Permits: West	$l n$
61	PMI	Purchasing Managers' Index	$l v$
62	NAPM new ordrs	Napm New Orders Index	$l v$
63	NAPM vendor del	Napm Vendor Deliveries Index	$l v$
64	NAPM Invent	Napm Inventories Index	$l v$
65	Orders: cons gds	Mfrs' New Orders, Consumer Goods And Materials	$\Delta l n$
66	Orders: dble gds	Mfrs' New Orders, Durable Goods Industries	$\Delta l n$

67	Orders: cap gds	Mfrs'New Orders, Nondefense Capital Goods	$\Delta \ln$
68	Unf orders: dble	Mfrs'Unfilled Orders, Durable Goods Indus.	$\Delta \ln$
69	M&T invent	Manufacturing And Trade Inventories	$\Delta \ln$
70	M&T invent/sales	Ratio, Mfg. And Trade Inventories To Sales	$\Delta \ln$
71	M1	Money Stock: M1	$\Delta^2 \ln$
72	M2	Money Stock: M2	$\Delta^2 \ln$
73	Currency	Money Stock: Currency held by the public	$\Delta^2 \ln$
74	M2 (real)	Money Supply: Real M2 (AC)	$\Delta \ln$
75	MB	Monetary Base, Adj For Reserve Requirement Changes	$\Delta^2 \ln$
76	Reserves tot	Depository Inst Reserves:Total, Adj For Reserve Req Chgs	$\Delta^2 \ln$
77	Reserves nonbor	Depository Inst Reserves:Nonborrowed,Adj Res Req Chgs	$\Delta \frac{\Delta x_t}{x_{t-1}}$
78	C&I loan plus	Commercial and Industrial Loans at All Commercial Banks	$\Delta^2 \ln$
79	DC&I loans	Change in Commercial and Industrial Loans at All Commercial Banks	$\ln$
80	Cons credit	Consumer Credit Outstanding - Nonrevolving	$\Delta^2 \ln$
81	Inst cred/PI	Ratio, Consumer Installment Credit To Personal Income	$\Delta \ln$
82	S&P 500	S&P's Common Stock Price Index: Composite	$\Delta \ln$
83	S&P: indust	S&P's Common Stock Price Index: & Industrials	$\Delta \ln$
84	S&P div yield	S&P's Composite Common Stock: Dividend Yield Real (S)	$\Delta \ln$
85	S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio Real (S)	$\Delta \ln$
86	Fed Funds	Interest Rate: Federal Funds	$\Delta \ln$
87	Comm paper	3-Month AA Financial Commercial Paper Rate	$\Delta \ln$
88	3 mo T-bill	Interest Rate: U.S.Treasury Bills,Sec Mkt,3-Mo.	$\Delta \ln$
89	6 mo T-bill	Interest Rate: U.S.Treasury Bills,Sec Mkt,6-Mo.	$\Delta \ln$
90	1 yr T-bond	Interest Rate: U.S.Treasury Const Maturities,1-Yr.	$\Delta \ln$
91	5 yr T-bond	Interest Rate: U.S.Treasury Const Maturities,5-Yr.	$\Delta \ln$
92	10 yr T-bond	Interest Rate: U.S.Treasury Const Maturities,10-Yr.	$\Delta \ln$
93	Aaa bond	Bond Yield: Moody's Aaa Corporate	$\Delta \ln$
94	Baa bond	Bond Yield: Moody's Baa Corporate	$\Delta \ln$
95	CP-FF spread	CP-FF spread (AC)	$\ln$
96	3 mo-FF spread	3 mo-FF spread (AC)	$\ln$
97	6 mo-FF spread	6 mo-FF spread (AC)	$\ln$
98	1 yr-FF spread	1 yr-FF spread (AC)	$\ln$
99	5 yr-FF spread	5 yr-FF spread (AC)	$\ln$
100	10 yr-FF spread	10 yr-FF spread (AC)	$\ln$
101	Aaa-FF spread	Aaa-FF spread (AC)	$\ln$
102	Baa-FF spread	Baa-FF spread (AC)	$\ln$
103	Ex rate: avg	Nominal Effective Exchange Rate, Unit Labor Costs (IMF)	$\Delta \ln$
104	Ex rate: Switz	Foreign Exchange Rate: Switzerland - Swiss Franc Per U.S.\$	$\Delta \ln$
105	Ex rate: Japan	Foreign Exchange Rate: Japan - Yen Per U.S.\$	$\Delta \ln$
106	Ex rate: UK	Foreign Exchange Rate: United Kingdom - Cents Per Pound	$\Delta \ln$
107	EX rate: Canada	Foreign Exchange Rate: Canada - Canadian \$ Per U.S.\$	$\Delta \ln$
108	PPI: fin gds	Producer Price Index: Finished Goods	$\Delta^2 \ln$
109	PPI: cons gds	Producer Price Index: Finished Consumer Goods	$\Delta^2 \ln$
110	PPI: int mat'ls	Producer Price Index:Intermediate Mat.Supplies & Components	$\Delta^2 \ln$
111	PPI: crude mat'ls	Producer Price Index: Crude Materials	$\Delta^2 \ln$
112	Spot market price	Spot market price index: bls & crb: all commodities	$\Delta^2 \ln$
113	PPI: nonferrous	Producer Price Index: Nonferrous Materials	$\Delta^2 \ln$
114	NAPM com price	Napm Commodity Prices Index	$\ln$
115	CPI-U: all	Cpi-U: All Items	$\Delta^2 \ln$
116	CPI-U: apparel	Cpi-U: Apparel & Upkeep	$\Delta^2 \ln$
117	CPI-U: transp	Cpi-U: Transportation	$\Delta^2 \ln$
118	CPI-U: medical	Cpi-U: Medical Care	$\Delta^2 \ln$
119	CPI-U: comm.	Cpi-U: Commodities	$\Delta^2 \ln$
120	CPI-U: dble	Cpi-U: Durables	$\Delta^2 \ln$
121	CPI-U: services	Cpi-U: Services	$\Delta^2 \ln$
122	CPI-U: ex food	Cpi-U: All Items Less Food	$\Delta^2 \ln$
123	CPI-U: ex shelter	Cpi-U: All Items Less Shelter	$\Delta^2 \ln$
124	CPI-U: ex med	Cpi-U: All Items Less Medical Care	$\Delta^2 \ln$
125	PCE defl	Pce, Impl Pr Defl:Pce (BEA)	$\Delta^2 \ln$
126	PCE defl: dble	Pce, Impl Pr Defl:Pce; Durables (BEA)	$\Delta^2 \ln$
127	PCE defl: nondble	Pce, Impl Pr Defl:Pce; Nondurables (BEA)	$\Delta^2 \ln$
128	PCE defl: service	Pce, Impl Pr Defl:Pce; Services (BEA)	$\Delta^2 \ln$
129	AHE: goods	Avg Hourly Earnings of Prod or Nonsup Workers Private Nonfarm - Goods-Producing	$\Delta^2 \ln$

130	AHE: const	Avg Hourly Earnings of Prod or Nonsup Workers Private Nonfarm - Construction	$\Delta^2 \ln$
131	AHE: mfg	Avg Hourly Earnings of Prod or Nonsup Workers Private Nonfarm - Manufacturing	$\Delta^2 \ln$
132	Consumer expect	U. Of Mich. Index Of Consumer Expectations (UM)	$\Delta \ln$

The column marked by Tran contains the method of transformation applied to each variable to ensure stationarity. $\ln$ stands for level, $\ln$ log, $\Delta \ln$ difference in log, $\Delta^2 \ln$ double difference in log and $\Delta \frac{\Delta x_t}{x_{t-1}}$ growth rate.

Notes

¹See Hastie, Tibshirani, and Friedman (2008) for the details of these two methods of variable selection.

²In both methods, Cochrane and Piazzesi (2005) forward rate factor is always controlled in the predictive regression. This is meant to find out those variables with predictive power independent of an already known predictor, Cochrane and Piazzesi's factor.

³Recently, Ando and Bai (2016) analyze a similar model. Their model allows a set of observed variables as well as group factors. A novel feature of their model is that the group membership of each individual is determined by the data and avoids arbitrary grouping of the data. It is an interesting future research topic to extend their model to include global factors.

⁴The squares and cubes are always selected with less frequencies than their original variable due to the design of the selection procedure.

⁵The t-values are computed by the Newey-West robust standard errors which are obtained using 18 lags. While it is a standard practice to use a robust standard errors based on HAC estimation with autocorrelated time series, Cattaneo and Crump (2014) shows that the Newey-West standard errors can be misleading in typical predictive regressions on stock returns and bond returns. See also Bauer and Hamilton (2015) for related issues.

⁶We acknowledge that an anonymous referee points this out.

⁷This is similar to the method developed by Bai and Ng (2006) to evaluate latent and observed factors. However, F has to be estimated in our case and the inferential theory in Bai and Ng (2006) is not directly applicable.

References

- Ando, T., and J. Bai. 2015. "Asset Pricing with a General Multifactor Structure." *Journal of Financial Econometrics* 13: 556–604.
- Ando, T., and J. Bai. 2016. "Panel Data Models with Grouped Factor Structure Under Unknown Group Membership." *Journal of Applied Econometrics* 31: 163–191.
- Ang, A., and M. Piazzesi. 2003. "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics* 50: 745–787.
- Bai, J.. 2003. "Inferential Theory for Factor Models of Large Dimensions." *Econometrica* 71: 135–272.
- Bai, J., and S. Ng. 2002. "Determining the Number of Factors in Approximate Factor Models." *Econometrica* 70: 191–221.
- Bai, J., and S. Ng. 2006. "Evaluating Latent and Observed Factors in Macroeconomics and Finance." *Journal of Econometrics* 131: 507–537.
- Bauer, M. D., and J. D. Hamilton. 2015. "Robust Bond Risk Premia." FRB San Francisco Working Paper 2015-15.
- Cattaneo, M. D., and R. K. Crump. 2014. "Comment on HAC Corrections for Strongly Autocorrelated Time Series." *Journal of Business and Economic Statistics* 32: 324–329.
- Choi, I., D. Kim, Y. Kim, and N. Kwark. A Multi-level Factor Model: Identification, Asymptotic Theory and Applications. 2014; mimeo, Sogang University.
- Clark, T. E., and M. W. McCracken. 2001. "Tests of equal forecast accuracy and encompassing for nested models." *Journal of Econometrics* 105: 85–110.
- Cochrane, J. H., and M. Piazzesi. 2005. "Bond Risk Premia." *American Economic Review* 95: 138–160.
- Cooper, I., and R. Priestley. 2011. "Time-Varying Risk Premiums and the Output Gap." *The Review of Financial Studies* 22: 2801–2833.
- Dias, F., M. Pinheiro, and A. Rua. 2013. "Determining the Number of Global and Country-Specific Factors in the Euro Area." *Studies in Nonlinear Dynamics and Econometrics* 17: 573–617.
- Fama, E. F., and R. H. Bliss. 1987. "The Information in Long-Maturity Forward Rates." *American Economic Review* 77: 680–692.
- Gregory, A. W., A. C. Head, and J. Raynauld. 1997. "Measuring World Business Cycles." *International Economic Review* 38: 677–701.
- Ghysels, E., C. Horan, and E. Moench. 2014. "Forecasting Through the Rear-View Mirror: Data Revisions and Bond Return Predictability." Staff Report, No. 581, FRB New York, Working paper.
- Hastie, T., R. Tibshirani, and J. Friedman. 2008 *The Elements of Statistical Learning, Data Mining, Inference, and Prediction*, 2nd ed Berlin: Springer.
- Huang, J., and Z. Shi. 2009. "Determinants of Bond Risk Premia." Manuscript, Penn State University.
- Joslin, S., M. Priebsch, and K. J. Singleton. 2014. "Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks." *Journal of Finance* 69: 1197–1233.
- Jurado, K., S. C. Ludvigson, and S. Ng. 2015. "Measuring Uncertainty." *The American Economic Review* 105: 1177–1216.
- Kose, M. A., C. Otrok, and C. H. Whiteman. 2003. "International Business Cycles: World, Region, and Country-Specific Factors." *American Economic Review* 93: 1216–1239.
- Ludvigson, S. C., and S. Ng. 2009. "Macro Factors in Bond Risk Premia." *Review of Financial Studies* 22: 5027–5067.
- Ludvigson, S. C., and S. Ng. 2011. "A Factor Analysis of Bond Risk Premia." In: edited by A. Ulah, and D. Giles. *Handbook of Empirical Economics and Finance*, 313–372. London: Chapman and Hall.

- Moench, E., S. Ng, and S. Potter. 2013. "Dynamic Hierarchical Factor Models." *Review of Economics and Statistics* 95: 1811–1817.
- Newey, W. K., and K. D. West. 1987. "A Simple, Positive Semi-definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica* 55: 703–708.
- Piazzesi, M., and E. Swanson. 2006. "Futures Prices as Risk-Adjusted Forecasts of Monetary Policy." FRB San Francisco Working Paper 2006-23.
- Wang, P. 2008. "Large Dimensional Factor Models with a Multilevel Factor Structure: Identification, Estimation and Inference." mimeo, Hong Kong University of Science and Technology.
- Wright, Jonathan H. 2011. "Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset." *American Economic Review* 101: (4): 1514–34.

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