Bayesian analysis of structural correlated unobserved components and identification via heteroskedasticity – Supplementary appendix

Mengheng Li[a,b*] *Ivan Mendieta-Muñoz*[c]

[a]*Economics Discipline Group, University of Technology Sydney, Sydney, Australia*
[b]*Centre for Applied Macroeconomic Analysis, Australian National University, Canberra, Australia*
[c]*Department of Economics, University of Utah, Salt Lake City, USA*

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Abstract

This supplementary appendix uses the same notations as in the main text, unless stated otherwise. Section 1, 2 and 3 provide proofs for Proposition 1, 2 and 3, respectively. Section 4 details the sampling procedure, including methods for system parameters that are not included in the main text due to limited space. Section 5 provides extra evidence on the stability of the structural matrix A.

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*Corresponding author: M. Li, University of Technology Sydney, UTS Business School, Ultimo NSW 2007, Australia. mengheng.li@uts.edu.au

1 Proof of Proposition 1

From the autocovariance function of the VARIMA$(p, 1, d)$ representation, $\mathcal{X}$ only involves the coefficient matrices of the VAR(p) cycles. Let C_{N^2} denote the commutation matrix such that for any $N \times N$ matrix M for which the relation $C_{N^2}\text{vec}(M) = \text{vec}(M')$ holds, so $\mathcal{X}$ is given by

$$\mathcal{X} = \begin{bmatrix} D_N^+ \left(I_{N^2} + \sum_{i=1}^{p} B_{c,i} \otimes B_{c,i}\right) D_N & 2D_N^+ & 2D_N^+(I_{N^2} + I_N \otimes B_1) \\ \left(-I_N \otimes B_{c,1} + \sum_{i=1}^{p-1} B_{c,i} \otimes B_{c,i+1}\right) D_N & -I_{N^2} & -C_{N^2} - I_N \otimes B_{c,1} + I_N \otimes B_{c,2} \\ \left(-I_N \otimes B_{c,2} + \sum_{i=1}^{p-2} B_{c,i} \otimes B_{c,i+2}\right) D_N & \mathbf{0}_{N^2 \times N^2} & -I_N \otimes B_{c,2} + I_N \otimes B_{c,3} \\ \vdots & \vdots & \vdots \\ \left(-I_N \otimes B_{c,p-1} + B_{c,1} \otimes B_{c,p}\right) D_N & \mathbf{0}_{N^2 \times N^2} & -I_N \otimes B_{c,p-1} + I_N \otimes B_{c,p} \\ (-I_N \otimes B_{c,p}) D_N & \mathbf{0}_{N^2 \times N^2} & -I_N \otimes B_{c,p} \end{bmatrix}.$$

To show that the $(N^2p + \frac{1}{2}N^2 + \frac{1}{2}N) \times \frac{1}{2}(5N^2 + N)$ matrix $\mathcal{B}$ is of full rank, it suffices to show that via elementary row operations the transformed matrix has $\frac{1}{2}(5N^2+N)$ non-zero rows, so that β is uniquely determined and so is Ω. Notice that $\text{vec}(\Omega_c)'$ has $N(N-1)/2$ elements that are shown up twice; but since $\mathcal{Y}$ and $\mathcal{X}$ are implied by the CUC model, it automatically guarantees such a structure.

Let $\mathcal{X}_{ij}^+$ denote the ij-th block of matrix $\mathcal{X}$ detailed above, $i = 1, ..., p+1$ and $j = 1, 2, 3$; let $\mathcal{X}^*$ denote its transformation via elementary row operations with block $\mathcal{X}_{ij}^*$. If $\mathcal{X}^+$ has rank $\frac{1}{2}(5N^2 + N)$, we should be able to construct $\mathcal{X}^*$ such that $\mathcal{X}_{33}^*$ is of full rank N^2. According to Assumption 1, we can construct a coefficient sequence ρ_i for $i = 3, ..., p+1$ with

$$\rho_i = \begin{cases} -c_{i-1}, & \text{for } i = 3, \\ -\sum_{j=2}^{i-1} c_j, & \text{for } 4 \leq i \leq p+1. \end{cases}$$

This allows for $\mathcal{X}_{33}^*$ to be constructed by

$$\mathcal{X}_{33}^* = \sum_{i=3}^{p+1} \rho_i \mathcal{X}_{i3}^+ = I_N \otimes \bar{B},$$

which is of full rank N^2 under Assumption 1. So we align $\mathcal{X}_{3j}^*$ with $\sum_{i=3}^{p+1} \rho_i \mathcal{X}_{ij}^+$ for $j = 1, 2$. Then we construct

$$\mathcal{X}_{2j}^* = \mathcal{X}_{2j}^+ + \sum_{i=3}^{p+1} (\rho_i + 1)\mathcal{X}_{ij}^+, \quad j = 1, 2, 3.$$

And the first row block of $\mathcal{X}^*$ is the same as that of $\mathcal{X}^+$.

According to Morley et al. (2003) and Trenkler and Weber (2016), $\mathcal{X}^*$ has a rank deficit only if there exists a $\frac{1}{2}(N+1)N \times 1$ vector f_1 and a $N^2 \times 1$ vector f_2 such that

$$(f_1'(\mathcal{X}_{13}^* + 2D_N^+\mathcal{X}_{23}^*) - f_2'\mathcal{X}_{33}^*)D_N = \mathbf{0}_{1\times\frac{1}{2}N(N+1)}, \tag{1}$$

$$f_1'(\mathcal{X}_{11}^* + 2D_N^+\mathcal{X}_{21}^*) - f_2\mathcal{X}_{31}^* = \mathbf{0}_{1\times\frac{1}{2}N(N+1)}. \tag{2}$$

It can be easily verified that $\mathcal{X}_{13}^* + 2D_N^+\mathcal{X}_{23}^* = 2D_N^+\mathcal{X}_{33}^*$; so (1) gives $f_2' = 2f_1'D_N^+$. With the constructed full rank $\mathcal{X}_{33}^* = (I_N \otimes \bar{B})$, it follows from (2) that we need

$$f_1'\big(D_N^+[(I_N - \sum_{i=1}^p B_{c,i}) \otimes (I_N - \sum_{i=1}^p B_{c,i})]D_N\big) = \mathbf{0}_{1\times\frac{1}{2}N(N+1)}$$

to ensure rank deficit. But this is not possible under the condition that the VAR(p) cycles are stable. This means the rank of $\mathcal{X}^*$ must be full, and so is $\mathcal{X}^+$.

2 Proof of Proposition 2

It follows from Assumption 1 that the following decomposition holds

$$\Omega_1^{-1} = C'C, \quad \Omega_2 = C'\mathrm{diag}(\omega_{1,2}, ..., \omega_{K,2})C.$$

Lanne et al. (2010) show that C, with diagonal elements $c_{11}, ..., c_{KK}$, is unique up to row order and sign change if and only if for all $i, j \in \{1, ..., K\}$, $i \neq j$ we have $\omega_{i,2} \neq \omega_{j,2}$.

This result also applies to our setting by defining $A = \mathrm{diag}(c_{11}, ..., c_{KK})^{-1}C$, so that A has unit diagonal. This means that A is a normalised version of C and its row ordering and sign are determined since the normalisation is done via its diagonal elements. By replacing C with $\mathrm{diag}(c_{11}, ..., c_{KK})A$ we have $\Omega_i^{-1} = A'\Sigma_i^{-1}A$ for $i = 1, 2$, or $\Omega_i = A^{-1}\Sigma_i A^{-1\prime}$.

3 Proof of Proposition 3

Similar to the discussion above, writing $\Omega_1^{-1} = C'C$ yields

$$\Omega_t = C'\mathrm{diag}(\sigma_{1,t}^2/\sigma_{1,1}^2, ..., \sigma_{K,t}^2/\sigma_{K,1}^2)C, \quad t = 2, ..., T.$$

Based on Bertsche et al. (2018), C with diagonal elements $c_{11}, ..., c_{KK}$ is identified up to row order and sign change *a.s.* under the random walk specification. Defining $A = \text{diag}(c_{11}, ..., c_{KK})^{-1}C$ (so that it has unit diagonal), and replacing C by $\text{diag}(c_{11}, ..., c_{KK})A$, we have $\Omega_t^{-1} = A'\Sigma_t^{-1}A$. Uniqueness is achieved by respecting that A is a normalised version of C and its row ordering and sign are determined. The result also carries over to the case where there exists one structural shock having constant volatility, as proven by Bertsche et al. (2018).

4 Details of the sampling procedure

Sampling $\delta_{i,j}$.

Firstly, notice that if $\delta_{i,j}$ is $N(0, \gamma_j)$-distributed, a new draw $\delta_{i1}^* = (\delta_{i,1}, ..., \delta_{i,i-1}, \delta_{i,i+1}, ..., \delta_{i,N+i-1}, \delta_{N+i+1}, ..., \delta_{i,k})'$ can be generated from $N(\mu_{i1}^\delta, \Psi_{i1}^\delta)$, where

$$\Psi_{i1}^\delta = \left(\sum_{t=2}^{T} \frac{\tilde{x}_t \tilde{x}_t'}{(\delta_{i,N+i} - 1)^2 \sigma_{i,s_{t-1}}^2 \omega_{i,s_{t-1}}} + \text{diag}(\gamma_1, ...\gamma_{i-1}, \gamma_{i+1}, ...\gamma_{N+i-1}, \gamma_{N+i+1}, ..., \gamma_K) \right)^{-1},$$

$$\mu_{i1}^\delta = \Psi_{i1}^\delta \sum_{t=2}^{T} \frac{\tilde{x}_t(\tau_{i,t}^* - \delta_{i,N+i} y_{i,t}^*)}{(\delta_{i,N+i} - 1)^2 \sigma_{i,s_{t-1}}^2 \omega_{i,s_{t-1}}}.$$

If the initialisation of unobserved components is omitted, the new draw is accepted with probability of one, but not from the correct conditional posterior. Notice that in

$$\text{vec}(V_{c,1}) = (I_{Np} - B_c^* \otimes B_c^*)^{-1} \text{vec} \left(\sum_{i=1}^{2} \pi_i R_c \Omega_i R_c' \right),$$

B_c^* and Ω_i, $i = 1, 2$ are functions of A, and thus of δ_{i1}^* conditional on other parameters, so with initialisation considered, the draw is accepted with probability

$$\min \left\{ \frac{|V_{c,1}(\delta_{i1}^*)|^{-1/2} \exp(-\frac{1}{2} x_{c,1}' V_{c,1}(\delta_{i1}^*)^{-1} x_{c,1})}{|V_{c,1}(\delta_{i1})|^{-1/2} \exp(-\frac{1}{2} x_{c,1}' V_{c,1}(\delta_{i1})^{-1} x_{c,1})}, 1 \right\},$$

where δ_{i1} (without asterisk) is the previous draw in the Markov chain and $x_{c,1}$ denotes the cycle components in x_1, *i.e.* $x_{c,1} = (c_1, ..., c_{2-p})'$.

Secondly, defining $\tilde{y}_{i,t} = \tau_{i,t}^* - \delta_{i,1}\tau_{1,t} - ... - \delta_{i,i-1}\tau_{i-1,t} - \delta_{i,i+1}\tau_{i+1,t} - ... - \delta_{i,N+1}c_{1,t} -$

$... - \delta_{i,K} c_{N,t}$, the proposed transformation

$$\begin{aligned}\tau^*_{i,t} =& \delta_{i,1}\tau_{1,t} + ... + \delta_{i,i-1}\tau_{i-1,t} + \delta_{i,i+1}\tau_{i+1,t} + ... + \delta_{i,N}\tau_{N,t} \\ &+ \delta_{i,N+1}c_{1,t} + ... + \delta_{i,N+i}y^*_{i,t} + ... + \delta_{i,K}c_{N,t} + (\delta_{i,N+i} - 1)e_{i,t-1},\end{aligned}$$

becomes

$$\tilde{y}_{i,t} = \delta_{i,N+1}y^*_{i,t} + (\delta_{i,N+i} - 1)e_{i,t-1}.$$

Ignoring initialisation, the conditional posterior follows

$$p(\delta_{i,N+1}|.) \propto (\delta_{i,N+1} - 1)^{-T+1} \exp\left(-\frac{1}{2}\sum_{t=2}^{T} \frac{(\tilde{y}_{i,t} - \delta_{i,N+i}y^*_{i,t})^2}{(\delta_{i,N+i} - 1)^2 \sigma^2_{i,s_{t-1}} \omega_{i,s_{t-1}}} - \frac{1}{2}\frac{\delta^2_{i,N+i}}{\gamma_{N+i}} \right). \quad (3)$$

This non-standard univariate distribution can be well-approximated by a Student's t-proposal with mean μ^δ_{i2} and scale parameter Ψ^δ_{i2} equal to the mode and curvature around the mode of the above density function.[1] That is, we apply Newton's method

$$\delta^{(n+1)}_{i,N+i} = \delta^{(n)}_{i,N+i} - \frac{p'(\delta^{(n)}_{i,N+i}|.)}{p''(\delta^{(n)}_{i,N+i}|.)}$$

to find the mode iteratively, with $\delta^{(1)}_{i,N+i} = \text{Cov}(\tilde{y}_{i,t}, y^*_{i,t})/\text{Var}(y^*_{i,t})$ and

$$\begin{aligned} p'(\delta_{i,N+i}|.) =& -(T-1)\frac{1}{\delta_{i,N+i} - 1} + (\delta_{i,N+i} - 1)^{-3} \sum_{t=2}^{T} \frac{(\tilde{y}_{i,t} - \delta_{i,N+1}y^*_{i,t})^2}{\sigma^2_{i,s_{t-1}}\omega_{i,s_{t-1}}} \\ &+ (\delta_{i,N+i} - 1)^{-2} \sum_{t=2}^{T} \frac{(\tilde{y}_{i,t} - \delta_{i,N+1}y^*_{i,t})y^*_{i,t}}{\sigma^2_{i,s_{t-1}}\omega_{i,s_{t-1}}} - \frac{1}{\gamma_{N+i}}\delta_{i,N+i}, \\ p''(\delta_{i,N+i}|.) =& (T-1)(\delta_{i,N+i} - 1)^{-2} - 3(\delta_{i,N+i} - 1)^{-4} \sum_{t=2}^{T} \frac{(\tilde{y}_{i,t} - \delta_{i,N+1}y^*_{i,t})^2}{\sigma^2_{i,s_{t-1}}\omega_{i,s_{t-1}}} \\ &- 4(\delta_{i,N+i} - 1)^{-3} \sum_{t=2}^{T} \frac{(\tilde{y}_{i,t} - \delta_{i,N+1}y^*_{i,t})y^*_{i,t}}{\sigma^2_{i,s_{t-1}}\omega_{i,s_{t-1}}} - \frac{1}{\gamma_{N+i}}, \end{aligned}$$

until some convergence criterion is met. Let μ^δ_{i2} denote the mode and define $\Psi^\delta_{i2} = -1/p''(\mu^\delta_{i2}|.)$. A new draw $\delta^*_{i,N+i}$ is generated from a Student' t-distribution $T(\mu^\delta_{i2}, \Psi^\delta_{i2}, \nu)$,

[1] More efficient proposals such as a mixture of Student's t-distributions can be easily constructed (Basturk et al., 2017) due to the fact that we deal with a univariate distribution. We find that a simple Student's t-proposal suffices.

with the degrees of freedom ν arbitrarily chosen. The draw is accepted with probability

$$\min\left\{\frac{|V_{c,1}(\delta^*_{i,N+i})|^{-1/2}\exp\left(-\frac{1}{2}x'_{c,1}V_{c,1}(\delta^*_{i,N+i})^{-1}x_{c,1}\right)p(\delta^*_{i,N+i}|.)T(\delta_{i,N+i};\mu^\delta_{i2},\Psi^\delta_{i2},\nu)}{|V_{c,1}(\delta_{i,N+i})|^{-1/2}\exp\left(-\frac{1}{2}x'_{c,1}V_{c,1}(\delta_{i,N+i})^{-1}x_{c,1}\right)p(\delta_{i,N+i}|.)T(\delta^*_{i,N+i};\mu^\delta_{i2},\Psi^\delta_{i2},\nu)},1\right\},$$

where $p(\delta_{i,N+i}|.)$ is the density kernel in (3) and $T(\delta_{i,N+i};\mu^\delta_{i2},\Psi^\delta_{i2},\nu)$ is the density of constructed Student's t-proposal evaluated at $\delta_{i,N+i}$. Once $\delta_{i,j}$ and $\delta_{i,N+i}$ are generated, A_{i-} is computed using

$$\delta_{i,j} = -\frac{A_{ij}}{1-A_{i(N+i)}} = -A_{ij}(1-\delta_{i,N+i}), \quad j \in \{1,...,i-1,i+1,...,K\},$$

as given in the main text.

Sampling Φ, $\sigma^2_{i,1}$, $\omega_{i,2}$, $\mathtt{P}$, γ_Φ and γ_i.
We sample Φ independently equation-by-equation. Based on $B = A^{-1}\Phi(L_p)$, we notice that only the sampling of Φ_{cc} is needed (*i.e.*, the autoregressive coefficient matrix for the VAR(p) cycles). Using

$$A\dot{x}_{t+1} = \Phi x_t + e_t, \quad \dot{x}_{t+1} = (\tau'_{t+1}, c'_{t+1})', \tag{4}$$

standard Bayesian calculation shows that posterior draws of Φ'_{i-} can be generated from $N(A_{i-}\mu^\Phi_i, \Psi^\Phi_i)\mathbb{1}_{\{||B^*_c||<1\}}$, where

$$\Psi^\Phi_i = \left(\sum_{t=1}^{T-1} x'_t\Sigma^{-1}_{s_t}x_t + \frac{1}{\gamma_\Phi}L^{-1}\right)^{-1},$$
$$\mu^\Phi_i = \Psi^\Phi_i\left(\sum_{t=1}^{T-1} x'_t\Sigma^{-1}_{s_t}\dot{x}_{t+1} + \frac{1}{\gamma_\Phi}L^{-1}W\right),$$

and $||B^*_c||$ denotes the largest eigenvalue in absolute value of B^*_c so that the indicator function $\mathbb{1}_{\{||B^*_c||<1\}}$ guarantees that the VAR(p) cycles are stationary. Taking initialisation into account, the draw is accepted with probability

$$\min\left\{\frac{|V_{c,1}(\Phi^*_{i-})|^{-1/2}\exp(-\frac{1}{2}x'_{c,1}V_{c,1}(\Phi^*_{i-})^{-1}x_{c,1})}{|V_{c,1}(\Phi_{i-})|^{-1/2}\exp(-\frac{1}{2}x'_{c,1}V_{c,1}(\Phi_{i-})^{-1}x_{c,1})},1\right\}.$$

For $i = 1, ..., K$, the volatility parameter $\sigma^2_{i,1}$ can be sampled from

$$IG\left(\alpha_v + \frac{T}{2}, \beta_v + \sum_{t=1}^{T-1} \frac{(A_{i-}\dot{x}_{t+1} - \Phi_{i-}x_t)^2}{2\omega_{i,s_t}}\right)$$

and the variance ratio $\omega_{i,2}$ can be sampled from

$$IG\left(\alpha_\omega + \frac{T_2}{2}, \beta_\omega + \sum_{t\in T_2} \frac{(A_{i-}\dot{x}_{t+1} - \Phi_{i-}x_t)^2}{2\sigma^2_{i,s_t}}\right).$$

These draws are accepted with probability

$$\min\left\{\frac{|V_{c,1}(\sigma^{2*}_{i,1}\omega^*_{i,j})|^{-1/2}\exp(-\frac{1}{2}x'_{c,1}V_{c,1}(\sigma^{2*}_{i,1}\omega^*_{i,j})^{-1}x_{c,1})}{|V_{c,1}(\sigma^2_{i,1}\omega_{i,j})|^{-1/2}\exp(-\frac{1}{2}x'_{c,1}V_{c,1}(\sigma^2_{i,1}\omega_{i,j})^{-1}x_{c,1})}, 1\right\}, \quad j = 1, 2.$$

The posterior draws of transition probability are sampled via a 2-dimensional Dirichlet distribution. Specifically, we draw $\mathtt{P}'_{1-}$ and $\mathtt{P}'_{2-}$ from

$$Dir_2(e_1+\sum_{t=2}^{T}\mathbb{1}_{\{s_{t-1}=1,s_t=1\}}, e_2+\sum_{t=2}^{T}\mathbb{1}_{\{s_{t-1}=1,s_t=2\}}),\ \ Dir_2(e_2+\sum_{t=2}^{T}\mathbb{1}_{\{s_{t-1}=2,s_t=1\}}, e_1+\sum_{t=2}^{T}\mathbb{1}_{\{s_{t-1}=2,s_t=2\}}),$$

respectively, where $\sum_{t=2}^{T}\mathbb{1}_{\{s_{t-1}=j,s_t=i\}}$ counts the number of transitions from volatility regime j to i. The new draw is accepted with probability

$$\min\left\{\frac{|V_{c,1}(\mathtt{P}^*)|^{-1/2}\exp(-\frac{1}{2}x'_{c,1}V_{c,1}(\mathtt{P}^*)^{-1}x_{c,1})}{|V_{c,1}(\mathtt{P})|^{-1/2}\exp(-\frac{1}{2}x'_{c,1}V_{c,1}(\mathtt{P})^{-1}x_{c,1})}, 1\right\}.$$

Based on the updated transition probability, the index process of two Markov regimes S_T is sampled using the forward filter and backward simulation smoother of Chib (1996) with initialisation taken into account. Through $e_t = A\dot{x}_{t+1} - \Phi x_t$ and $e_t \sim N(0, \Sigma_{s_t})$, the algorithm utilises $p(S_T|e_1, ..., e_T) = \prod_{t=1}^{T} p(s_t|Y_T, s_{t+1}, ..., s_T, \mathtt{P})$ and $p(s_t|Y_T, s_{t+1}, ..., s_T, \mathtt{P}) \propto p(s_t|e_t)p(s_{t+1}|s_t, \mathtt{P})$ by a forward recursion that determines $p(s_t|e_t)$ and a backward recursion that draws posterior samples of S_T.

Finally, the posterior draws of shrinkage parameters are directly generated from an inverse gamma distribution due to conjugacy. We draw γ_Φ from $IG(\alpha_\gamma + \frac{K^2p}{2}, \beta_\gamma + \frac{1}{2}\sum_{i=1}^{K}(\Phi_{i-} - A_{i-}W)(\Phi_{i-} - A_{i-}W)')$ and γ_i from $IG(\alpha_\gamma + \frac{K-1}{2}, \beta_\gamma + \frac{1}{2}A'_{-i}A_{-i})$ for $i = 1, ..., K$.

Sampling $\sigma_{i,t}$

Under SV, we have $A_{i-}\dot{x}_{t+1} = \Phi_{i-}x_t + e_{i,t}$ as in (4) with $e_{i,t} \sim N(0, \sigma^2_{i,t})$ for $i = 1, ..., K$. We adopt the method of Kim et al. (1998) using a 7-component Gaussian mixture to draw from $p(\log \sigma_{i,t} | \{e_{i,t}\}_{t=1}^{T-1}, \rho_i, \boldsymbol{s}_T)$ and $p(s_t | \{\log \sigma_{i,t}\}_{t=1}^{T-1}, \{e_{i,t}\}_{t=1}^{T-1})$ iteratively, where $s_t = \{1, ..., 7\}$ here is a tabulated indicator auxiliary process selecting the mixing component.

5 Robustness check

Table 1 reports the posterior mean estimate of the structural matrix A in the SCUC model using the data up until the GFC. This serves as a robustness check in order to corroborate if the inclusion of this recent period introduces any changes in the A matrix. The results do not suggest significant differences from what is reported in the main text. This is so because even if the high volatility brought about by the GFC is ignored, the data still shows two distinct volatility states, namely the high volatility state in the 1980s and the low volatility state after the "Great Moderation".

Table 1: Structural matrix A obtained from the SCUC model for the US Phillips curve prior to the Great Financial Crisis

Two volatility states Markov regime switching

	$\tau_{1,t}$	$\tau_{g,t}$	$\tau_{2,t}$	$c_{1,t}$	$c_{2,t}$
$\tau_{1,t}$	1	0.17 −3.6	−0.04 −8.4	0.46 **−24.8**	0.10 −4.2
$\tau_{g,t}$	0.14 **−11.2**	1	−0.55 **−30.4**	0.26 **−14.2**	−0.22 **−17.3**
$\tau_{2,t}$	0.09 −3.0	0.27 **−18.3**	1	−0.38 **−12.7**	0.22 **−15.8**
$c_{1,t}$	−0.37 **−23.6**	−0.22 **−19.5**	0.25 **−31.4**	1	−0.61 **−28.1**
$c_{2,t}$	0.22 **−25.6**	−0.01 −0.1	0.09 −4.4	−0.23 **−21.0**	1

Stochastic volatility

	$\tau_{1,t}$	$\tau_{g,t}$	$\tau_{2,t}$	$c_{1,t}$	$c_{2,t}$
$\tau_{1,t}$	1	0.10 −6.4	−0.06 3.4	0.37 **−33.2**	0.04 −2.3
$\tau_{g,t}$	0.08 −9.2	1	−0.52 **−18.2**	0.34 **−26.9**	−0.30 **−11.5**
$\tau_{2,t}$	0.11 −6.8	0.22 **−32.7**	1	−0.42 **−24.1**	0.31 **−30.3**
$c_{1,t}$	−0.23 **−18.2**	−0.21 **−24.0**	0.18 **−22.9**	1	−0.54 **−33.8**
$c_{2,t}$	0.20 −2.3	−0.04 1.1	0.14 −9.2	−0.18 **−17.2**	1

Reported is the posterior mean estimate of the structural matrix A obtained from the SCUC model, using data prior to the GFC. Below the posterior mean is the SDDR for testing $H_0 : A_{ij} = 0$, with boldface numbers indicating strong evidence against H_0. The upper panel shows estimates identified considering the two-volatility-state Markov regime switching, while the bottom panel shows estimates identified considering stochastic volatility.

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