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The progress in development of the Planck-Balance 2 (PB2): A tabletop Kibble balance for the mass calibration of E2 class weights

Fortschritte in der Entwicklung einer Planck-Waage (Planck-Balance 2 – PB2): eine Tabletop-Kibble-Waage zur Kalibrierung von Gewichten der Klasse E2

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Abstract: In this paper we present the progress in development of a table-top version of the Kibble balance under the name Planck-Balance 2 (PB2). The PB2 is developed as a collaboration effort between the Technische Universität Ilmenau (TU Ilmenau) and Physikalisch-Technische Bundesanstalt (PTB) aiming for automatized mass calibration of the set of weights in the range from 1 mg to 100 g within the required uncertainties as stated by OIML recommendation R111 for weights of E2 class. We describe the design and the operational performance of the PB2 system in detail, the results of rigorous investigations of the error sources and subsequent improvements made since the beginning of the project in early 2017, the measurement data with the corresponding relative uncertainties and the preliminarily obtained uncertainty budget.

Keywords: Weights of E2 class, mass metrology, Kibble balance, Planck balance, traceability, uncertainty, calibration.

Zusammenfassung: In diesem Artikel wird der Entwicklungsstand einer table-top Version der Kibble-Waage unter dem Namen Planck-Waage 2 (PB2) dargestellt. Die PB2 ist ein Kooperationsprojekt zwischen der Technischen Universität Ilmenau (TU Ilmenau) und der Physikalisch-Technischen Bundesanstalt (PTB) mit dem Ziel automatisierter Massenkabrierungen von Gewichten im Bereich

von 1 mg bis 100 g (mit Messunsicherheiten, wie in der OIML R111 für Gewichte der Klasse E2 angegeben). Eine detaillierte Beschreibung des Aufbaus und der Leistungsfähigkeit des PB2-Systems wird vorgestellt sowie die Ergebnisse der Untersuchungen der Fehlerquellen, die Messdaten mit den entsprechenden relativen Unsicherheiten und das vorläufig ermittelte Unsicherheitsbudget.

Schlagwörter: Gewichte der Klasse E2, Massemetrologie, Kibble-Waage, Planck-Waage, Rückführbarkeit, Unsicherheit, Kalibrierung.

1 Introduction

Since the year 2019 the unit of the mass, the kilogram, in the International System of Units (SI) [1] is defined by a natural constant, namely through the fixed numerical value of the Planck constant. According to the present definition, the value of the Planck constant is $h = 6.626\,070\,15 \times 10^{-34}$ J s, or expressed in base units $\text{kg m}^2 \text{s}^{-1}$, where the meter is defined in terms of the speed of light c and the second by the frequency $\Delta\nu_{\text{Cs}}$ of the hyperfine transition of an unperturbed cesium-133 atom to the ground state [1]. There exist two well established methods for practical realization of the kilogram according to the current definition. One method is based on counting the atoms of a silicon sphere by X-ray crystal density method [2]. The second, the Kibble (Watt) balance [3], is based on a precise virtual comparison of mechanical and electrical powers obtained from a two-step experiment both steps based on electromagnetic interaction and with direct traceability to macroscopic quantum effects: the Josephson effect and the quantum Hall effect. The latter method of determining the mass and dissemination of the quantum-based kilogram can be realised at the highest level by the direct measurements of quantum-electrical based quantities [4].

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Previously, for more than a century, all mass standards had been traced to the single artifact defined as the International Prototype of the Kilogram (IPK) made from a platinum-iridium alloy (Pt90 %, Ir10 %) and stored at BIPM (Bureau International des Poids et Mesures) in Sèvres, France. The dissemination was done by comparison measurements [5] using a special class of apparatuses known as mass comparators [6]. Multiples and sub-multiples of 1 kg are realised as a mass scale via several comparisons as described in [7]. After the redefinition this way of dissemination of the mass units via conventional mass standards, i. e. weights at fixed nominal values will of course be continued. The new kilogram definition opens up the possibility to realize methods for determining a mass of any value in terms of the Planck constant without the use of any other mass standard. This will be possible with the use of instrumentation-based methods rather than by comparisons with mass standards [4]. Therefore, a new class of the specially designed apparatuses would potentially simplify the calibration procedures and minimize the necessary time and, as a consequence, the respective economic burden. Additionally, using the Kibble balance technology would allow to calibrate mass standards of any arbitrary value (e. g. 3.247 g) directly, i. e. without the need of interpolation between standard mass values (e. g. 1 g, 2 g, 5 g, 10 g).

A number of groups worldwide are already working on developing such balances [8, 9, 10, 11, 12, 13] at different uncertainty levels and with different operation ranges. The TU Ilmenau and the PTB were developing in a joint collaborative project two table-top sized Kibble balances for industrial applications, called “Planck-Balance 1” (PB1) for calibration of masses even under vacuum conditions with measurement uncertainties that correspond to the weights of E1 class according to OIML R 111-1 [14] and “Planck-Balance 2” (PB2) for weights of E2 class in air, as expected for mass calibration laboratories, respectively.¹ In this paper, the current state of the design and the performance of the PB2 system is described in detail. The paper presents in addition the improvement process, i. e. the results of rigorous investigations of the error sources and subsequent improvements of the system and its performance made during three years of collaborative work.

2 Principle of operation

The principle of precise comparison of virtual electrical and mechanical powers used in the Watt balance, nowa-

days, in honor of Bryan Kibble, who has originally introduced the method, called Kibble balance, is extensively studied since the first reports in late 70s [15]. By the Kibble balance, the mass calibration is traced back to the value of the Planck constant via electrical measurements instead of a comparison with mass standards [3]. The operation principle is based on performing two sequential measurements, named as force/static mode and velocity/dynamic mode. From the velocity mode the electromagnetic force factor² of the magnet and coil system (integrated in the core of the experimental setup) is determined and combined with the measurements from the force mode to determine the mass. These measurements are described by

$$F = m \cdot g = Bl \cdot I = Bl \cdot \frac{U_f}{R}, \quad (1)$$

and

$$U_v = Bl \cdot v, \quad (2)$$

respectively. The weight as a mechanical force F of Eq. (1) acting along the direction of the local free-fall acceleration g on the mass m to be determined, is counterbalanced by the electromagnetic force generated by an electric current I applied to a coil with length l situated in a magnetic field with flux density B . The current can be measured as voltage drop U_f over a precision resistor with resistance R . So far the system works like a common electromagnetic force compensation system. For the second mode, the velocity mode, the voltage U_v is induced when the same coil is moved in the same magnetic field with velocity v (see Fig. 1). In the case of the PB2, the link between the measured mass and the Planck constant can be established by calibrating the voltmeters and the precision resistor used for the measurement of the induced voltage and the voltage drop over a precision resistor against the programmable Josephson voltage standard (PJVS) and the quantum Hall resistor (QHR) [3].

It is worth to note that according to Eq. (1) and Eq. (2) it follows that this measurement principle can serve also to establish the standard of the force and realize the Newton directly traceable to the Planck constant without the need to obtain the value of the local free-fall acceleration at the site of the measurements.

There are a number of Kibble balances in different National metrology institutes that operate in accordance to this principle for realizing the kilogram traceable to the

¹ Progress on the PB1 will be published elsewhere soon.

² This factor can be found with different notations in the literature, such as, *Bl factor*, *geometric factor*, *force factor* or *flux integral*.

Figure 1: Schematic representation of the PB2 operation in force (a) and velocity (b) modes. An external voice coil system is attached to the load carrier, and functions as the weighing sensor in the Planck-Balance. The inner voice coil is the actor and drives the oscillation used to determine the Bl of the external voice coil in the velocity mode.

Planck constant. They have been initially developed to determine the Planck constant as closely traceable to the IPK as possible, and only later, once the numerical value of the Planck constant was fixed, have started to perform the reverse measurements. Therefore, they were developed to operate at one specific nominal mass with an exceptionally high precision performance and low relative measurement uncertainties around 1×10^{-8} . The redefinition of the kilogram opens chances to develop smaller and cost-effective table-top versions of Kibble balances and perform the calibration of any arbitrary mass directly traceable to the Planck constant, since it is no longer necessary to start at a mass of 1 kg as “transfer point” for establishing a dissemination chain. However, due to several technological limitations it should always be evaluated the trade-off between development effort, time, financial investment, the actual conditions and the complexity of the measurement process against the desired uncertainties of the mass calibration. For this reason, our aim in developing the PB2 is to achieve typical measurement uncertainties for mass calibration as expected at calibration laboratories that would be compatible with the requirements of the OIML R 111-1 for weights of E2 class. Another goal was to develop the PB2 with reduced complexity similar to conventional analytical balances and with an option to measure mass artifacts at the 1 mg to 100 g level in the laboratory without access to any calibrated masses, however with direct traceability to the Planck constant. Additionally, within the PB2 development concept a software tool, called *Virtual Planck-Balance* (VPB) is included that applies procedures from statistical analysis to allow a better supervision of the performance of the Planck-Balance directly by the operator at the mass calibration laboratory level.

In accordance with the operational terminology of the Kibble balance, as mentioned earlier, the PB2 uses two measurement modes – a velocity mode (V) and a force

mode (F) – which are used in a manner creating V-F-V measurement cycle, similar to common weighing procedures known from metrological guidelines for canceling out effects of the linear drifts or changes of the ambient conditions on the result. The typical measurement time for the single mode of V or F is about 30 s to 50 s. In order to achieve low measurement uncertainty, it is possible to increase the number of VFV cycles (details will be discussed in Section 5.1).

3 Design and components of the PB2 setup

Although the design of the PB2 system is much simpler than most of the other Kibble balances it is necessary to provide the same components of required metrology modules which is a combination from different metrological disciplines. Thus, in this section of the paper we will introduce the components of PB2 separated into mechanical, electromagnetic (coil and magnet assembly [16]), optical, electrical, digital processing and statistical data analysis.

The concept of PB2 was developed on a modular basis, i. e. it should be possible to change/integrate components with an acceptable effort of time and money. In general it was the aim to use commercially available components as far as possible. The general setup of PB2 is depicted in Fig. 2. In addition to commercially available off-the-shelf components like the laser interferometer (3) and electromagnetic force compensation analytical balance (EMFC load cell – 2), the PB2 system (see also Fig. 3) consists of a closed-circuit magnet and a custom manufactured coil actuator (4) attached to the load carrier. The geometrical factor of the magnet system and the coil assembly, Bl

Figure 2: a) Photograph and b), c) sketch of the PB2 showing the size of the setup. Below, a flow chart indicates how the parameters contribute to the final evaluation of a mass m . 1 – Weight, 2 – Load cell with internal voice coil, 3 – Laser interferometer, 4 – External voice coils. (The sketch shows a pile of three voice coil systems, which, in the future, will be integrated. Currently only one voice coil is acting, see Fig. 3 and Fig. 4).

(measuring actuator), in the actual Kibble balance experiment is determined through the force and velocity modes. Supporting peripheral electronics were installed to record the internal position signal of the load cell and to supply electrical current to the two voice coil actuators. The coil integrated inside of the load cell, which is rigidly fixed to the proportional beam, we name internal coil, and the one rigidly fixed to the bottom side of the load carrier external coil.

The internal coil (drive actuator) is used to excite the external coil (measurement actuator) in the velocity mode and is connected to a current source of type 3245A [17] in the force mode in order to produce a constant offset force. This is necessary for the reduction of the coil current effect (to be described later in Section 6.3.5). Furthermore, the measurement mirror is attached rigidly to the load carrier

in order to determine the displacement that is used to calculate the velocity of the mirror and the measurement coil.

During the course of the PB2 development we have used different external coils due to the necessity of meticulous mechanical alignments of the coil in the magnetic field of the magnet assembly and the adjustments of the electrical measurements to the more desirable measurement ranges. The resistance value of these coils range approximately from 3 k Ω to 5 k Ω as a result of the material properties of the wire and its geometry. The number of turns is totaling in the range from 3000 to 5500 in the volume comprising the coil by a height of 10 mm, and inner minimal and outer maximal diameter of 29 mm and 32 mm, respectively. These coils are custom manufactured by a commercial vendor on the base of copper wire with a diameter of 0.05 mm as a baked varnish coil. The choice of the

magnet was made in favor of the more traditionally used material – SmCo – that has approximately 20 % lower remanence in comparison to the NdFeB, however only about one third of the temperature coefficient of that of NdFeB. See Fig. 3 for an overview of the coil and magnet assembly and [16] for a detailed description.

Initially, the current source 3245A was used in the force mode to apply the current that is necessary to compensate the weight force, and in the velocity mode as an AC current source for the excitation of the coil. Later, however, it turned out that the 3245A operates with low latency and lacks the external trigger option for synchronizing the applied electrical current and the measurement voltages with an external common time base. Thus, a cus-

Figure 3: Closed-circuit dual magnet system used in the Planck-balance consisting of (1) a ferromagnetic flux guide, (2) a coil and (3) two permanent magnets which are generating (4) a radial magnetic field in the air gap.

tomized peripheral electronic unit was installed. This peripheral electronic unit consists of a fast digital-to-analog converter (DAC), which is augmented by a power amplifier that drives the external coil current of the measurement actuator in the force mode. The DAC is part of a digital signal processing unit (DSP) from the company dSPACE on which the data acquisition and signal processing controller for the load cell was implemented in both, the force mode with a PID controller and for the velocity mode. The DSP currently does not allow the use of an external clock signal; however, it was chosen also because it provides flexibility in rapid prototyping and implementation of necessary measurement routines.

In order to provide a traceable time base that is crucial for the velocity mode of measurements, the internal clock of the function generator (for generating a sine signal) [18] is determined by a frequency counter [19] that, in turn, is connected to the reference clock. Thus, a time base using traceable standard is integrated for the traceable determination of the amplitude trajectory in the velocity mode, the measurements of the multimeters and the interferometer. They all are triggered traceable to a common 10 MHz reference clock. Since the signal frequency is specified by the dSPACE system, when the excitation signal is specified, its control loop frequency was measured using a frequency counter which is connected to the same time reference. In the future, it is aimed (the progress is underway) to replace this comparatively expensive unit – that also offers

Figure 4: Illustration of the interferometer setup. (1) Moving measurement mirror fixed to the load carrier on the yz-plane, (2) reference mirror and (3) custom manufactured 90° prism are fixed to the mechanically static reference frame of the PB2, (4) external measurement coil fixed to the bottom side of the load carrier on the xz-plane, (5) interferometer head (clamping and fixing mechanisms are not shown for better visibility). Inset: Side view (xy-plane) on the measurement and reference half-cube mirrors, and the 90° prism.

Figure 5: A signal flow diagram representing the functional interactions between the units of the PB2 system.

limited resolution – by a compact, custom-made electronics which is a combination of ARM microprocessors, field programmable gate array (FPGA) chips and AD/DA converters.

A homodyne differential interferometer system [20] measures the displacement of the coil (depicted in Fig. 4). The actual optical path length of the traveling laser beam to the reference and the measurement mirrors and back to the laser head is shorter than it is illustrated in Fig. 4 and most of the rigidly connected mechanisms are omitted. The distance between the test and reference beam is 21 mm.

The diagram shown in Fig. 5 describes the signal flow during the operation of the PB2 for both, the velocity and the force mode. This structure was partially reshaped during the course of the development by adding/eliminating some units (for example added – climate module, control hardware, eliminated – current source 3245A) or by supplementing the measurements by additional metrology equipment for temporarily investigation purposes (for examining the tilt angles during velocity mode – three-beam interferometer, for setting the load carrier perpendicular to the gravitational acceleration direction – a liquid pool and an autocollimator was integrated). Several precision optical alignment cuboids (7.5 mm × 15 mm × 15 mm) are used with a reflective coating on all sides.

Initially, these alignment cuboids were chosen as application specific, custom made (fitting to our PB2 design specifications) uncoated precision alignment cuboids, based on a glass material of the H-K9L type. Later the cuboids were coated on all sides with a reflective material using vapor deposition technology. Then, the angles between the surfaces of each optical alignment cuboids were measured and well characterized. According to the calibration certificate they agree to better than $21.8 \mu\text{rad}$ with a right angle corresponding to a cosine error of 0.24×10^{-9}

(see Fig. 4). They are attached to the load carrier of the balance or the mechanically static reference of the PB2 and serve multiple purposes, such as: fixed reference mirror, moving measurement mirror, or alignment tool for the load carrier. In order to enable long-term measurements without user/human intervention and for ensuring the necessary cable shielding for the different connections of the PB2 in velocity and force mode a digitally automated in-house custom developed relay switching unit was used.

In PB2 the test weights can be lifted and lowered remotely by an automated weight lifter to minimize disturbances caused by an operator. Furthermore, a vacuum compatible piezo based linear motor of type N-310 from company Physik Instrumente is used in the weight lifter system that allows the PB2 system to operate in vacuum. Due to its working principle of a walking motion of alternately deflected piezo actuators, it does only generate mechanical noise during the active lifting and lowering process while no measurement of compensation current is performed. Additionally, it does not generate magnetic fields that could disturb the measurement actuator.

The weighing pan is designed as a fork structure as shown in Fig. 4, while a fitting fork structure is used to lift the weight under test. The weight lifter and the linear piezo actuator are mounted on a separate stage that can be mechanically adjusted in six degrees of freedom relative to the main mechanical structure of the PB2 by which a continual and contactless operation with minimal gap between the fork fingers is maintained. The special design of the weight lifter allows the operation of the PB2 with a wide range of sizes and shapes of the mass standards including wire weights. Additionally, by replacing both fork structures with three spherical contact pins, the setup can easily be adapted for weights such as spherical silicon type artifacts.

4 Measurement procedures

4.1 Signal processing

In the current setup the signal processing of the PB2 is maintained by a hybrid analog and digital system (see Fig. 5 and ref. [21]), consisting of several custom-made circuitries as well as a customized set-up of commercially available devices. The controller is implemented on a digital signal processor (DSP) system. The control algorithm runs on a processor board of type DS1006, which is enhanced by an analog to digital converter (ADC) board of type DS2004 and a digital to analog converter (DAC) board of type DS2102, both operating with 16-bit resolution. The current PIDT1-controller is implemented with a sampling frequency $f_s = 10$ kHz that provides an output voltage U_{drv} in order to control the measured position sensor voltage U_p to a given set point value. The output voltage of the DSP is converted by an amplifier circuitry into a controlled current that is proportional to U_{drv} . When applied to the internal coil of the balance actuator, the output current can be utilized to excite the load carrier with a sinusoidal motion that is necessary in the velocity mode for calibration of the force factor, Bl , of the external coil. After this calibration, the output current is connected via latching relays to the external coil and is then used to balance the system in the force mode.

In addition to the control algorithm, several different signal generators are implemented on the DSP, which provide chirp signals for testing purposes and the sinusoidal modulation of the set point position in the dynamical operation mode “velocity mode”. Since the internal clock of the DSP represents the time base for the excitation frequency, one of the DACs outputs a square wave that changes its voltage level at each sampling period of the DSP and therefore yields a frequency of $f_s/2$. The actual frequency of this signal is measured by a frequency counter that is disciplined via a stable reference clock of frequency $f_{ref} = 10$ MHz. The same reference frequency is supplied to the waveform generator that provides a square wave signal with the frequency f_{trig} . The square wave signal is used to trigger the measurement of the laser interferometer and the digital multimeters, with a sampling frequency of 1 kHz. One multimeter measures the induced voltage during the velocity mode, while the other one measures the necessary compensation current during the force mode as a voltage drop U_f across a pre-calibrated reference shunt resistor [22] that is connected in series with the coil and the current amplifier output.

4.2 Force mode

Several measurement campaigns have been made during the course of PB2 development with different nominal mass values. In this article, we present results of two of the latest measurement campaigns with weights of nominal mass values as given in Table 1, which have been made after identifying most of the sources of uncertainty and minimizing their influences. When the velocity mode is performed with specially adjusted parameters (see previous section), the system undergoes the force mode of the measurements by restructuring the measurement infrastructure by the latching relays. This procedure of switching from velocity (V) to force (F) mode follows the conventional measurement cycle which in our case we term as VFVFV... VFV. The force mode cycle consists of two measurement sub-states depending on the status of weight at the load carrier, namely loaded (A) and unloaded (B). The position of the external coil is servo controlled in the weighing position during both states. The weight lifter is used to place the weight piece on the fork structure attached to the load carrier while the balance further controls the coil in the weighing position. Thus, the necessary compensation current in the force mode is determined as the difference of the compensation (control) currents of the loaded and the unloaded state of the balance. Since the coil current affects the magnetic field of the actuator, this current difference depends on the absolute values of both compensation currents [23, 24]. The error caused by this effect can be reduced by adjusting the coil position to be in the apex of the magnetic flux density of the magnet system. The current PB2 system does not provide yet the possibility to adjust the immersion depth of the coil and relies on the manufacturing tolerances of the voice coil actuator and the mountings of the coil. Therefore, the actual Bl profile was acquired experimentally by determining this geometric factor at different coil positions. As shown in Fig. 6, the apex was not exactly met and the Bl changes relatively with approximately $0.5 \times 10^{-6} \mu\text{m}^{-1}$, which is sufficient for comparison of the Bl values determined in force and velocity mode because the repeatability of the coil positioning is in the order of nanometers.

In order to reduce the error that is caused by the coil current effect [3] (see also Section 6.3.5) a tare weight is generated virtually by means of the internal coil actuator. The current flowing through this actuator is adjusted in a way that when the load carrier is unloaded, the coil produces a force equivalent to half the weight, but in the upwards direction. This means that the external voice coil actuator must push the lever arm also upwards, in order

Figure 6: Relative change of B_l depending on the coil position.

to maintain it at the equilibrium position. In the loaded state the external coil must pull the lever arm with the same amount of force in the downwards direction, i. e. the absolute value of the force between loaded and unloaded state are equal, but the sign of the electric current – and hence, of the force direction – is different. This procedure of electrically adjusting the offset values has an advantage over the usage of standard mass artefacts as tare weights. Primarily, it is fast to implement, and it can readily be adjusted for any other arbitrary negative or positive mass value and avoids complications in handling physical tare masses mechanically. To adapt this internal offset current I_i to the changing B_l of the internal drive actuator, an adjustment procedure is therefore performed regularly. Incrementally small current values ΔI_i are added to the previously used offset current I_{i0} and for both currents through the internal coil actuator, the differences ΔI_m of the absolute values of the necessary compensation current in loaded (I_A) and unloaded (I_B) state are then determined as

$$\Delta I_m = |I_B| - |I_A|. \quad (3)$$

From these measurements the new internal current I_i is calculated by linear interpolation to obtain the ΔI_m of zero as

$$I_i = I_{i0} + \frac{\Delta I_m(I_{i0}) \cdot \Delta I_i}{\Delta I_m(I_{i0}) - \Delta I_m(I_{i0} + \Delta I_i)}. \quad (4)$$

This procedure is repeated each time before a new set of force mode measurements is taken for on-line compensation of temperature effects and the general drift of the internal coil actuator.

4.3 Velocity mode

4.3.1 Non-linearities of B_l

In the velocity mode the coils attached to the balance oscillate along the y -axis in the magnetic field generated by their permanent magnets. The working frequency is typically 4 Hz with an amplitude of 20 μm , but for the sake of investigations frequencies ranging from about 0.5 Hz to 10 Hz and amplitudes between 20 μm to 40 μm have also been applied. The velocity v of the coil motion is measured using an interferometer and a stable clock, and the induced voltages in the coils U_v are measured using a digital multimeter (3458A [25], with sampling frequency 1 kHz and aperture time 0.7 ms) in DC mode to determine the B_l from Eq. (2). Contrary to the common approach that most of the known Kibble balance systems use, we have taken a conceptual shift in implementing the velocity mode of the measurements by moving the coil sinusoidally and measuring an AC rather than a DC signal generated in the velocity mode [26]. A similar approach is applied in [27], however, with oscillating the magnet rather than the measurement coil. To our best knowledge, the idea was first described in [28].

Since the magnetic field of the permanent magnet has only an approximate linearity the induced voltage will be subject to a distortion. As a consequence, the value of the B_l as obtained from the oscillation $B_l(y_{0,VM})$ around, say y_0 , through the magnetic field in the velocity mode, will not coincide with the value of the B_l at y_0 , at which the force mode will be performed (see Fig. 7). The measured B_l in the velocity mode will correspond to $y_{0,VM}$, which is lower than B_l at y_0 . In fact, the B_l from the velocity mode will always be lower if the magnetic field is not constant or linear, and therefore, a correction has to be applied. In order to obtain the true value of the B_l at the position y_0 , we first measure the B_l as a function of the vertical coil position y in the force mode. The result is depicted in Fig. 6. By applying a quadratic fit to the obtained data the shape of the magnetic field can be recovered for the whole motion range of the coil. As a next step we simulate numerically the induced voltage that would be obtained when oscillating through a B_l of the measured shape. We assumed the velocity to be undisturbed, and calculate a B_l from the input velocity amplitude and the output induced voltage amplitude considering the non-linear B-field, as obtained beforehand. In this way the relative error can be calculated and a correction applied. Note that only the shape of the B_l – i. e. the normalized values – is of importance (as it is obtained from Fig. 6), since we need only the relative correction; the absolute correction is applied to the B_l as mea-

Figure 7: The $Bl(y_{0,VM})$ as obtained from the velocity mode does not coincide with the $Bl(y_0)$ at the position y_0 , which is the mean position of the oscillation range, and at which position the force mode will be applied. $Bl(y_{0,VM})$ refers to the value at $y_{0,VM}$, which is by Δy_0 off from y_0 .

sured in the velocity mode. Furthermore, the knowledge on the exact mass value that is used to determine the shape of our Bl is irrelevant. To summarize, the procedure is, first to determine the Bl in the velocity mode from our measurement data, and then correct for the non-linear shape of the Bl to get the value at y_0 , which is then considered for the determination of the mass in the force mode.

4.3.2 Data analysis of harmonic signals

Initially, assuming that both position and voltage measurements are ideal single-component sine waves with amplitudes S and U_{ind} , and initial phases φ_s and φ_u , the oscillation can be described by

$$s(t) = S_0 + S \sin(\omega t + \varphi_s), \quad (5)$$

where S_0 is the dc offset and $\omega = 2\pi f_{sig}$ is the angular frequency with f_{sig} denoting the oscillation frequency. The coil velocity therefore can be obtained as the analytical derivative of Eq. (5) with respect to the time t as

$$v(t) = \omega S \cos(\omega t + \varphi_s) \quad (6)$$

and the induced voltage as

$$u(t) = U_{ind} \cos(\omega t + \varphi_u). \quad (7)$$

In the real measurement data of the induced voltage and the mechanical oscillation, higher order harmonics are present in the signals after using Fourier analysis [29]. This mainly arises from the nonlinear Bl over the motion range. Therefore, the signal of the induced voltage $u(t)$ with time t can be described by using a single-component

sine wave with amplitude U_1 superimposed by higher order harmonics:

$$u(t) = U_0 + \sum_{n=1}^N U_n \sin(n\omega t + \varphi_{un}) \quad (8)$$

where U_n and φ_{un} are the amplitude and the initial phase of the n^{th} -order harmonic of the induced voltage, respectively. U_0 denotes a DC offset, and N is the number of higher order harmonics, with $N \in \mathbb{N}^+$.

The oscillation $s(t)$ can also be described by a sum of N harmonic terms:

$$s(t) = S_0 + \sum_{n=1}^N S_n \sin(n\omega t + \varphi_{sn}) \quad (9)$$

where S_0 is a DC offset, S_n the amplitude of the n^{th} -order harmonic, φ_{sn} the phase of n^{th} -order harmonic. This can be also presented as a linear combination of $2N$ shifted sine waves with A_n and B_n as amplitudes of in-phase and quadrature components as

$$s(t) = S_0 + \sum_{n=1}^N [A_n \sin(n\omega t) + B_n \cos(n\omega t)]. \quad (10)$$

The amplitude S_n of the n^{th} -order harmonic of $s(t)$ can be determined as $S_n = \sqrt{A_n^2 + B_n^2}$. In a typical measurement the position data are obtained directly from a commercial laser interferometer (see Section 6.6) at a sample rate of 1 kHz to 10 kHz. After that the maximum position amplitude S is obtained by fitting a sine wave to the time-position data. The maximum velocity is later calculated analytically as $2 \cdot \pi \cdot f_{sig} \cdot S$.

Assuming that the Bl is constant along the whole coil motion range it can be computed by dividing the measured fundamental amplitude of the induced voltage by the fundamental amplitude of the coil velocity as

$$Bl = \frac{U_1}{\omega S_1} = \frac{U_1}{2\pi f_{sig} S_1}. \quad (11)$$

In the PB2 system, the oscillation frequency f_{sig} can be accurately measured with a frequency counter. Therefore, the accuracy of identifying the Bl depends on the estimation of fundamental amplitudes U_1 and S_1 . The sine fitting algorithm is applied to the measurement data for estimating these amplitudes (the induced voltage and the coil motion). The performance of a three-parameter sine fitting algorithm has been tested with numerical simulations and correlation analyses against some perturbations, such as additive Gaussian white noise, quantization error, and frequency error [10]. The multiharmonic sine fitting algorithm has the analogous performance.

5 Preliminary results

The PB2 aims at calibrating weights of accuracy class E2 for a mass range from 1 mg up to 100 g. In order to evaluate the performance and accuracy of our setup two different measurements campaigns have been performed. During the first measurement campaign a calibrated mass standard of nominal weight of 20 g was used to evaluate the force factor Bl in the force mode. This force factor was then compared to the force factor obtained from the velocity mode. The observed difference then was compared to the combined standard uncertainty. In accordance to calibration certificate provided by PTB, the 20 g mass standard was chosen for this evaluation because it has the smallest uncertainty in our set of available mass standards.

In a second measurement campaign weight pieces of different nominal values are used. In Table 1 the nominal values for both measurement campaigns are listed. The respective measurement results for the force factor are then compared to the one obtained for the 20 g mass standard, which serves as a reference value. Observed deviations from the reference value point to unrevealed systematic effects in the force mode. At the same time the second measurement campaign also serves as a confirmation of

- a) the consistency of the obtained results for automated mass calibration of the set of weights in the range from 1 mg weight piece up to at least 5 decades of magnitude higher values within the maximum permissible errors recommended by OIML for the weights of E2 class; and
- b) for demonstration of the fact that, once the performance of table-top PB2 setup at the typical mass laboratory conditions is at the required level of expanded uncertainty for calibrating mass values of E2 class of weight pieces, the calibration process can be already made directly traceable to the Planck constant within SI unit system without need to reference with any known calibrated masses values.

5.1 20 g measurement

Before the measurements, several adjustments are made for the proper operation of system. The main adjustments discussed in this work are as follows:

- (i) Correction for non-linear Bl – in both, velocity mode (see Section 6.8.1 and Fig. 11) and force mode (see Section 6.8.2 and Fig. 6).
- (ii) Abbe error – the location of the laser spot does not coincide with the center axis of the coil. This yields an Abbe error during the measurements of the coil

displacement. Details of the corrections are discussed in Section 6.9.1.

- (iii) Vertical Alignment – in order to determining the absolute mass value, the force measurement direction should be aligned in parallel to the vector of the gravitational acceleration (see Section 6.9.2).
- (iv) Corner load error – the centers of mass of the mass artifacts should be placed at the same location on the load carrier (see Section 6.9.3).

For validating the results, the Bl was measured in both modes, the velocity mode and the force mode. For the force mode, a calibrated E1 mass standard of nominal value 20 g was used for a good traceability. The Bl s have then been compared for the sake of verifying whether there is a systematic offset between both modes that is higher than the estimated combined measurement uncertainty. In Section 6 a detailed uncertainty evaluation will be presented.

The final representative measurement results of this campaign are shown in Fig. 8. Blue triangles show continuous measurements of the force (F) mode with 20 g and red squares show the velocity (V) mode, each mode repeated 26 times and lasting in total 65 hours. A set of 50 loading (A) and unloading (B) sub-states during the single force mode of measurements were performed by averaging the measured values for 20 s after reaching the settling time of 10 s for each sub-state. Thereby, providing an average value and the calculated combination of the standard deviations ($\sqrt{[std(Bl_F)/\sqrt{n}]^2 + [std(Bl_V)/\sqrt{n}]^2}$) of periodically repeated 50 measurements of ABAB...ABA sub-states from single force mode measurements, assuming additionally procedural and data processing conditions described in Sections 4.2 and 6.3.3.

The velocity mode was performed at oscillation frequencies ranging from 0.5 Hz to 10 Hz with at least 45 periods following the analysis provided in Sections 4.3 and 4.3.2. In this particular campaign the force and velocity modes comprise a total of 1300 and 3120 measurements, respectively. The evaluation of both modes shows a relative difference of 4.93×10^{-7} . This offset is smaller than the combined measurement uncertainty of 2.47×10^{-6} , but higher than the calibration uncertainty of the mass standard, which amounts to 2.5×10^{-7} .

5.2 Comparison of a set of mass standards

In addition to the measurement with a 20 g mass standard, measurements were also done with a set of E1 mass standards whose nominal values are shown in Table 1 in order to identify the non-linearity and the overall character-

Figure 8: Comparison of the B/l as obtained from force mode and velocity mode with a calibrated 20 g mass standard. Each blue triangle represents one B/l factor obtained by one ABA cycle in force mode, similarly, each red square represents one B/l factor resulting from one velocity mode with 45 periods of the excitation signal. The values are used to calculate a B/l value that is corrected for the influences of temperature, tilt oscillation, index of refraction, air buoyancy and etc. The final result is the difference of the mean of all B/l_V and the mean of all B/l_F .

istics of the system in a wide mass range. With each mass an average number of 47 sets of force mode measurements with ten ABA cycles and the same number of sets of velocity mode with excitation frequencies ranging from 2 Hz to 6 Hz were conducted. The number of sets varied between 22 and 133 to perform the measurements in a stable and quiet environment which was best achievable overnight and during weekends. In order to not confuse drift effects of the systems sensitivity with a nonlinear behavior, the order of different mass value measurements was chosen in non-sequential random manner.

Table 1: Nominal mass values (in g) used in the measurements.

Meas. #1	20								
Meas. #2	0.001	0.01	0.1	0.5	1	2	5	20	50

All mass standards were chosen of accuracy class E1, meaning that the true mass value lies within its maximum permissible error (MPE), as defined in the guideline OIML R111-1 [14]. No additional calibration has been performed before the campaign.

In Fig. 9 a graphical summary of the measurement results is given. Black dots denote the MPE for E1 mass standards, while red squares are for MPE of E2 mass standards. Green triangles show the Type A uncertainty for the measurements that are evaluated as the square root of the sum of the squared standard deviations of mean value of the B/l measurements in force mode and velocity mode. It can be seen that except for 1 mg and 10 mg, the statistical uncertainty is lower than the respective MPE of E1, whereas

for all mass standards it lies below the MPE of E2. This observation cannot be done for the offset between the measured and the nominal mass of the mass standards. Only for 50 g and 5 g (the 20 g is used as a reference, thus there is no offset) the offset is smaller than the respective MPE of E2. Since the velocity mode is independent of the mass to be calibrated, we conclude that the force mode suffers from unknown systematic effects. Further investigation is underway to reveal the source of error.

6 Uncertainty evaluation and traceability

In this section a preliminary uncertainty analysis of the measurements made with a 20 g weight piece is provided. Some additional details of the results of uncertainty evaluation presented in this section can be found in [11] and [30].

For the PB2 setup a special vacuum chamber was developed in which some measurements under the medium vacuum conditions (10^{-1} mbar to 10^{-2} mbar) were made to test the performance of the system and investigate the environmental influences. However, in this paper all the presented measurements were made in air, and therefore the air buoyancy force acting on the weight has been taken into account and was corrected as described in the OIML R 111-1 [14]. For this purpose, the density of the air was determined by measuring the air temperature, humidity and pressure with a calibrated climate module of type Sartorius YCM20MC that is included in the PB2 system as

Figure 9: Comparison of the B/I as obtained from a set of mass standards compared to the B/I as obtained from a 20 g mass standard.

another modular component. Additionally, a calibrated Pt100 temperature sensor is immersed into the actuator magnet (see Fig. 3, the hollow space along the magnet's central axis) in order to track the temperature change because of environment and the self-heating of the actuator's magnet.

6.1 Mass

6.1.1 Calibration

We used several highly polished reference standard weight pieces made from special stainless steel with nominal values as provided in Table 1.

The 20 g weight in the upper row was a knob weight of class E1, additionally calibrated at PTB. The calibration certificate shows the value 20.000 012 g and an uncertainty of 5 μ g, with $k = 2$. This value will be included in the next Section 6.1.2 to give a combined uncertainty for the mass including buoyancy effects.

In the second row are presented the nominal values of the masses of a commercial set of standard knob weights and flat polygonal sheet weight pieces of class E1 (purchased short before the measurement campaign) all made of stainless steel. Here, no further calibration was undertaken, however, we know that the true mass does not exceed from its nominal mass to more than MPE of E2.

6.1.2 Air buoyancy

In the force mode, the gravitational force acting on a calibrated weight of 20 g is compared with an electromag-

netic force. When the measurement environment is in air, the gravitational force is partially compensated by the air buoyancy, and the mass m_{cal} must be corrected by

$$m = m_{cal} \left(1 - \frac{\rho_a}{\rho} \right), \quad (12)$$

where ρ_a is the air density, and ρ is the mass density. m_{cal} is the calibration value of the mass.

The density ρ and the volume V of the mass depend on the temperature T , and thus Eq. (12) can be written as

$$m = m_{cal} - \rho_a V(T). \quad (13)$$

The air buoyancy correction is one of the major contributions to the uncertainty budget of the mass. According to Eq. (13), the standard uncertainty associated with the corrected mass m is estimated as

$$u(m) = \left(\left(\frac{\partial m}{\partial m_{cal}} \right)^2 u^2(m_{cal}) + \left(\frac{\partial m}{\partial \rho_a} \right)^2 u^2(\rho_a) + \left(\frac{\partial m}{\partial V} \right)^2 u^2(V) \right)^{\frac{1}{2}}, \quad (14)$$

where $u(m_{cal})$ is the mass calibration uncertainty, $u(\rho_a)$ is the standard deviation of the air density, and $u(V)$ is the standard deviation of the mass volume.

The density of moist air ρ_a is evaluated by using the CIPM formula [31] as

$$\rho_a = \frac{p M_a}{Z R_g T} \left[1 - x_v \left(1 - \frac{M_v}{M_a} \right) \right], \quad (15)$$

where p is the air pressure in Pa, T is the temperature in K, x_v is the mole fraction of water vapor, M_a is the molar mass of dry air in g mol^{-1} , M_v is the molar mass of water in g mol^{-1} , Z is the factor of compressibility, and R_g is

Table 2: Standard uncertainty of the air density.

	Mean value	Uncertainty of sensor	Standard uncertainty of air density, kg m^{-3}
Temperature	22.39 °C	0.05 °C	2.03×10^{-4}
Air pressure	95 437.48 Pa	50 Pa	5.90×10^{-4}
Relative humidity	36.08 %	3 %	3.61×10^{-4}
CO ₂	450×10^{-6}	100×10^{-6}	4.62×10^{-5}
Formula			2.47×10^{-5}
Combined			7.22×10^{-4}

the molar gas constant in $\text{J mol}^{-1} \text{K}^{-1}$. The molar gas constant R_g , the molar mass of dry air M_a and the molar mass of water M_v are constant values with a fixed uncertainty. Z and x_v are functions of the ambient conditions dependent from pressure, temperature and relative humidity. M_a is further improved by using the mole fraction of carbon dioxide x_{CO_2} . Therefore, the standard uncertainty of the air density $u(\rho_a)$ is determined by the measurement uncertainty of the ambient sensors and the known values of R_g , M_v , and x_v .

The ambient conditions in the measurement room are varying over time and therefore, the nominal values of ambient condition are determined by using the average values from the measurement sensors during the whole measurement time. These nominal values and the associated measurement uncertainties are listed in Table 2. The air density ρ_a of 1.12 kg m^{-3} is calculated under these nominal values of temperature, pressure, humidity and CO₂ content. The corresponding contributions of the measurement sensors are also estimated in Table 2. Since the relative standard uncertainty of the air density formula itself is given by 2.2×10^{-5} [31], its standard uncertainty is $2.47 \times 10^{-5} \text{ kg m}^{-3}$. Finally, the combined standard uncertainty of the air density $u(\rho_a)$ is obtained to be $7.22 \times 10^{-4} \text{ kg m}^{-3}$.

The temperature variations with time have an influence on the volume of the weight. The volume $V(T_{\text{ref}})$ of a weight can be obtained at a reference temperature T_{ref} . The actual volume must be corrected for the measured temperature T from the sensor as

$$V(T) = V(T_{\text{ref}}) [1 + \alpha_V (T - T_{\text{ref}})], \quad (16)$$

where α_V is the volume expansion coefficient, and $T_{\text{ref}} = 20^\circ \text{C}$.

The standard uncertainty of the volume $u(V)$ is determined by the calibration uncertainty of the mass volume $V(20^\circ \text{C})$ and the measurement uncertainty of the temperature sensor. From the mass calibration certificate, the volume $V(20^\circ \text{C})$ is $2.4961 \text{ cm}^3 \pm 0.0006 \text{ cm}^3$ ($k = 2$). The measurement uncertainty of the temperature sensor is 0.05 K .

According to the GUM [32], the combined uncertainty of the volume $u(V)$ is $3 \times 10^{-10} \text{ m}^3$.

Thus, according to Eq. (14), the combined standard uncertainty of the mass $u(m)$ is $3.1 \times 10^{-3} \text{ mg}$, and the relative uncertainty is 1.6×10^{-7} .

6.2 Local gravity

The value of the local gravitational acceleration must be known at the reference point of the measurements and at the time instances when the measurements are made. In the case of our measurement, the gravitational acceleration was measured by the Federal Agency for Cartography and Geodesy (BKG) with an absolute gravimeter A10 #012 in a specific laboratory location. Furthermore, the value was transferred by the measurements with a relative gravimeter CG5 40496/00044 to the laboratory measurement table of PB2 (made of non-ferromagnetic massive granite slabs with a weight of over 500 kg). The value for the reference point is obtained as

$$9.810\,171\,58 \text{ m s}^{-2} \pm 0.21 \mu\text{m s}^{-2},$$

with a vertical gravity gradient of

$$(-3.22 \mu\text{m s}^{-2} \pm 0.03 \mu\text{m s}^{-2}) \text{ m}^{-1}$$

between 0.226 m and 0.732 m height above the reference point.

For this mean value, the corrections regarding earth tides, air pressure influence and polar motion are considered. For the measurement laboratory of PB2 at TU Ilmenau the temporal variations of the value for the earth tides, the changes in air pressure and the pole movement are identified to be always smaller than $\pm 2.00 \mu\text{m s}^{-2}$ (relative uncertainty 2×10^{-7}). If desired, the value of the local gravitational acceleration can be determined more accurately at the time of measurement by monitoring the gravitational variations, e. g. by use of a relative gravimeter. As an alternative way, tidal corrections can be done by means of software based tidal predictions [33, 34]. For the air pressure

effects a general admittance factor of $0.003 \mu\text{m s}^{-2} \text{hPa}^{-1}$ can be applied, and for the polar motion the current orientation of the rotation axis can be obtained from the International Earth Rotation and Reference Systems Service (IERS) [35]. Applying all those corrections the remaining relative uncertainty for the local gravity value remains well below 1×10^{-8} . For the measurements described in this article no corrections were applied and a relative uncertainty of 2×10^{-7} was assumed.

6.3 Electrical

6.3.1 Resistance

We used in an interchangeable manner several 4 terminal standard precision resistors of model Vishay VHA518-11ZT [22] with different nominal values covering 3 decades from 10Ω to $10 \text{ k}\Omega$. For the measurements presented in this paper we integrated in the electrical circuit of the measurement scheme resistors with 500Ω and $1 \text{ k}\Omega$ nominal values. The 500Ω resistor was used for the 20 g mass measurement campaign. It was calibrated at PTB against a quantum Hall resistance standard with a relative measurement uncertainty of 2×10^{-9} ($k = 1$) under a calibration temperature of 23°C . The value of resistance is influenced by the temperature, and the typical temperature coefficient of the resistance is $\pm 0.2 \times 10^{-6} \text{ K}^{-1}$, which is given in the data sheet. The value of the resistance must be corrected with respect to the real working temperature. The working temperature is measured by a temperature sensor with a measurement uncertainty of 0.05 K . As the temperature is not directly measured at the resistor, it is assumed that the measurement uncertainty of temperature is 0.1 K . Therefore, the relative measurement uncertainty for the resistor due to temperature change can be estimated as 2×10^{-8} . The time between the calibration of the resistor at PTB and the measurement campaign with PB2 at TU Ilmenau is less than a month. No history of the resistor was available, therefore the possible drift rate was taken from the supplier's specifications datasheet. This states a drift of about 50 ppb in 100 days. The uncertainty attributed to the drift is thus estimated to 17 ppb. It is known that due to mechanical shocks or sudden temperature changes the value of the resistor can undergo changes. Therefore, the resistor(s) was integrated in a protective box by which a direct mechanical shocks are avoided. Additionally, for the transportation (by a private car, as was also done for the 3458A multimeters, see next section) a special care was taken to wrap the resistor (box) inside the protective material, that also provided a passive thermal con-

trol against direct influences caused by temperature variations.

The combined relative measurement uncertainty of the resistance becomes 2.63×10^{-8} . As a remark, the retrace behaviour of the resistors has not been studied yet, but is on the to-do list for future investigations where we will additionally collect more data on drift of the resistance value.

6.3.2 Voltage

Calibrated 3458A multimeters are used to measure the induced voltage in the measurement coils and the voltage drop across the calibrated precision shunt resistor. In the first phase of the developments their calibration was regularly made for different levels of nominal voltage against the Fluke 5720A Calibrator [36], which, in turn, has been calibrated 3 times during the course of 4 months at PTB against the PJVS system for 10 V, 1 V and 0.1 V measurement ranges at about 23°C temperature (see Fig. 10).

The 5720A Calibrator along with an extra 3458A multimeter, used to monitor the voltage stability (drift) without unclamping the connector-plugs or turning the power off completely, were transported to PTB Braunschweig and back to the measurement laboratory of PB2 in TU Ilmenau (distance of about 270 km, one way) connected to external portable power source supplied by the battery of the car. In turn, a negligible alteration of the gain factors have been observed which were mainly attributed to the relative humidity of the laboratory environment and the differences of the atmospheric pressure between Braunschweig and Ilmenau. During the second phase of the PB2 development project the 3458A multimeters and the 5720A calibrator have been already calibrated by us using an own AC and DC PJVS system located directly in the PB2 measurement laboratory at TU Ilmenau. For economical reasons, the 5720A calibrator was still used continuously during all the course of the development to obtain interpolated values of the calibration coefficients for the multimeters due to non-regularly operation of the PJVS system. Since, for the continuous operation with the PJVS system the cryoprobe with the array chip are required to be immersed in liquid helium, we have refilled the liquid helium only when a major measurement campaigns were made. To reduce the costs of the PB2 setup further and for the maintenance of this necessary electrical infrastructure, the progress is underway to utilize voltage reference sources on the basis of Zener references LTZ1000 that are already built in-house and are metrologically well characterized.

Figure 10: Timeline of the voltage calibration with four main 3458A multimeters in use for measuring 1 V voltage at 1 V measurement range against the 5720A Calibrator. Red dots represent the gain factors of the 5720A Calibrator calibrated against PTB's PJVS for 1 V voltage range.

6.3.3 Voltage measurement of DC signal

During the force mode measurement, the so-called ABA cycles are performed consecutively to determine the Bl_F . The voltages of the unloaded (U_A) and loaded (U_B) balance are measured with the same magnitude but different sign, i. e. $U_A = -U_B$. The voltage U_F for the Bl_F determination is calculated by the difference between U_A and U_B as

$$U_F = |U_A - U_B|. \quad (17)$$

The measured amplitudes of U_A and U_B were about 271 mV and -271 mV, respectively, with standard deviations of about 417 μ V. Since the measurement uncertainties of U_A and U_B were the same, i. e. $u(U_A) = u(U_B)$, the relative standard uncertainty of the voltage U_F can be estimated as

$$\frac{u(U_F)}{U_F} = \frac{(u^2(U_A) + u^2(U_B))^{\frac{1}{2}}}{|U_A - U_B|} = \frac{u(U_A)}{\sqrt{2}|U_A|}, \quad (18)$$

which is derived from Eq. (17). The estimation of $u(U_A)$ is sufficient to evaluate the relative uncertainty of U_F according to Eq. (18). Putting the numbers for U_A and the standard deviation from the measurement, as given above, into Eq. (18) would give only a Type A uncertainty contribution, which however, is already included in the Type A uncertainty of the whole measurement. In order to estimate the Type B contribution to the voltage measurement, a more detailed analysis of the digital multimeter is necessary (see Table 3). The measurement of U_A is in the measurement range of 1 V, and the temperature deviation between ambient temperature (average temperature

Table 3: Preliminary uncertainty budget for the unloaded voltage measurements (or loaded, obtained for the force mode of measurements for 1 V measurement range) in μ V V^{-1} with $k = 1$.

Component	Estimate $\times 10^{-6}$	Distribution	Uncertainty $\times 10^{-6}$
Calibration	0.3	Rectangular	0.17
Gain	0.03	Gaussian	0.03
Linearity	0.3	Rectangular	0.17
Combined			0.25

22.39 °C) and calibration temperature (23 °C) is within 1 °C and remain within the 0.6 °C during the campaign. In such a case, the calibration error for max. ± 0.6 °C is given as 0.3 μ V V^{-1} [37]. The multimeter 3458A used in the measurements was calibrated against a Josephson voltage standard before the campaign, so the uncertainties for gain and linearity were taken from these reports. The preliminary uncertainty budget for the unloaded (or loaded) voltage measurement is evaluated in Table 3, and the obtained combined relative uncertainty of U_A is 0.25×10^{-6} . Other minor contributions to the uncertainty budget for the voltage measurement, not included here and yet subject to further investigations, are given in the PTB report [37] and the data sheet of the 3458A multimeter [25].

6.3.4 Voltage measurement of AC signal

In a similar manner as the uncertainty of the DC voltage measurements for the force mode was estimated the uncer-

tainty of voltage measurements of AC signals can be performed. Here, the induced voltage amounts to 90 mV for a signal frequency of 4 Hz, and a geometric factor of 180 T m. The combined relative standard uncertainty of 0.25×10^{-6} (see Table 3) can be taken as the final uncertainty estimate in the velocity mode. Sampling noise, jitter noise and Johnson noise are not treated here, as they enter into type A uncertainty.

6.3.5 Coil current effect

All force mode measurements with the PB2 system are done while adjusting it with the internal actuator in order to have the same absolute value of compensation current as mentioned in Section 4.2. Nonetheless, due to limited resolution of the current source and changes of the Bl of the internal actuator the current values I_A and I_B in load-on- and load-off-state still have a slightly different absolute value. The remaining uncertainty due to the coil current effect was estimated by identifying the parameters α and β of the coil current effect [23, 24] from measurements of force and velocity mode with different mass standards in a range of 0.1 g to 50 g that are shown in Fig. 9. From the linear regression (see Section 6.3.6) to the data $\alpha = -5.45 \times 10^{-1} \text{ A}^{-1}$ and $\beta = 2.24 \text{ A}^{-2}$ was obtained. Considering the measured remaining differences between the absolute values of I_A and I_B , the coil current effect contributes a relative uncertainty of 1.3×10^{-6} during the determination of Bl with a 20 g mass standard. In future, this uncertainty contribution can be reduced by correcting the measurement data with the identified coil current parameter, but further investigations are necessary since for the 20 g standard the residuals of the fit are in the same order as the magnitude of correction.

6.3.6 Force deviation

In order to obtain small residuals from the regression, an unknown force error ΔF_e was introduced into the formula

$$\frac{Bl_F}{Bl_V} - 1 = \frac{\alpha}{2} (I_B + I_A) + \frac{\beta}{2} (I_B^2 + I_A^2) + \frac{\Delta F_e}{Bl_V (I_B - I_A)} \quad (19)$$

that describes the error of the Bl_F that is determined in force mode relative to Bl_V which is obtained in the velocity mode and that was also used to identify the parameter α and β in Section 6.3.5. The best fit was achieved with $\Delta F_e = 0.21 \mu\text{N}$, which represents a relative error of 1.1×10^{-6} for the measurement with a 20 g standard. The origin of

this offset is not yet clear. This is why it contributes fully to the uncertainty budget.

6.4 Data analysis

The general procedure to analyse the harmonic (AC) signals by multi-parameter sine fitting has been described above. The uncertainties that arise from Additive Gaussian White Noise (AGWN), from harmonic distortion, and time jitter have been studied in detail in [10]. In summary, all these error contributions in the data analysis are found to be negligible.

6.5 Frequency error

As stated above, the frequency will be accurately measured with a frequency counter. Here we estimate the bias if the measured frequency (that is used in the linear fit) deviates from the true frequency. This error was estimated – as the other errors were – numerically, by means of a Monte Carlo simulation. The main result is that the relative bias follows a linear form in respect with the relative frequency error. To be more specific, in order to hold the relative bias below 1×10^{-8} , the frequency has to be measured with a relative uncertainty of the same order of magnitude. Therefore, we consider the frequency error to be negligible in our system.

6.6 Length

A homodyne differential interferometer system (SP2000-DI) [20] that provides a specified displacement measurement resolution of 20 pm with an optically compensated path, measures the displacement of the coil (depicted in Fig. 4). The system employs a stabilized He-Ne Laser with a calibrated vacuum wavelength of 632.991 235 2 nm and a specified relative uncertainty of 2×10^{-8} ($k = 2$).

6.6.1 Air refractive index

When the coil motion is measured in air, the vacuum wavelength λ_{vac} from the laser interferometer is changed due to the air refractive index n . The wavelength λ_{Air} in air can be calculated according to Snell's law of refraction as

$$\lambda_{Air} = \frac{\lambda_{vac}}{n}. \quad (20)$$

Table 4: Uncertainty budget of air refractive index with $k = 1$.

	Nominal or mean value	Uncertainty of sensor	Relative uncertainty
Wavelength	632.9912352 nm	1×10^{-8} (relative)	7.28×10^{-14}
Temperature	22.39 °C	0.05 °C	4.41×10^{-8}
Pressure	95437.48 Pa	50 Pa	1.33×10^{-7}
Relative humidity	36.08 %	3 %	2.94×10^{-8}
CO ₂	450×10^{-6}	100×10^{-6}	1.34×10^{-8}
Formula			1.20×10^{-8}
Combined			1.44×10^{-7}

As the displacement is measured with the interferometer, the amplitude $\hat{S}_{meas}$, estimated using the samples of displacement, must be corrected for the air refractive index n , which leads to an increase in Bl_V . Thus, the corrected amplitude $\hat{S}_{corr}$ becomes

$$\hat{S}_{corr} = \frac{\hat{S}_{meas}}{n}. \quad (21)$$

The air refractive index n is calculated using the Ciddor formula [38], which depends on the vacuum wavelength λ_{vac} , the air temperature T , the atmospheric pressure p , the relative humidity h_r and the carbon dioxide content [39]. The measurement uncertainties of the environmental sensors are listed in Table 4, and the corresponding contributions to the uncertainty of the air refractive index n are estimated therein. The combined relative uncertainty amounts to 1.44×10^{-7} .

6.7 Aperture time

When sampling an AC signal with a digitizer, the acquisition time is much less than the period of the measured signal. The effective data acquisition time, which is even shorter than the sampling time, is called aperture time t_{Aper} which acts like a filter. Compared to the original signal, both, initial phase and amplitude of the sampled AC signal, hence are biased. In the velocity mode, the initial phase is not of interest, and thus only the estimated amplitude must be corrected as (see [26])

$$\hat{x} = \hat{x}_{meas} \frac{2\pi f t_{Aper}}{2 \sin(2\pi f t_{Aper}/2)}, \quad (22)$$

where $\hat{x}$ and $\hat{x}_{meas}$ are the amplitudes of original and sampled signal, respectively.

A laser interferometer is used to measure the coil motion. The sampling frequency (50 MHz) is very big relative to the signal frequency, such that the relative bias of the displacement amplitude amounts to be about 2.8×10^{-13}

[26], therefore in our calculations it is negligible. The induced voltage is digitized by the multimeter with the sampling frequency $f_s = 1$ kHz, and the aperture time is 0.7 ms. When the signal frequency f_{sig} is between 0.5 Hz and 10 Hz, the relative bias of U_{ind} lies between 2×10^{-7} and 8×10^{-5} . This is a well quantified error and is corrected after the sine fitting is applied. The time jitter of the aperture time is 5 ns [37], and the relative uncertainty of amplitude is smaller than 1×10^{-9} . The contribution of the aperture time to the uncertainty is negligible.

6.8 Correction for non-linear Bl

6.8.1 Velocity mode

As already mentioned in Section 4.3, the Bl , as obtained from the amplitudes of velocity and induced voltage, does not coincide with the Bl -value at the position $y = 0$. The method of correction has been described above. However, here we estimate the magnitude of the correction as well as the remaining uncertainty. Since the curvature of the Bl changes along y the resulting error will also change slightly as a function of the position (see Fig. 11).

Figure 11: Relative error due to the non-linear Bl as a function of the mean oscillation position. The amplitude of oscillation is 20 μm .

The relative correction is about 7.3×10^{-7} . The remaining uncertainty can be estimated from the uncertainty in the quadratic fit parameter, which in this case is about 13.5 %. In our case no correction has been applied, so that the complete correction value is considered in the uncertainty budget.

6.8.2 Force mode

From Fig. 6 we can calculate the relative gradient of Bl at the position $y = 0$ which is the position where the PID controller directs the coil for the ‘mass on’ and ‘mass off’ cycles. It has a value of about $6.5 \times 10^{-7} \mu\text{m}^{-1}$. Given a positioning repeatability of approximately 0.1 nm the relative uncertainty due to the non-linear Bl in the force mode amounts to negligible 6.5×10^{-11} . Finally, as a relative uncertainty contribution due to the non-linear Bl we have found 7.3×10^{-7} .

6.9 Mechanics

6.9.1 Abbe error

As the usage of our closed-circuit magnet system does not allow to fix the mirror directly on the coil, one has to rely on a rigid frame that is connecting the external coil and the measurement mirror. Additionally, in the existing PB2 setup, the laser spot is not located on the center axis of the coil therefore yielding an Abbe error during the measurements of the coil displacement. The tilt motions dependent amplitude variation of the load carrier, as shown in Fig. 13(a) cause an Abbe error [21] as a function of excitation frequency. These tilt motions were identified with the help of an additional three beam interferometer [40]. For this, the displacement of three points (the reflection spots) on a mirror that is placed on to the load carrier surface (see Fig. 12) was measured by means of the spatially separated three beam laser system. Thus, the tilt about the x -axis (later referred as “Torsion”) and about the z -axis (referred as “Nodding”) can be calculated along the measured displacement in y -direction. The lever position and therefore the displacement of the load carrier is controlled by the internal coil actuator of the balance and its position sensor. Since the position sensor yields a nonlinear relation between the displacement in measures of meter and the position voltage, a polynomial that represents this relation is identified in a preliminary experiment on the same setup. The obtained polynomial can be used to linearize the position control of the load carrier displacement in order to reduce the harmonic distortion of the measured sinusoidal signals.

Figure 12: Schematic representation of the tilt angle measurement set-up.

The procedure for these tilt angle corrections is as follows. The three displacement signals l_i are measured and used to calculate the complex amplitude of the mean displacement located in the middle of the three laser spots and the amplitudes of nodding and torsion angles φ_z and φ_x , respectively:

$$\varphi_x = (l_1 - l_2) / b \quad \text{and} \quad \varphi_z = (l_3 - l_2) / b, \quad (23)$$

where $b = 12$ mm represents the distance between two adjacent laser spots. All complex amplitudes are calculated from the measurement data by applying a sine fit as proposed in [26].

As depicted in Fig. 13(a), tilt angle amplitudes of up to 80 nrad occur in the current PB2 setup. The graph in Fig. 13(b) shows the data collected during two days of measurements for the Bl determinations by alternating the oscillation frequency between increasing and decreasing modes. Overall there were done 380 determinations of Bl at 20 different frequencies and the mean value and the standard deviation of each subset of data is shown in the graph. The change of the position of the laser spot on the mirror surface relative to the center of the coil due to a tilt of the load carrier, which is schematically shown in Fig. 14, can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(\varphi_z) & \sin(\varphi_z) & 0 \\ -\sin(\varphi_z) \cdot \cos(\varphi_x) & \cos(\varphi_z) \cdot \cos(\varphi_x) & -\sin(\varphi_x) \\ -\sin(\varphi_z) \cdot \sin(\varphi_x) & \cos(\varphi_z) \cdot \sin(\varphi_x) & \cos(\varphi_x) \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}. \quad (24)$$

Figure 13: (a) Measured tilt angle amplitudes during sinusoidal excitation. (b) BI determination with (red) and without (blue) tilt correction.

Figure 14: Relative position of mirror surface and coil and additional displacement due to load carrier tilt.

Since the interferometer measures the displacement in y -direction, only the difference of this coordinate y to its initial value y_0 is considered:

$$y - y_0 = -\sin(\varphi_z) \cdot \cos(\varphi_x) \cdot x_0 + (\cos(\varphi_z) \cdot \cos(\varphi_x) - 1) \cdot y_0 - \sin(\varphi_x) \cdot z_0. \quad (25)$$

The observed tilt amplitudes are in the range of several nanoradians, thus Eq. (25) can be simplified to

$$y - y_0 = -\varphi_z \cdot x_0 - \varphi_x \cdot z_0. \quad (26)$$

Thus, with the knowledge of the distances x_0 and z_0 between the center axis of the coil and laser spots of the interferometer, the complex displacement amplitude $\underline{S}$ can be obtained and corrected as

$$\underline{S}' = \underline{S} - z_0 \cdot \underline{\varphi}_x - x_0 \cdot \underline{\varphi}_z, \quad (27)$$

where $\underline{\varphi}_x$ and $\underline{\varphi}_z$ represent the complex amplitudes that were obtained by sine fitting to the measured tilt angles. x_0 and y_0 are taken from the technical drawings of our PB2 setup (that has an actual below $\pm 10 \mu\text{m}$ manufacturing tolerance) and an estimated uncertainty of 0.5 mm due to the shift of the laser spot during alignment of the interferometer. Applying this correction to the measured displacement amplitude for the frequency range of 2 Hz to 10 Hz the dependency of the BI calculated with Eq. (11) have been significantly reduced (see Fig. 13(b)). In the range of 0.5 Hz to 10 Hz , in which the measurements with the PB2 system are usually performed, the relative correction of the determination of BI due to the tilt amounts a magnitude of 3.1×10^{-5} . The statistical uncertainty of this correction is 5.04×10^{-7} , while the estimated uncertainty of the determination of x_0 and z_0 provides an additional uncertainty of 8.61×10^{-7} for the determination of the BI . Therefore, the combined uncertainty contribution due to tilt motions of the load carrier is 9.98×10^{-7} . For the lower frequency range there is still a frequency dependency visible that cannot be corrected by the tilt angle measurement and needs further investigation. Along the increasing excitation frequency in this range the magnitude of this dependency appears to be in the same order as the standard deviation in determining the BI . The main reason for this increase is the fact that the amplitude of the induced voltage decreases with decreasing the oscillation frequency within the mechanically limited displacement amplitude of $40 \mu\text{m}$. For the further investigations of the oscillation frequency influence on the BI -determination, a reduction of the standard deviation of the induced voltage amplitude measurement is necessary. In order to reduce these uncertainties, the progress is underway in developing and integrating a viable approach

Figure 15: (a) Schematic of the alignment axis with an alignment cuboid, (b) horizontal displacement of the load carrier.

that utilizes a well-known and well-characterized sine signal provided by an accurate signal generator (3245A) and referenced by PJVS or in best case directly by an AC PJVS with sufficient settling time as presented in [21, 41].

6.9.2 Vertical alignment

Additionally, the force measurement system should be aligned parallel to the vector of the gravitational acceleration, g . This is necessary, because we are determining an absolute mass value, rather than measuring a mass difference (i. e. a calibration against a known reference standard). For alignment purposes, another mirror cuboid is attached to the load carrier of the balance (Fig. 15(a)). The angles between the surfaces of the cuboid were identified beforehand and do not deviate more than $21.8 \mu\text{rad}$ from their nominal value of 90° . During the alignment process, a second interferometer is used to measure the orthogonal displacement of the side of the mirror cuboid, thus measuring the horizontal displacement, while the load carrier is driven up and down by an excitation of the lever. The beams of the interferometer need to be aligned to better than 0.4 mrad so that it can work properly within the specifications given by the manufacturer. The uncertainty that is contributed by the remaining possible misalignment of the beam adds a value of 1.15×10^{-6} to the relative uncertainty of the determination of Bl , which is ten times greater than the contribution due to the misalignment of the mirror surfaces of 1.1×10^{-7} . From the ratio of the horizontal to vertical displacement, later measured by the PB2 internal interferometer, the trajectory of the load carrier, which is guided by a parallel lever system and its misalignment are identified. The measurements of the horizontal displacement are shown in Fig. 15(b), which sums up a set of six

repeated measurements for both axes in x - and z -direction. These horizontal displacements correspond to an angular misalignment of 2.8 mrad and 2.2 mrad , respectively. The resulting cosine errors of 3.9×10^{-6} and 2.4×10^{-6} are therefore known with a combined uncertainty of 1.16×10^{-6} and are corrected during the data analysis [11].

With an autocollimator and the reflective surface of a liquid pool (e. g. alcohol or water with a droplet of washing-up liquid in order to reduce the tension of the water surface), the normal vector of the top surface of the mirror cuboid can be adjusted relative to g . This is done by initially aligning the autocollimator along g by means of the liquid pool, which is placed on the load carrier, and then the liquid pool is replaced by the mirror cuboid. Since the orientation of the trajectory relative to the mirror surface is known from the horizontal displacement, the trajectory adjustment follows the adjustment of the top surface.

6.9.3 Corner load error

The commercial load cell has a specified corner load error of maximum 0.2 mg for a load of 100 g and a displacement radius of 15 mm , i. e. the relative error can amount to 1.4×10^{-7} per millimeter. The centers of mass of the different mass artifacts are placed to within about 1 mm at the same location on the load carrier. Assuming a rectangular distribution we can attribute a relative measurement uncertainty of 7.7×10^{-8} due to the corner load error.

6.10 Time

As a time reference a GPS disciplined oven stabilized quartz oscillator of type RF Suisse RS CGGO-10 is used.

Table 5: Combined uncertainty budget given in 1×10^{-6} (ppm) with $k = 1$ for the measurement of a 20 g mass standard.

Source of uncertainty	Relative uncertainty
Time standard	negligible
<i>Velocity mode</i>	
Type A	0.19
Interferometer	0.18
Voltage	0.25
Data analysis	negligible
Frequency error	negligible
<i>Length</i>	
Length standard	0.01
Air refractive index	0.15
Aperture time (interferometer and voltage)	negligible
Non-linear Bl	0.73
<i>Force mode</i>	
Type A	0.20
Mass (in air)	0.16
Gravity	0.20
Resistor	0.03
Voltage	0.25
Vertical alignment	1.16
Abbe	1.00
Corner load	0.08
Coil current	1.30
Parasitic force	1.10
Non-linear Bl	negligible
Total combined	2.47

A typical stability of 1×10^{-10} is specified by the company. The time interval error is thus negligible.

6.11 Combined uncertainty

Finally, combining all the investigated uncertainty contributions (see Table 5) for the 20 g-measurement we obtained the total combined uncertainty of 2.47 ppm.

7 Virtual Planck-Balance

The Planck-Balance is a complex measurement system, containing many possible sources of uncertainty. Moreover, these uncertainties can interact in a very complicated manner, which, in certain scenarios, cannot be fully mapped by linear derivatives of the measurement function to high accuracy. Therefore, a careful study of the uncertainty budget becomes time consuming. Our aim, however, is to finally provide a device that is easy to use. This applies

also to the uncertainty estimation of the calibration process. Thus, in order to simplify the process of uncertainty evaluation for a mass measured with the Planck-Balance, and to increase the accuracy of its estimation, a Digital Twin of the Planck-Balance, the so-called Virtual Planck-Balance (VPB) is under development. The concept is borrowed from the Virtual Coordinate Measurement Machine (VCMM), which is a Metrological Digital Twin for the uncertainty estimation for Coordinate Measurement Machines [42, 43].

In order to estimate the uncertainty of a measurement process, three ways are viable. 1) Establish an analytical uncertainty budget according to the GUM [32]. This method was used especially for the evaluation of the uncertainty contributions presented in the previous chapters. 2) An experimental determination, which requires a reference standard, as described for CMMs in [44]. In our case, this method has been used only to check whether our system suffers from a systematic offset. Specifically, we have applied it in our measurements in order to compare and verify the obtained results with the calibrated 20 g mass standard. For a final application of the Planck-Balance, this way is not viable, since we aim at calibrating unknown mass standards without referencing to a mass artifact. 3) An evaluation based on numerical simulations. This is what we are aiming for with the Virtual Planck-Balance in its final stage of development.

All three ways of uncertainty evaluation require good models that describe the influencing factors (this has been done above). The next step in simulating the measurement process is to combine those models with random numbers that vary each influencing parameter before a new virtual measurement is started, i. e. setting up a Monte Carlo (MC) simulation. During the project run-time, this has been established to a certain degree only, and needs further improvement.

In Fig. 16 the graphical user interfaces are shown that were developed for the force mode (a) and velocity mode (b) separately with MATLAB. The following step would be to combine both tools to a single one, describing the final measurement process of a mass calibration. In this version only parts of the aforementioned uncertainty contributions, and their respective models, were known. Thus, the uncertainty distribution that was calculated by means of this version did not match to the one that was obtained by a real long-term measurement of the Planck-Balance (which provided a strong indication that the model is yet not precise enough).

After having elaborated more sophisticated models, based on new investigations of the Planck-Balance, currently a new version of the Virtual Planck-Balance is un-

Figure 16: Graphical user interfaces for the Virtual Planck-Balance (VPB): (a) for force mode (b) for velocity mode.

der development, completely rewritten in C++ language. The reason for redesigning the Virtual Planck-Balance is that PTB is investigating the possibility to apply the concept of the VCM to other measurement devices as well. Therefore, the code of the VCM has been carefully analysed and multiple building blocks have been established to split up the Virtual Device in blocks that can be used in an unaltered manner in other Virtual Devices, such as the statistical ‘tool box’, and blocks that are unique for each new Device, such as the measurement models. The whole simulation process will be revised and improved. Thus, to be more concrete, the core idea of the VCM-concept, which is to be transferred to VPB, is to replicate the whole measurement process within the simulation. The temporal component is a central part of a real measurement and therefore has to be considered in the simulation. Usually, a Monte Carlo simulation is performed with fixed means of the values to be propagated. Following the VCM-concept, ‘noise’ is added to data that are obtained from real measurements. This way we can think about the given value being time dependent as it is represented by a series of multiple values. Depending on the strategy of evaluation, for example, if during the measurement we observed large drifts (which were not corrected), we may expect to obtain a larger uncertainty in comparison to a measurement with smaller drifts. The contribution of the described effects to the total uncertainty of the measurement may be referred to as *uncertainty contribution of the measurement process*.

The simulation consists of multiple nested loops, where the outermost of them is the loop necessary to perform MC simulation. During each simulation run the whole measurement will be virtually repeated. The sec-

ond to outer loop represents a loop over multiple cycles containing alternating groups of measurements for velocity and force mode. The two innermost loops are executed successively, where each of them represents the measurement process of a single group of multiple velocity and force mode measurements.

This explicit separation of different time scales in terms of multiple loops, allows to separately treat long-term and short-term influences. This is necessary, because the treatment of both influences on the same time scale could lead to an erroneous uncertainty for non-linear systems. This way, for example, the local gravitational acceleration is randomized on the outermost loop and the induced voltage on the most-inner loop.

The already existing physical models of the PB described in previous chapters can be directly used for the creation of the VPB. For existing models and their respective parameters, the following question has to be asked: On what time-scale does a given parameter remain unchanged and when does it have to be re-randomized? When this time-scale is identified, the procedure for determination of the underlying distribution and its parameters can be adapted. Additionally, new models describing the behaviour of the balance during different phases of the mass calibration measurements still have to be determined and integrated.

Finally, when this Virtual Planck-Balance is able to map the true uncertainty of the measurement process, only a few real measurements need to be performed. Those measurements provide a mean value for the measurement. The attributed uncertainty, however, will be provided by the Monte Carlo simulation. This will also save measurement time.

The VPB is being developed with a long-term perspective to be easy to integrate into a commercial software of the PB. This will ease the transition between the ongoing research-phase and industrial application. Additionally, as the software is designed to be modifiable and extensible, future commercialization by multiple manufacturers, the same as in the case of VCMM, is made possible.

8 Conclusions and outlook

In this work, we have presented the progress in the development of the Planck-Balance 2 setup and the preliminary obtained operational performance results in mass calibration. The PB2 is a compact table-top version of the Kibble balance for industrial applications, aiming to perform mass calibration for the weights of E2 class in the range from 1 mg to 100 g within the maximum permissible errors and the associated uncertainties as recommended by International Organization of Legal Metrology (OIML). This balance is developed within the collaboration established between the PTB and TU Ilmenau, and operates in a specialized mass measurement laboratory in the Institute of Process Measurement and Sensor Technology at the TU Ilmenau. The presented results are the outcome of approximately three years of investigation since the beginning of the project.

Initially, a comparison of the electromagnetic force factors of PB2 setup, as is obtained from the force and velocity mode of measurements, is carried out in air with a 20 g weight of class E1. This was used as a reference mass and was prior calibrated by PTB in accordance to the conventional subdivision and mass comparison method by down-scaling it from a 1 kg artifact. For the measurement of a 20 g mass, a combined relative uncertainty of 2.5×10^{-6} ($k = 1$) has been achieved with the PB2. Then the comparison of combined relative measurement uncertainties of each individual electromagnetic force factor as is obtained from the measurements using a set of mass standards are made in reference to the result obtained for the 20 g mass standard. It has been shown that PB2 is capable of realizing the SI unit of mass for the mass values ranging over five decades (presented 1 mg to 50 g, Fig. 9) in accordance to the final act of the 2019 redefinition of the SI base units. It has been shown also, that PB2 is capable of realizing the SI unit of mass for the mass values ranging over five decades (presented 1 mg to 50 g) in accordance to the ‘mise en pratique’ [1] of the redefined kilogram. The determined uncertainties are in the same order of magnitude as

needed for calibrations of E2 weights according to OIML R111.

Several upcoming improvements of the PB2 setup are discussed in this work that are aiming at achieving the following competing objectives. Firstly, our main objective is to provide a system for mass calibration that is, at the same time, a realization of the unit kilogram. Furthermore, we aim at a further reduction of the total relative uncertainty budget necessary to calibrate mass standards of class E2. For this, we have already identified several sources of errors and outlined the options for reducing their influences. For example, as the main uncertainty contributions, there are the alignment process and the Abbe error. For this end, some minor mechanical, electrical, optical and data processing design alterations have yet to be finalized until PB2 is released for operation in common mass calibration laboratories. Secondly, it is the economic reasoning, which can be described in terms of the required uncertainties of the mass calibration to be achieved against the costly technologies which should be integrated in the PB2 setup. As an example, it should be obvious that a PJVS system (despite providing us immensely big research and development platform) will not be a part of the PB2 system, due to its enormous cost. Contrariwise, when aiming for a system (PB1) that would be capable of mass calibration according to E1 specification then the use of PJVS is required.

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