Self-Organized Formation of Spatial and Temporal Dissipative Structures in Semiconductors

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Nonlinear current transport behavior during avalanche breakdown of germanium comprises the spontaneous symmetry-breaking evocation of both spatially inhomogeneous and temporally unstable dissipative structures in the formerly homogeneous semiconductor. Such kind of nonequilibrium order-disorder transitions are discussed in terms of two simple models based on the new circle-map formalism of universal scaling behavior and the well-known Rashevsky-Turing theory of morphogenesis.

Recent intensive studies of nonlinear physical systems sustain the hope that, despite their rich variety of complex dissipative structures, they might possess universal features shared by entire classes of similar nonlinear dynamical processes. In particular, this hope was strikingly realized through the discovery of a new set of universality rules — independent of any dynamical details — characterizing different systems as they approach turbulence on distinct paths. Simultaneously, new global structures of scale-invariant or fractal objects could be discerned [1].

So far, it appears that the subject of such complex nonlinear behavior is dominated by theoretical investigations and computer studies, whereas experimental measurements on real physical systems represent the minority. Among the various objects which can be studied experimentally, solid-state turbulence in semiconductors appears particularly interesting. Since the spatio-temporal nonlinear transport phenomena in current-carrying semiconducting systems are associated with the motion of charge carriers, advanced electronic measuring techniques can directly be applied for the experimental observations. This fact represents a distinct experimental advantage for semiconductors as a study object for complex behavior compared to other physical systems. First experimental results on chaotic nonlinear dynamics of different semiconductor systems have been lately reported by

several groups [2–8]. In the following, we briefly summarize our recent experimental investigations on the spontaneous formation of both spatial and temporal dissipative structures in semiconducting germanium. Such kind of a symmetry-breaking nonequilibrium phase transition occurring in homogeneously doped germanium during impurity-impact-ionization-induced avalanche breakdown is then discussed in terms of a universal circle-mapping and a morphogenetic reaction-diffusion model.

Our experimental system consists of single-crystalline p-type germanium having typical dimensions of about $0.2 \times 2 \times 5$ mm^3 and properly arranged ohmic aluminum probe contacts. Similar to the corresponding phenomena in gaseous plasma discharges, impact ionization of the shallow impurities can be achieved in the bulk of the homogeneously doped semiconductor at low temperatures. In the temperature regime of liquid helium most of the charge carriers are frozen out at the impurities. Since the ionization energy is only about 10 meV, avalanche breakdown already takes place at electric fields of a few V/cm and persists until all impurities are ionized. The underlying nonequilibrium phase transition from a low conducting state to a high conducting state is directly reflected in strongly nonlinear regions of negative differential resistivity. Under variation of distinct control parameters (electric field, magnetic field, temperature, etc.) the resulting electric current flow displays a wide variety of spatio-temporal nonlinear transport behavior, comprising the spontaneous symmetry-breaking emergence of both spatially inhomogeneous and temporally unstable dissipative struc-

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tures in the formerly homogeneous semiconductor. One of the major issues of strong current interest is the question to what extent the break-up of spatial order during current filamentation is correlated with the onset of low-dimensional temporal chaos in the current oscillations.

More valuable insight into the complex spatial behavior of our semiconductor system can be provided by means of two-dimensional imaging of the current filament structures developing during avalanche breakdown. Utilizing low-temperature scanning electron microscopy [9], the images are obtained by scanning the specimen surface with an electron beam and by recording the beam-induced current change in the voltage-biased specimen as a function of the coordinate point (x, y) of the beam focus. Figure 1 clearly indicates the self-sustained formation of a typical current filament pattern in the nonlinear regime of the current-voltage characteristic. As reported elsewhere [10] in detail, the current flow becomes more and more homogeneous if the semiconductor system is driven further into its linear post-breakdown region at higher electric fields.

In addition to the spatially resolved observation of current filament structures, the above electron-beam scanning technique also provides information on the temporal response to the local beam-induced perturbation as a function of the beam coordinate. Time-resolved measurements indicate that the beam irradiation can stimulate current oscillations if the beam is focused upon the boundary region of a current filament [10]. The temporal profile of the beam-induced current oscillations suggests that it is the boundary region of the spatial current patterns where the spontaneous temporal current structures develop during avalanche breakdown. From these experiments, the filament configurations exhibiting chaotic temporal resistance behavior and, hence, the connection between the onset of temporal chaos and spatial disorder can directly be identified.

Self-generated current oscillations (with a relative amplitude of about 10^{-3} in the frequency regime $0.1 - 10\,\text{kHz}$) are found to be superimposed upon the steady d.c. current (of typically a few mA) in the strongly nonlinear post-breakdown regime of the measured current-voltage characteristic. By means of slightly varying the applied electric or transverse magnetic field, the temporal behavior of the current changes dramatically, displaying the typical uni-
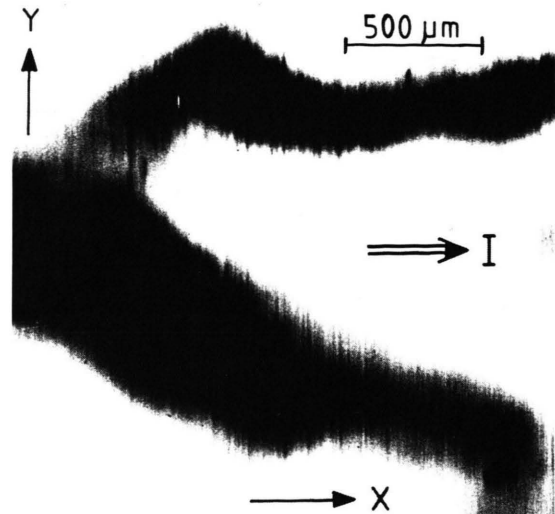

Fig. 1. Brightness-modulated image of the filamentary current flow in a homogeneously doped semiconductor during avalanche breakdown obtained by low-temperature scanning electron microscopy. The dark regions correspond to the filament channels extending along the x-direction.

versal scenarios of chaotic nonlinear systems [5]. The observed dynamical phenomena include the intermittent switching between different oscillation modes (Pomeau-Manneville scenario), the period-doubling cascade to chaos (Feigenbaum scenario), and the quasiperiodic route to chaotic oscillations (Ruelle-Takens-Newhouse scenario), as well as suppression of quasiperiodicity through frequency locking. Moreover, we observed a transition from an ordinary chaotic state to a strongly turbulent hyper-chaotic state of higher dimensionality. The apparent structural change of the underlying attractor between different dynamical states indicates a break-up of the semiconducting system from strongly coupled into more independent subsystems. In this way, new actively participating degrees of freedom are gained reflecting increasing dimensionality of the semiconducting system. The decreasing spatial correlation between various subsections of the same semiconductor crystal can be simply demonstrated by monitoring independently distinct partial voltage drops via probe contacts properly arranged along the crystal surface. Taking further into account the experimental evidence that the different crystal parts are spatially coupled by diffusive phonon propagation [11], our experiments deal with a chal-

lenging example of a macroscopic system consisting of spatially separated and coupled subsystems which by themselves show nonlinear dynamical behavior.

As an example, we concentrate in the following on the self-organized formation of quasiperiodic and mode-locked current instabilities. Due to the presence of competing fundamental oscillatory modes intrinsic to our semiconducting system, quasiperiodicity can be obtained at two incommensurate frequencies where the system trajectory traces out an invariant two-torus in phase space. Representing a 2-D projection of the characteristic quasiperiodic attractor, the phase portrait of Fig. 2a obtained from different partial voltage drops along the semiconductor crystal clearly indicates weak nonlinear coupling between the corresponding two spatially separated oscillation centers, displaying rather independent temporal behavior. By increasing the degree of nonlinear coupling through slightly varying the external magnetic field as an appropriate control parameter, the semiconductor system develops an increasing tendency to lock into commensurate motion where the ratio of the oscillatory frequencies is now a rational number. This phenomenon is known as frequency or mode locking, and it is generally present in dissipative dynamical systems [12]. A rather convenient example is given by two clocks that − hanging back to back on the wall − tend to synchronize their motion. The stronger nonlinear coupling between the two competing oscillatory modes giving rise to mode locking is demonstrated in the phase portrait of Fig. 2b for our semiconducting system, where the two originally incommensurate natural frequencies lock exactly into the rational ratio of the integers 2:3. Now the spatially separated oscillation centers display temporally resonant response behavior. Since mode locking relies on the interdependence of the frequency and the amplitude of nonlinear oscillators, the frequency ratios remain locked over a finite region of parameter space, although both the amplitudes and frequencies of the oscillations are changing. The amplitudes adjust in such a way as to keep the frequency ratio constant.

Distinct properties of such complex dynamical behavior found in real nonlinear dissipative systems can be modelled by simple discrete 1-D mappings of the circle onto itself [12]. These so-called "circle maps" are predicted to exhibit universal scaling
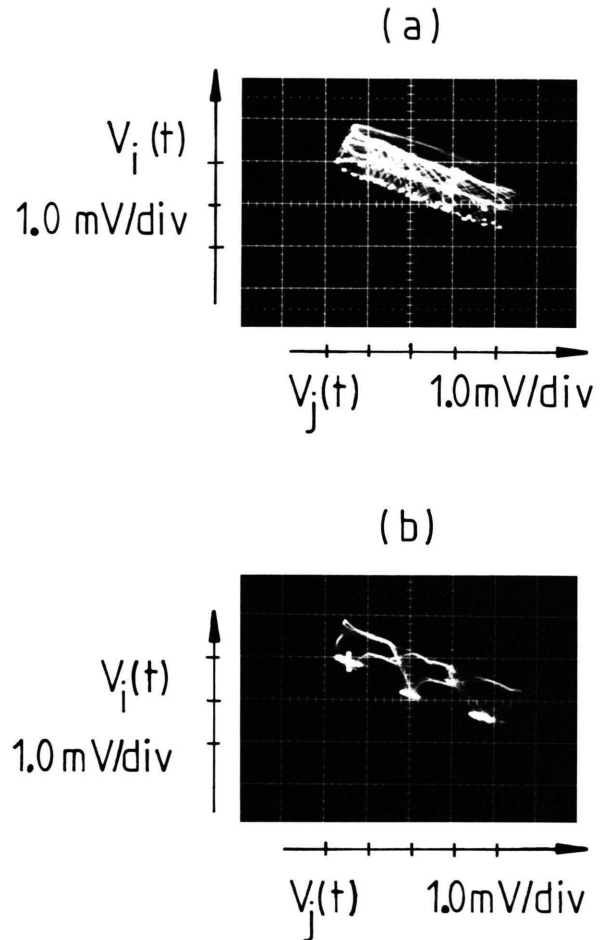

Fig. 2. Phase portraits of a quasiperiodic (a) and a mode-locked state (b) obtained by plotting two different inner voltage drops against one another. The bright spots at the lower edge of the phase portraits correspond to the points of the Poincaré cross-section.

behavior due to the characteristic self-similar staircase structure of the mode-locking intervals at the critical boundary where the map ceases to be an invertible diffeomorphism. Below criticality the mode-locked resonances form hornlike structures known as "Arnold tongues". Depending upon the strength of the nonlinear interaction between the oscillators, the widths of these tongues grow from zero (no coupling) as far as the tongues overlap at criticality. Correspondingly, the residual unlocked portions of quasiperiodic states decrease from unity to a Cantor set of zero measure. Just beyond the critical threshold of nonlinear coupling the invariant (quasiperiodic or mode-locked) two-torus of the

system trajectory in phase space breaks up by folding onto itself as a consequence of the overlap of resonances. Experimentally, the onset of the resulting chaotic state can be visualized by an apparently erratic jumping between different resonant modes. Following the above quasiperiodic approach to chaos via frequency locking in view of symmetry and structure formation processes, the spontaneous emergence of a highly symmetrical and well ordered mode-locking structure developing from a more disordered state of quasiperiodicity represents a typical disorder-order transition, confining the system trajectory flow to only a few degrees of freedom. Nevertheless the still open question arises whether the complex temporal behavior of the latter chaotic state might be interpreted as a new degree of disorder (ensuing the quasiperiodic state) or even as a higher form of order (ensuing the mode-locked state), perhaps.

Finally, we point out that such kind of symmetry-breaking nonequilibrium phase transitions occurring during avalanche breakdown of our semiconductor system can qualitatively be described by a simple phenomenological reaction-diffusion model [13] based on the well-known Rashevsky-Turing theory of morphogenesis [14]. The simplest version of Turing's model can be realized by a two-cellular symmetrical reaction-diffusion system, consisting of cross-inhibitorily coupled, potentially oscillating two-variable subsystems (4-D flow). Involving the concrete physical mechanism of impurity impact ionization coupled with energy relaxation and energy exchange of hot carriers through emitted phonons, the two morphogens of our model can be interpreted as the number density of moving charge carriers reflected in the electric current (activator) and the mean energy per carrier (inhibitor). The diffusive coupling is then governed by the long-range phonon propagation. It is emphasized that the relevance of the present 4-D model for our semiconductor system is essentially based on the spatial identification of the temporal dissipative structures as localized oscillation centers in the boundary region of current filaments [10]. Moreover, reflecting general behavioral characteristics of many different synergetic systems in nature, the above Turing model may acquire a rather universal significance.

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