Theoretical Efficiency of a Thermionic Diode

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(Z. Naturforschg. 20 a, 1588—1591 [1965]; received 11 August 1965)

A theoretical analysis of the efficiency of a thermionic diode is made in the highly efficient region of very low anode potential ($V_a < 1$ volt). Maximum efficiency is calculated as a function of the cathode temperature for optimum values of anode potential. The calculations are performed for different values of usable power density, anode temperature and emissivity of electrodes.

The analysis is based on the potential energy diagram of an electron shown in Fig. 1*. Energy in the form of heat is supplied to the electron at the FERMI level of the cathode. If the electron gains enough speed to overcome the potential energy bar-

Fig. 1. Potential energy diagram of an electron in a thermionic diode.

rier (eV_c), it will reach the anode. It will lose at least eV_a units of energy to the anode lattice (this energy will have to be removed by cooling the

anode). The electron will next lose eV_l units of energy while doing usable work at the load. Before returning to the FERMI level of the cathode it will lose eV_w units of energy at the cathode lead wire¹. The performance of such a diode was discussed by several workers²⁻⁵. It was shown that the efficiency (η) increases as the anode potential (V_a) decreases.

There is, however, an optimum value of V_a , beyond which a further decrease of V_a will lead to a decrease in the efficiency (due to an increase in the back emission). This optimum value depends in general on the anode temperature (T_a), and varies in the range 0.5 to 1.0 volt for $T_a < 500^\circ\text{C}$. In 1962, V_a values, smaller than 1.6 volts were considered to be impractical for the operation of thermionic diodes². Two years later CHAPMAN⁶ and KLUGE⁴ reported on thin film coated electrode materials having minimum V_a values in cesium vapor of 1.4 and 1.0 volt, respectively. With this, experimentalists have arrived at the doorsteps of the optimum range which was mentioned above. The present analysis covers this optimum range of low anode potential and high efficiency.

* Glossary:

- A = Coefficient in the RICHARDSON eq. (1) ($120 \text{ amp/cm}^2/^\circ\text{K}^2$).
- B = Function defined in eq. (6) (watt/cm^2).
- e = Charge of the electron ($1.602 \times 10^{-19} \text{ coul.}$).
- F = Function of T_c defined in eq. (4) (volts).
- H = Conductive heat loss at the cathode lead wire per unit area of the cathode (watt/cm^2).
- I = Current per unit area of the cathode (Amp/cm^2).
- k = BOLTZMAN constant ($1.3803 \times 10^{-23} \text{ Joule/}^\circ\text{K}$).
- L = LORENZ number ($2.45 \times 10^{-8} \text{ watt} \cdot \Omega/^\circ\text{K}$)⁶.
- P = Usable power density (power dissipated in the load per unit area of cathode) (watt/cm^2).
- Q = Power dissipated in the load and in the cathode lead wire per unit area of the cathode (watt/cm^2).
- T = Temperature ($^\circ\text{K}$).
- T_0 = Room temperature (300°K).
- V = Voltage (volt).

W = JOULE heat developed at the cathode lead wire per unit of the cathode (watt/cm^2).

ϵ = Emissivity.

η = Efficiency.

σ = STEFAN BOLTZMAN constant ($5.67 \times 10^{-12} \text{ watt/cm}^2/^\circ\text{K}^4$).

Subscripts: a Anode, c Cathode, l Load, w Cathode lead wire.

¹ Unlike anode lead wire, whose cross sections may be large, cathode lead wires must have a relatively small cross section, in order to reduce heat loss through conduction.

² N. S. RASOR, *Advan. Energy Conv.* **2**, 545 [1962].

³ J. M. HOUSTON and H. F. WEBSTER, *Advan. Electron Phys.* **17**, 125 [1962].

⁴ W. KLUGE, *Z. Naturforschg.* **19 a**, 595 [1964].

⁵ J. M. HOUSTON, *J. Appl. Phys.* **30**, 481 [1959].

⁶ R. A. CHAPMAN, *J. Appl. Phys.* **35**, 2832 [1964].

Fig. 2. Efficiency of thermionic diode, black ($\epsilon=1$) electrodes (V_a and V_c in volt, P in watt/cm²). (a) $T_a=300^\circ\text{K}$, (b) $T_a=500^\circ\text{K}$.

Fig. 3. Efficiency of thermionic diode, tungsten electrodes (V_a and V_c in volt, P in watt/cm²). (a) $T_a=300^\circ\text{K}$, (b) $T_a=500^\circ\text{K}$.

Analysis

The RICHARDSON equation² was used for the cathode and anode current.

$$I_i = A T_i^2 \exp \left[-\frac{e V_i}{k T_i} \right], \quad i = a, c. \quad (1)$$

The net current is equal to

$$I = I_c - I_a. \quad (2)$$

The efficiency of a thermionic diode, having a potential energy diagram, as shown in Fig. 1, is given by

$$\eta = \frac{I(V_c - V_a) - W}{\varepsilon \sigma (T_c^4 - T_a^4) + H + I_c(V_c + 2kT_c/e) - I_a(V_c + 2kT_a/e)}. \quad (3)$$

The terms $2kT_i/e$, $i = a, c$, represent the heat, carried by the electrons in the form of kinetic energy (sometimes referred to as the electron cooling terms). In case that $I_a = 0$ ($i \cdot e \cdot I = I_c$) it has been shown^{5,3} that the efficiency was maximized by the following relations:

$$\left. \begin{aligned} W &= [\eta/(2-\eta)]^{1/2} F(T_c) I, \\ H &= \frac{1-\eta}{\eta} W, \quad F(T_c) = (T_c^2 - T_0^2)^{1/2} L^{1/2}, \end{aligned} \right\} \quad (4)$$

It is assumed that these relations will approximately maximize η even when $I_a \neq 0$. Then eq. (3) and (4), after some algebra, reduce to

$$\eta = (2P - Q)/(P + B) \quad (5)$$

where

$$\begin{aligned} Q &= I(V_c - V_a), \\ B &= \varepsilon \sigma (T_c^4 - T_a^4) + I_c(V_a + 2kT_c/e) - I_a(V_a + 2kT_a/e). \end{aligned} \quad (6)$$

P is equal to the usable power density, that is, power dissipated at the load, namely

$$P = Q - [\eta/(2-\eta)]^{1/2} I F(T_c). \quad (7)$$

The pair of eqs. (5) and (7) is mathematically equivalent to

$$\eta = \mathfrak{h}_1(P, V_a, T_c, T_a). \quad (8)$$

Although the explicit form of $\mathfrak{h}_1$ is unknown, η can be calculated for any value of the independent variables by using eq. (5) and (7). The independent

variable V_a was made dependent by requiring that it will maximize η , namely

$$\partial \eta / \partial V_a = 0. \quad (9)$$

Thereby the problem was reduced, so that η can be evaluated from

$$\eta = (2P - Q)/(P + B) \quad (10)$$

with the constraints

$$P - Q + [\eta/(2-\eta)]^{1/2} I F(T_c) = 0, \quad (10a)$$

$$\eta \frac{\partial B}{\partial V_a} + \frac{\partial Q}{\partial V_a} = 0 \quad (10b)$$

which is the mathematical equivalent of

$$\eta = \mathfrak{h}_2(P, T_c, T_a). \quad (11)$$

η was calculated from eq. (10) with the aid of a computer code for the LGP-30 digital computer⁷. Final results for two cases are presented (Figs. 2, 3). In one case (Fig. 2) cathode and anode are assumed to be black ($\varepsilon = 1$) and in the other (Fig. 3) the cathode and anode materials are assumed to have the emissivity of tungsten⁸. The multiple reflection emissivity in the latter case was calculated from $\varepsilon = [1/\varepsilon(T_c) + 1/\varepsilon(T) - 1]^{-1}$, $T = [T_c T_a]^{1/2}$. (12)

Geometric mean temperature, rather than anode temperature is used, because of the non-grayness of the metallic surface⁹.

Fig. 4. Efficiency in the optimum V_a range (T_a in °K, V_a in volt).

⁷ Royal McBee Corporation, Westchester Ave., Port Chester, N. Y.

⁸ D. E. GRAY (Coordinating Ed.), Am. Inst. Phys. Handbook, McGraw-Hill Book Co., Inc., New York 1963.

⁹ H. GIEDT, Heat Transfer, Van Nostrand Co. Inc., 1957. Page 248.

Conclusions

The locus of the points of maximum efficiency (Fig. 3) is shown in Fig. 4 (solid lines). For comparison, curves of maximum efficiency for tungsten electrodes, as computed by Houston⁵, are also given (broken line) in Fig. 4. It is evident from Fig. 4 that if a diode could be made to work with an opti-

mum anode potential of 0.8–0.9 volts (Fig. 3), the efficiency of present day diodes ($V_a > 1.6$ volts) could perhaps be more than tripled.

Acknowledgements

The author is indebted to Dr. N. S. RASOR of the Thermo Electron Eng. Corp. for his constructive criticism, and to Mr. Y. BONEH of the Nuclear Research Center – Negev for his help with the computations.

Non-linearities and Discontinuities of the Photoemission from Multi-Alkali Cathodes at Nitrogen Temperatures*

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(Z. Naturforschg. 20 a, 1591–1599 [1965]; received 28 September 1965)

An account is given of measurements relating to the temperature dependence of the photoemission of multialkali cathodes at temperatures in the region of 80 °K. These measurements show intensity-dependent and spectrally dependent non-linearities with hysteresis in which pronounced discontinuities of the quantum yield of the photoemission occur up to a factor of 2. With a temperature constancy in the region of ± 0.01 °K these discontinuities are fully reproducible. Because of the extremely high temperature sensitivity of the effect interesting prospects are afforded when it is used for detecting thermal radiation on the basis of photoelectrons that can be utilized particularly well technically.

In discussing the increase of the photoemission from alkali halides and other polar crystals just below the fundamental-band absorption, whereby the photoelectrons are excited secondarily by excitons apparently produced by the light¹, F. SEITZ suggested that “such a process would depend on incident radiation intensity or would lead to hysteresis and irreversible changes”.

APKER and TAFT¹, on the other hand, established complete invariability of their results and thus strict linearity of the photoemission when the light intensity is varied by more than a factor of 10. In the many works that have appeared since then no doubts have been cast on their findings².

In the following an account is given of measurements of the photoemission from multi-alkali cathodes. These measurements show the dependence on the light intensity predicted by SEITZ (non-linearity, hysteretic properties and discontinuities) and are reproducible only by reactivation in keeping with the hysteresis. At the same time extremely high sen-

sitivity of the properties of the cathode on temperature is obtained. The latter was taken as the starting point of these investigations when a complete anomaly in the temperature dependence of the photoemission was found for the first time in 1960³ at multi-alkali cathode temperatures in the region of 110 °K. It was established that the quantum yield increased in a reproducible manner by a factor of 3 for photon energies of 2 eV when the temperature drops by 1 °K. This factor was determined from measurements in which, it may be pointed out by way of completion, not the photoemission with constant monochromatic light at various cathode temperatures was measured, but the spectral distribution at progressively changing temperatures.

The application of this temperature effect in a radiation detector has been proposed. Here the photoelectric current of the cathode, which is suitably illuminated in the range in question, is modified very finely when the radiation to be measured heats the cathode⁴. When continued appropriately

* Work supported by the Fraunhofergesellschaft (Contract No. T 532 I 203).

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¹ L. APKER and E. TAFT, Phys. Rev. **81**, 698 [1951].

² E. TAFT, H. PHILIPP, and L. APKER, Phys. Rev. **113**, 156 [1959].

³ G. FRISCHMUTH-HOFFMANN, P. GÖRLICH, and H. HORA, Z. Naturforschg. **15 a**, 1014 [1960].

⁴ H. HORA, DBPat. 1 137 876.