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On interpolative Hardy-Rogers contractive of Suzuki type mappings

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Abstract: In this paper, we obtain a fixed point theorem ω - ψ -interpolative Hardy-Rogers contractive of Suzuki type mappings. In the following, we present an example to illustrate the new theorem is applicable. Subsequently, some results are given. These results generalize several new results present in the literature.

Keywords: metric space, interpolative, Suzuki type contraction.

MSC: 47H10, 54H25

1 Introduction

The fixed point theory is one of the main research areas in nonlinear analysis. In metric spaces, this theory began with the Banach fixed point theorem. Banach [7] introduced Banach fixed point theorem known as the “Banach contraction principle”. This principle states that:

Theorem 1.1. [7] Let (X, d) be a complete metric space and let T be a mapping of X into itself such that

$$d(Tx, Ty) \leq \alpha d(x, y).$$

where $\alpha \in [0, 1)$ and $x, y \in X$. Then T has a unique fixed point.

There were many generalizations of this theorem. One of the generalizations was given by Kannan [20], which characterize the completeness of underlying metric spaces.

Theorem 1.2. [20] Let (X, d) be a complete metric space. A mapping $T : X \rightarrow X$ is said to be a Kannan contraction if there exists $\lambda \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq \lambda(d(x, Tx) + d(y, Ty))$$

for all $x, y \in X \setminus \text{Fix}(T)$. Then, T possesses a unique fixed point.

Kannan’s fixed point theorem [20] has been generalized in different ways by many authors. In 2018, Karapınar [18] used the interpolative approach to define the generalized Kannan-type contraction and proved a fixed point result on it.

A mapping $T : X \rightarrow X$ on (X, d) a complete metric space such that

$$d(Tx, Ty) \leq \kappa[d(x, Tx)]^\alpha \cdot [d(y, Ty)]^{1-\alpha}$$

where $\kappa \in [0, 1)$ and $\alpha \in (0, 1)$, for each $x, y \in X \setminus \text{Fix}(T)$. Thus T has a fixed point in X .

One another of the most interesting the Banach contraction principle generalizations of it was given by Hardy-Rogers[11].

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Theorem 1.3. [11] Let (X, d) be a complete metric space. The mapping $T : X \rightarrow X$ is called an interpolative Hardy-Rogers type contraction if there exist positive reals $\beta, \alpha, \gamma, \delta > 0$, with $\beta + \alpha + \gamma + \delta < 1$, such that

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) + \delta \left[\frac{1}{2} (d(x, Ty) + d(y, Tx)) \right]$$

for each $x, y \in X \setminus \text{Fix}(T)$. Then, the mapping T has a unique fixed point in X .

Following this, Karapınar *et al.* [12] introduced the following notion of interpolative Hardy-Rogers type contraction.

Theorem 1.4. [12] Let (X, d) be a complete metric space. The mapping $T : X \rightarrow X$ is called an interpolative Hardy-Rogers type contraction if there exist $\lambda \in [0, 1)$ and positive reals $\beta, \alpha, \gamma > 0$, with $\beta + \alpha + \gamma < 1$, such that

$$d(Tx, Ty) \leq \lambda [d(x, y)]^\beta \cdot [d(x, Tx)]^\alpha \cdot [d(y, Ty)]^\gamma \cdot \left[\frac{1}{2} (d(x, Ty) + d(y, Tx)) \right]^{1-\alpha-\beta-\gamma}$$

for each $x, y \in X \setminus \text{Fix}(T)$. Then the mapping T has a fixed point in X .

Some interesting results in this concept may be found in the work of [2, 4, 6, 10, 15].

In the following, we recollect the notions of ω -orbital admissible mappings. The concept of ω -orbital admissible mappings was introduced by Popescu as a clarification of the concept of α -admissible mappings of Samet *et al.* [24].

Definition 1.5. [22] Let T be a self map defined on X and $\omega : X \times X \rightarrow [0, \infty)$ be a function. T is said to be an ω -orbital admissible if for all $x \in X$, we have:

$$\omega(x, Tx) \geq 1 \Rightarrow \omega(Tx, T^2x) \geq 1.$$

In our theorem, the following condition has often been considered in order to avoid the continuity of the involved contractive mappings.

(R) A spaces (X, d) is defined ω -regular, if $\{x_n\}$ is a sequence in X which $x_n \rightarrow t \in X$ as $n \rightarrow \infty$ and satisfies $\omega(x_n, x_{n+1}) \geq 1$ for each n and then, we have $\omega(x_n, t) \geq 1$.

The existence results of fixed points of this sense maps have been extensively studied, see [3, 5, 9, 14, 19].

Describe using Ψ the set of all nondecreasing self-mappings ψ on $[0, \infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$. Regard that for $\psi \in \Psi$, we have $\psi(0) = 0$ and $\psi(t) < t$ for each $t > 0$, see; [1, 23].

Popescu [21] introduced two generalizations of a result given by Bogin [8] for a class of non-expansive mappings on complete metric spaces. The aim of his work was to replace the non-expansiveness condition with the weaker C -condition investigated by Suzuki [26]. Karapınar [13] introduced the definition of a non-expansive mapping satisfying the C -condition. Subsequently, the existence of fixed points of maps satisfying the C -condition has been extensively studied; see [16, 17, 25, 27, 28]. We state first the definition of a non-expansive map and a map satisfying the C -condition on a metric space.

Definition 1.6. A mapping T on a metric space (X, d) is called a non-expansive mapping if

$$d(Tx, Ty) \leq d(x, y)$$

for all $x, y \in X$.

Definition 1.7. A mapping T on a metric space (X, d) satisfies the C -condition if

$$\frac{1}{2} d(x, Tx) \leq d(x, y) \Rightarrow d(Tx, Ty) \leq d(x, y)$$

for each $x, y \in X$.

2 Main Results

First, let's start with the definition of ω - ψ -interpolative Hardy-Rogers contractive of Suzuki type.

Definition 2.1. Let (X, d) be a metric space. The mapping $T : X \rightarrow X$ is called an ω - ψ -interpolative Hardy-Rogers contractive of Suzuki type. If there exist $\psi \in \Psi$, $\omega : X \times X \rightarrow [0, \infty)$ and positive reals $\beta, \alpha, \gamma > 0$, with $\beta + \alpha + \gamma < 1$, such that

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \Rightarrow \omega(x, y)d(Tx, Ty) \leq \psi([d(x, y)]^\beta \cdot [d(x, Tx)]^\alpha \cdot [d(y, Ty)]^\gamma \cdot \left[\frac{1}{2}(d(x, Ty) + d(y, Tx))\right]^{1-\alpha-\beta-\gamma}) \quad (1)$$

for each $x, y \in X \setminus \text{Fix}(T)$.

Theorem 2.2. Let (X, d) be a complete metric space and the mapping $T : X \rightarrow X$ be an ω - ψ -interpolative Hardy-Rogers contraction of Suzuki type if T is ω -orbital admissible mapping and $\omega(x_0, Tx_0) \geq 1$ for some $x_0 \in X$. Therefore, the mapping T has a fixed point in X provided that at least one of the following properties holds:

- i. (X, d) is ω -regular;
- ii. T is continuous;
- iii. T^2 is continuous and $\omega(x, Tx) \geq 1$ where $x \in \text{Fix}(T^2)$.

Proof. Let $x_0 \in X$ satisfy $\omega(x_0, Tx_0) \geq 1$. Let $\{x_n\}$ be the sequence constructed by $T^n(x_0) = x_n$ for each positive integer n . Assume that $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}$, so $x_{n_0} = Tx_{n_0}$ that means x_{n_0} is a fixed point of T . Then, we can suppose that $x_n \neq x_{n+1}$ for each positive integer n . As T is ω -orbital admissible $\omega(x_0, Tx_0) = \omega(x_0, x_1) \geq 1$ implies that $\omega(x_1, Tx_1) = \omega(x_1, x_2) \geq 1$. Similarly, continuing this process,

$$\omega(x_n, x_{n+1}) \geq 1. \quad (2)$$

By substituting the assign $x = x_{n-1}$ and $y = Tx_{n-1} = x_n$ in (1), we obtain

$$\begin{aligned} \frac{1}{2}d(x_{n-1}, Tx_{n-1}) &= \frac{1}{2}d(x_{n-1}, x_n) \leq d(x_{n-1}, x_n) \Rightarrow \\ d(x_n, x_{n+1}) &\leq \omega(x_{n-1}, x_n)d(Tx_{n-1}, Tx_n) \\ &\leq \psi([d(x_{n-1}, x_n)]^\beta \cdot [d(x_{n-1}, Tx_{n-1})]^\alpha \cdot [d(x_n, Tx_n)]^\gamma \\ &\quad \cdot \left[\frac{1}{2}(d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1}))\right]^{1-\alpha-\beta-\gamma}) \\ &= \psi([d(x_{n-1}, x_n)]^\beta \cdot [d(x_{n-1}, x_n)]^\alpha \cdot [d(x_n, x_{n+1})]^\gamma \\ &\quad \cdot \left[\frac{1}{2}(d(x_{n-1}, x_{n+1}) + d(x_n, x_n))\right]^{1-\alpha-\beta-\gamma}) \end{aligned} \quad (3)$$

thus, using $\psi(p) < p$ for $p > 0$

$$\begin{aligned} d(x_n, x_{n+1}) &\leq \psi([d(x_{n-1}, x_n)]^\beta \cdot [d(x_{n-1}, x_n)]^\alpha \cdot [d(x_n, x_{n+1})]^\gamma \\ &\quad \cdot \left[\frac{1}{2}(d(x_{n-1}, x_n) + d(x_n, x_{n+1}))\right]^{1-\alpha-\beta-\gamma}) \\ &< [d(x_{n-1}, x_n)]^\beta \cdot [d(x_{n-1}, x_n)]^\alpha \cdot [d(x_n, x_{n+1})]^\gamma \\ &\quad \cdot \left[\frac{1}{2}(d(x_{n-1}, x_n) + d(x_n, x_{n+1}))\right]^{1-\alpha-\beta-\gamma} \end{aligned} \quad (4)$$

then, assume that

$$d(x_{n-1}, x_n) < d(x_n, x_{n+1}) \quad (5)$$

for every $n \in \mathbb{N}$, then,

$$\frac{1}{2} (d(x_{n-1}, x_n) + d(x_n, x_{n+1})) \leq d(x_n, x_{n+1})$$

$$[d(x_n, x_{n+1})]^{\alpha+\beta} < [d(x_{n-1}, x_n)]^{\alpha+\beta}.$$

which is a contradiction, hence we get that for all $n \in \mathbb{N}$,

$$d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n). \quad (6)$$

Then, the positive sequence $\{d(x_{n-1}, x_n)\}$ is non-increasing sequence with positive terms so, we attain that there exists $j \geq 0$ such that $\lim_{n \rightarrow \infty} d(x_{n-1}, x_n) = j$. Accordingly, we get

$$\frac{1}{2} (d(x_{n-1}, x_n) + d(x_n, x_{n+1})) \leq d(x_{n-1}, x_n).$$

Further, by (3)

$$[d(x_n, x_{n+1})]^{1-\gamma} \leq \psi[d(x_{n-1}, x_n)]^{1-\gamma}.$$

or equivalent

$$d(x_n, x_{n+1}) \leq \psi(d(x_{n-1}, x_n)).$$

Thence, by repeating this condition, we can write,

$$d(x_n, x_{n+1}) \leq \psi(d(x_{n-1}, x_n)) \leq \psi^2(d(x_{n-2}, x_{n-1})) \leq \dots \leq \psi^n(d(x_0, x_1)). \quad (7)$$

We claim that $\{x_n\}$ is a fundamental sequence in (X, d) . Then, we shall use the triangle inequality with (7), we can find

$$\begin{aligned} d(x_n, x_{n+l}) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \dots + d(x_{n+l-1}, x_{n+l}) \\ &\leq \psi^n(d(x_0, x_1)) + \psi^{n+1}(d(x_0, x_1)) + \dots + \psi^{n+l-1}(d(x_0, x_1)) \\ &\leq \sum_{k=n}^{\infty} \psi^k(d(x_0, x_1)) \end{aligned} \quad (8)$$

Letting $n \rightarrow \infty$ in (8), we get that $\{x_n\}$ is a fundamental sequence in (X, d) . Regarding by (X, d) is completeness, there exists $t \in X$ such that

$$\lim_{n \rightarrow \infty} d(x_n, t) = 0 \quad (9)$$

We show that the point t is the fixed point of T . If i. holds, that is (X, d) is ω -regular, then $\{x_n\}$ verify (2), that is $\omega(x_n, x_{n+1}) \geq 1$ for every $n \in \mathbb{N}$, we get $\omega(x_n, t) \geq 1$ We assert that

$$\frac{1}{2} d(x_n, Tx_n) \leq d(x_n, t) \text{ or } \frac{1}{2} d(Tx_n, T(Tx_n)) \leq d(Tx_n, t)$$

for every $n \in \mathbb{N}$. Assuming

$$\frac{1}{2} d(x_n, Tx_n) > d(x_n, t) \text{ and } \frac{1}{2} d(Tx_n, T(Tx_n)) > d(Tx_n, t),$$

on the account of the triangle inequality, we obtain

$$\begin{aligned} d(x_n, x_{n+1}) &= d(x_n, Tx_n) \leq d(x_n, t) + d(t, Tx_n) \\ &< \frac{1}{2} d(x_n, Tx_n) + \frac{1}{2} d(Tx_n, T(Tx_n)) \\ &= \frac{1}{2} d(x_n, x_{n+1}) + \frac{1}{2} d(x_{n+1}, x_{n+2}) \\ &\leq \frac{1}{2} d(x_n, x_{n+1}) + \frac{1}{2} d(x_n, x_{n+1}) \\ &= d(x_n, x_{n+1}) \end{aligned}$$

which is contradiction. Therefore, for every $n \in \mathbb{N}$, either

$$\frac{1}{2}d(x_n, Tx_n) \leq d(x_n, t) \quad (10)$$

or

$$\frac{1}{2}d(Tx_n, T(Tx_n)) \leq d(Tx_n, t) \quad (11)$$

holds. In case that (10) holds, we get

$$\begin{aligned} d(x_{n+1}, Tt) &\leq \omega(x_n, t)d(Tx_n, Tt) \\ &\leq \psi([d(x_n, t)]^\beta \cdot [d(x_n, Tx_n)]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(x_n, Tt) + d(t, Tx_n))]^{1-\alpha-\beta-\gamma}) \\ &= \psi([d(x_n, t)]^\beta \cdot [d(x_n, x_{n+1})]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(x_n, Tt) + d(t, x_{n+1}))]^{1-\alpha-\beta-\gamma}) \\ &< [d(x_n, t)]^\beta \cdot [d(x_n, x_{n+1})]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(x_n, Tt) + d(t, x_{n+1}))]^{1-\alpha-\beta-\gamma}. \end{aligned} \quad (12)$$

If the second condition, (11) holds, we have

$$\begin{aligned} d(x_{n+2}, Tt) &\leq \omega(x_{n+1}, t)d(T^2x_n, Tt) \\ &\leq \psi([d(Tx_n, t)]^\beta \cdot [d(Tx_n, T^2x_n)]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(Tx_n, Tt) + d(t, T^2x_n))]^{1-\alpha-\beta-\gamma}) \\ &= \psi([d(x_{n+1}, t)]^\beta \cdot [d(x_{n+1}, x_{n+2})]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(x_{n+1}, Tt) + d(t, x_{n+2}))]^{1-\alpha-\beta-\gamma}) \\ &< [d(x_{n+1}, t)]^\beta \cdot [d(x_{n+1}, x_{n+2})]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(x_{n+1}, Tt) + d(t, x_{n+2}))]^{1-\alpha-\beta-\gamma}. \end{aligned} \quad (13)$$

Therefore, letting $n \rightarrow \infty$ in (12) and (13), we get that $d(t, Tt) = 0$, that is $t = Tt$.

In case that the assumption ii. is true, that is the mapping T is continuous,

$$Tt = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} x_{n+1} = t.$$

If the last assumption, iii. holds, as above, we have $T^2t = \lim_{n \rightarrow \infty} T^2x_n = \lim_{n \rightarrow \infty} x_{n+2} = t$ and we want to show that, also $Tt = t$. Assuming on the contrary, that $t \neq Tt$, since

$$\frac{1}{2}d(Tt, T^2t) = \frac{1}{2}d(Tt, t) \leq d(Tt, t)$$

by (1) we get

$$\begin{aligned} d(t, Tt) &\leq \omega(Tt, t)d(T^2t, Tt) \\ &\leq \psi([d(Tt, t)]^\beta \cdot [d(Tt, T^2t)]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(Tt, Tt) + d(Tt, T^2t))]^{1-\alpha-\beta-\gamma}) \\ &< [d(Tt, t)]^\beta \cdot [d(Tt, t)]^\alpha \cdot [d(t, Tt)]^\gamma \\ &\quad \cdot [\frac{1}{2}(d(Tt, t))]^{1-\alpha-\beta-\gamma} \\ &\leq d(Tt, t), \end{aligned}$$

which is a contradiction. Consequently, $t = Tt$, that is, t is a fixed point of the mapping T . $\square$

Example 2.3. Let $X = [0, 2]$ and $d : X \times X \rightarrow [0, +\infty)$ be the usual metric on $\mathbb{R}$. The mapping $T : X \rightarrow X$ be defined as

$$Tx = \begin{cases} \frac{7}{8}, & \text{if } x \in [0, 1] \\ \frac{x}{3}, & \text{if } x \in (1, 2] \end{cases}$$

Further, let $\omega : X \times X \rightarrow [0, \infty)$, where

$$\omega(x, y) = \begin{cases} 3, & \text{if } x \in [0, 1] \\ 1, & \text{if } x = 0, y = 2 \\ 0, & \text{otherwise} \end{cases}$$

The mapping T is not continuous but, since $T^2 = \frac{7}{8}$ we have that T^2 is a continuous mapping. Let the function $\psi \in \Psi$ defined as $\psi(t) = \frac{t}{4}$ and we choose $\beta = \frac{1}{2}$, $\alpha = \frac{1}{3}$ and $\gamma = \frac{1}{7}$. Then, we have to check that (1) holds. We have to examine the following cases:

Case 1: For $x, y \in [0, 1]$ we obtain $d(Tx, Ty) = 0$, so (1) holds.

Case 2: For $x = 0$ and $y = 2$,

$$\begin{aligned} \frac{1}{2}d(0, T0) &= \frac{7}{16} < 2 = d(0, 2) \Rightarrow \\ \omega(0, 2)d(T0, T2) &= 0.208333333 \leq 0.351427142 \\ &= \frac{1}{4}(2)^{\frac{1}{2}} \cdot \left(\frac{7}{8}\right)^{\frac{1}{3}} \cdot \left(\frac{4}{3}\right)^{\frac{1}{7}} \left(\frac{43}{48}\right)^{\frac{1}{42}} \\ &= \psi([d(0, 2)]^{\frac{1}{2}} \cdot [d(0, T0)]^{\frac{1}{3}} \cdot [d(2, T2)]^{\frac{1}{7}} \cdot \\ &\quad [\frac{1}{2}(d(0, T2) + d(2, T0))]^{\frac{1}{42}}). \end{aligned}$$

For all other cases (1) holds, since $\omega(x, y) = 0$.

As a result, the assumptions of Theorem 2.2 is satisfied, also the mapping T has a fixed point, that is $x = \frac{7}{8}$.

Corollary 2.4. Let (X, d) be a complete metric space and T be a self mapping on X , such that

$$\begin{aligned} \frac{1}{2}d(x, Tx) \leq d(x, y) \text{ implies} \\ d(Tx, Ty) \leq \psi([d(x, y)]^\beta \cdot [d(x, Tx)]^\alpha \cdot [d(y, Ty)]^\gamma \\ \cdot [\frac{1}{2}(d(x, Ty) + d(y, Tx))]^{1-\alpha-\beta-\gamma}) \end{aligned} \quad (14)$$

for each $x, y \in X \setminus \text{Fix}(T)$, where $\psi \in \Psi$ and positive real $\beta, \alpha, \gamma > 0$, with $\beta + \alpha + \gamma < 1$. Then T possesses a fixed point in X .

Proof. In Theorem 2.2 is sufficient to get $\omega(x, y) = 1$ for proof. □

Further, taking $\psi(p) = p\lambda$, with $\lambda \in [0, 1)$ in Corollary (2.4), we obtain the following consequence.

Corollary 2.5. Let (X, d) be a complete metric space and T be a self mapping on X such that

$$\begin{aligned} \frac{1}{2}d(x, Tx) \leq d(x, y) \text{ implies} \\ d(Tx, Ty) \leq \lambda[d(x, y)]^\beta \cdot [d(x, Tx)]^\alpha \cdot [d(y, Ty)]^\gamma \\ \cdot [\frac{1}{2}(d(x, Ty) + d(y, Tx))]^{1-\alpha-\beta-\gamma} \end{aligned} \quad (15)$$

for each $x, y \in X \setminus \text{Fix}(T)$, where positive real $\beta, \alpha, \gamma > 0$, with $\beta + \alpha + \gamma < 1$. Then, the mapping T possesses a fixed point in X .

References

- [1] R.P. Agarwal, E. Karapınar, D. O'Regan, A.F. Roldán López de Hierro, *Fixed Point Theory in Metric Type Spaces*; Springer: Cham, Switzerland, 2015.
- [2] T. Abdeljawad, R.P. Agarwal, E. Karapınar, P.S. Kumari. Solutions of the nonlinear integral equation and fractional differential equation using the technique of a fixed point with a numerical experiment in extended b -metric space. *Symmetry*, 11(5), 686, 2019.
- [3] H. Afshari, S. Kalantari, H. Aydi, Fixed point results for generalized $\alpha - \psi$ -Suzuki-contractions in quasi- b -metric-like spaces. *Asian-Eur. J. Math.* 11, 1850012, 2018.
- [4] H. Aydi, E. Karapınar, A.F. Roldán López de Hierro, ω -interpolative Ćirić-Reich-Rus-type contractions. *Mathematics*, 7(1), 57, 2019.
- [5] H. Aydi, E. Karapınar, B. Samet, Fixed points for generalized (α, ψ) -contractions on generalized metric spaces. *J. Inequal. Appl.*, 2014, 229, 2014.
- [6] H. Aydi, C.M. Chen, E. Karapınar. Interpolative Ćirić-Reich-Rus type contractions via the Branciari distance. *Mathematics*, 7(1), 84, 2019.
- [7] S. Banach, Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fund. Math.*, 3, 133-181, 1922.
- [8] J. Bogin, A generalization of a fixed point theorem of Goebel, Kirk and Shimi. *Can. Math. Bull.* 19, 7-12, 1976.
- [9] C.M. Chen, A. Abkar, S. Ghods, E. Karapınar, Fixed point theory for the α -admissible Meir-Keeler type set contractions having KKM* property on almost convex sets. *Appl. Math. Inf. Sci.*, 11, 171-176, 2017.
- [10] Y.U. Gaba, E. Karapınar. A new approach to the interpolative contractions. *Axioms*, 8(4), 110, 2019.
- [11] G.E. Hardy, T.D. Rogers, A generalization of a fixed point theorem of Reich. *Can. Math. Bull.* 16, 201-206, 1973.
- [12] E. Karapınar, O. Alqahtani, H. Aydi, On interpolative Hardy-Rogers type contractions. *Symmetry*, 11(1), 8, 2018.
- [13] E. Karapınar, I.M. Erhan, U. Aksoy, Weak ψ -contractions on partially ordered metric spaces and applications to boundary value problems. *Boundary Value Problems*, 2014(1), 1-15, 2014.
- [14] E. Karapınar, Discussion on (α, ψ) -contractions on generalized metric spaces. *Abstr. Appl. Anal.* 2014, 962784, 2014.
- [15] E. Karapınar, Revisiting simulation functions via interpolative contractions. *Applicable Analysis and Discrete Mathematics*, 13(3), 859-870, 2019.
- [16] E. Karapınar, K. Tas, Generalized (C) -conditions and related fixed point theorems. *Comput. Math. Appl.* 61, 3370-3380, 2011.
- [17] E. Karapınar, Remarks on Suzuki (C) -condition. In: Luo A., Machado J., Baleanu D. (eds) *Dynamical Systems and Methods*, 2012, (pp. 227-243). Springer, New York, NY.
- [18] E. Karapınar, Revisiting the Kannan type contractions via interpolation. *Advances in the Theory of Nonlinear Analysis and its Application*, 2(2), 85-87, 2018.
- [19] E. Karapınar, B. Samet, Generalized α, ψ -contractive type mappings and related fixed point theorems with applications. *Abstr. Appl. Anal.*, 2012, 793486, 2012.
- [20] R. Kannan, Some Results on Fixed Point, *Bull. Calcutta Math. Soc.* 60, 71-76, 1968.
- [21] O. Popescu, Two generalizations of some fixed point theorems. *Comput. Math. Appl.* 62, 3912-3919, 2011.
- [22] O. Popescu, Some new fixed point theorems for α -Geraghty contractive type maps in metric spaces. *Fixed Point Theory Appl.* 2014, 190, 2014.
- [23] I.A. Rus, Generalized Contractions and Applications, *Cluj University Press: Cluj-Napoca, Romania*, 2001.
- [24] B. Samet, C. Vetro, P. Vetro, Fixed point theorems for $\alpha - \psi$ -contractive type mappings. *Nonlinear Anal.*, 75, 2154-2165, 2012.
- [25] P. Salimi, E. Karapınar, Suzuki-Edelstein type contractions via auxiliary functions, *Math. Probl. Eng.*, 2013, 8 pages, 2013.
- [26] T. Suzuki, Some results on recent generalization of Banach contraction principle. In: *Proc. of the 8th Int. Conf. of Fixed Point Theory and Its Appl.*, 751-761, 2007.
- [27] T. Suzuki, A generalized Banach contraction principle that characterizes metric completeness, *Proc. Amer. Math. Soc.*, 136, 1861-1869, 2008.
- [28] T. Suzuki, A new type of fixed point theorem in metric spaces, *Nonlinear Anal.-TMA*, 71, 5313-5317, 2009.