

# Scaling Properties of Traffic-flow Data

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By computing the probability distributions of the velocity difference between cars a time-delay  $\tau$  apart, the scaling properties of traffic flow can be analysed. These data display scaling behaviour thus confirming earlier results that have found  $1/f^\alpha$ -noise in traffic flow. Furthermore, the applicability of the scaling analysis for describing the two-point statistics of traffic flow is demonstrated, leading to an additional test for the dynamical properties of microscopic and macroscopic traffic flow models.

## 1. Background

There are several references that report the existence of  $1/f^\alpha$ -noise in traffic flow, mostly based on theoretical work or computer simulations of various microscopic and macroscopic models [1–4]. There exists an additional claim about the intermittent behaviour of traffic flow [3], based on a standard model in traffic flow theory (the transition from congested flow to uncongested flow can be understood as a queue discharge [5]), where the intermittence concerns switches between a congested flow regime and a free flow regime. (The theoretical models are mostly for one-lane traffic, however, other results exist also [6].) Empirical evidence for  $1/f^\alpha$ -noise is reported in [7], unfortunately, the results found there are based on a somewhat limited statistics.

Recently, with the support of local authorities, we were able to collect a large amount of high-resolution traffic-flow data, where the velocities and arrival times were recorded of any car that passed an induction loop detector. This gives us the opportunity to do a reliable statistical analysis, which we are going to report in the following. The analysis will be done in the spirit of analysing the scaling properties of turbulent flow [8, 9], which can be shown to yield sensible results even in completely different fields [10]. The appropriate tool is the computation of the probability distribution of the velocity differences  $\Delta_\tau v = v(t - \tau) - v(t)$  for various values of  $\tau$ . These distributions  $P_\tau(\Delta_\tau v)$

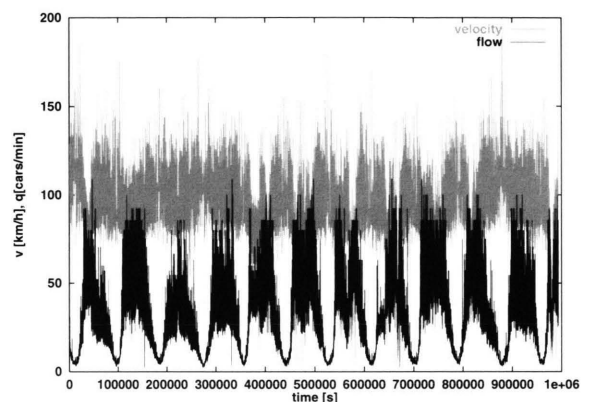


Fig. 1. Averaged velocity (measured in km/h) and flow (measured in cars/min), averaged over twenty cars as function of time showing the strong non-stationarity of traffic flow.

provide a more comprehensive account on the statistical features of the time-series under consideration compared with the power spectrum, which is a function of  $\langle(\Delta_\tau v)^2\rangle$  alone. In general, the  $P_\tau(\Delta_\tau v)$  give a complete characterization of the two-point statistics of the time-series, and can be used to provide a further opportunity for the testing of traffic flow models, as it is sensitive to dynamical aspects of traffic flow and not only to statical ones as the fundamental diagram (the measurement of the relation between flow and average velocity).

Unfortunately, traffic flow is highly non-stationary as can be seen in Fig. 1, where the time-averaged flow is shown for the full data-set of length  $\approx 10^6$  sec of a two-lane road, so some caution is needed about the results derived. In order to suppress effects stemming

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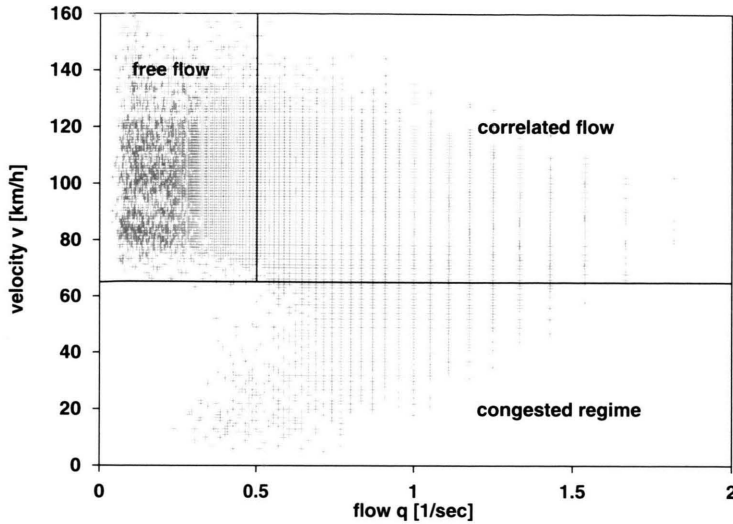


Fig. 2. Fundamental diagram as computed from the data in Fig. 1, with lines separating the different regimes used in the subsequent analysis. The separation lines are drawn at  $v_c = 65$  km/h and  $q_c = 0.5$  cars/sec.

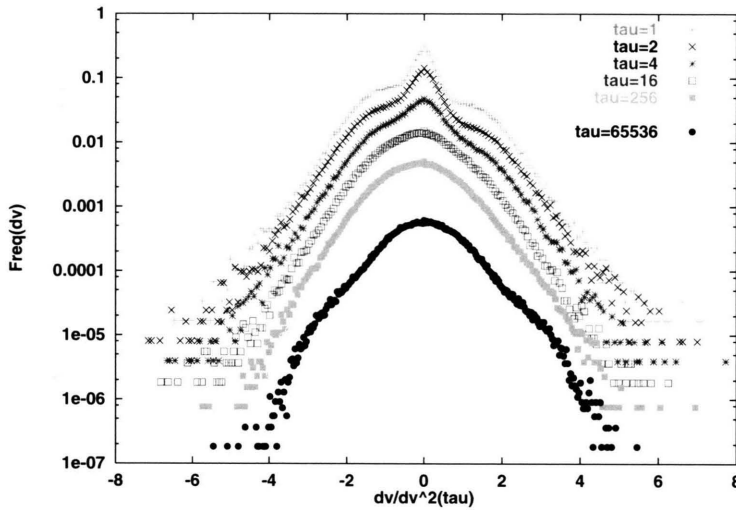


Fig. 3. Frequency distribution  $P_\tau(\Delta v)$  for both lanes. The curves are shifted in  $y$ -direction with respect to each other for clarity. All distributions are normalized to a standard deviation of 1.

from non-stationarity we compute conditional probability distributions, based on the following (rough) classification of traffic flow. When plotting the averaged flow  $q$  (averaged over 20 cars in the data of Figs. 1 and 2) versus the corresponding average velocity  $v$  in this interval, as is shown in Fig. 2, three regimes can be identified: (i) a free flow regime, where cars interact only occasionally, defined by  $v \geq 65$  km/h and  $q \leq 0.5$  cars/sec, (ii) a regime of highly correlated flow, where the system is dominated by cars driving with the same speed in small queues and with small headways ( $v \geq 65$  km/h and  $q \leq 0.5$  cars/sec), and (iii) a congested regime with medium or small flows

at large densities ( $v < 65$  km/h). We are aware of further investigations made about the congested regime [11], however since the data are very sparse there, we do not subdivide the fundamental diagram in Fig. 2 further.

The data used in the subsequent analysis are the velocities of single cars crossing an induction loop detector, collected at Köln-Nord over more than one week. The measurements are truncated to 8-bit integers ranging from 0 to 255 km/h, and have an accuracy of approximately 3 %. Additionally to the velocities, the type of a car (truck, passenger car) and the time when a car crosses the detector is recorded. The data-

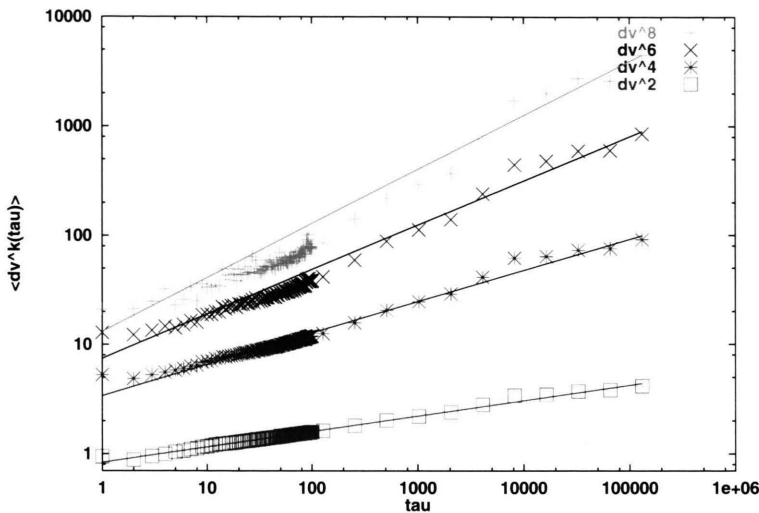


Fig. 4. Scaling of the even moments  $\langle (\Delta_\tau v)^n \rangle$  as function of  $\tau$  for the full data-set, showing the scaling behaviour. For larger moments, there is a raising noise in the data-points, however a reliable fit is still possible.

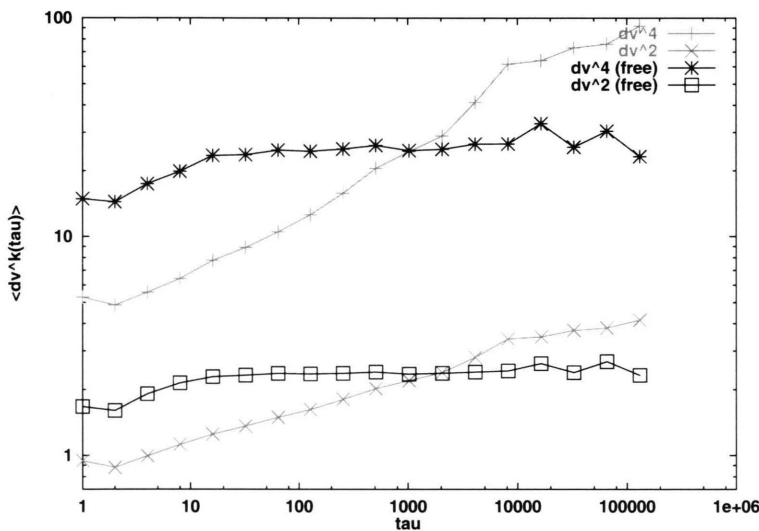


Fig. 5. Comparison between data from the free-flow regime compared to the full data-sets. As expected, there is no scaling at all in the free-flow regime, because the velocities of cars at different time-lags are independently drawn from an underlying distribution, the distribution of their desired velocities.

set contains a total of 515,429 data-points from two lanes, which will be merged in the following to avoid the overloading of our presentation.

## 2. Analysis of the Data

In Fig. 3, we have plotted the probability distribution function  $P_\tau(\Delta_\tau v)$  for various values of  $\tau$ . Note that  $\tau$  refers to a real time that is measured in *sec*, and not to the car index. We have estimated the  $P_j(v_{i-j} - v_i)$  with respect to the car index  $j$  and found that this hides the scaling behaviour. The  $\Delta v$ -axis is normalized with  $\langle (\Delta v)^2 \rangle$  which facilitates the recognition of intermittence. The distribution is definitely non-Gaussian for small values of  $\tau$ , and there

were only small differences between the two lanes. Those differences can be explained by differences in the velocity distribution, e. g. the mean velocities on the left lane are larger than on the right lane. The peak in  $P_\tau(\Delta_\tau v)$  at small values of  $\Delta v$  and for small  $\tau$  can be explained as follows. When the flow exceeds a certain value, traffic organizes itself by building-up small queues of cars driving together with small headways, but approximately equal velocities, which is a prominent feature of the system in the above mentioned correlated regime. The peak vanishes for  $\tau = 5 \dots 8$ , as a further analysis (results not shown) suggests. This corresponds to 10 ... 16 cars at maximum flow 2 cars/sec, which can be interpreted as the average queue-length.

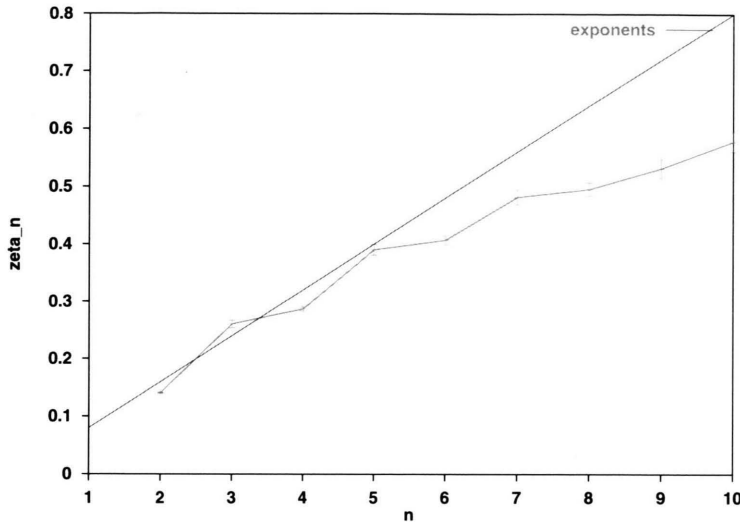


Fig. 6. Plot of  $\zeta_n$  as function of  $n$ , where  $\zeta_n$  is the exponent of  $\langle(\Delta_\tau v)^n\rangle$ . There are deviations from a straight line, indicating multi-fractal behaviour.

In Fig. 4 we have plotted the scaling functions  $\langle(\Delta_\tau v)^n\rangle$  for the even moments  $n = 2, 4, 6, 8$ , together with the least mean square fit results for these curves. The data for the odd moments are more noisy and are not shown here. From the scaling behaviour  $\langle(\Delta_\tau v)^n\rangle \propto \tau^{\zeta_n}$ , the resulting exponent for the second moment is  $\zeta_2 = 0.141 \pm 0.001$ , corresponding to a  $1/f^\alpha$ -noise with  $\alpha = 1.14$ . The results shown in Fig. 5 demonstrate, that this result is not trivial. Here we have used data from the free-flow regime only and compared them to the data from the complete data-set showing that the scaling behaviour is not present in the free-flow regime. That is what one would expect, because in the free-flow regime cars are independent from each other except at very small values of  $\tau$ , so that the moments of  $p_\tau(\delta_\tau v)$  are not dependent on  $\tau$ .

In Fig. 6, finally, the scaling exponents  $\zeta_n$  are plotted versus  $n$ , showing multifractal behaviour because  $\zeta_n \propto n$  is not fulfilled.

### 3. Conclusion

We have shown here, that traffic flow shows scaling behaviour. Thus we confirm older results [7], which have reported  $1/f^\alpha$ -noise in traffic flow. Note that the exponent  $\alpha = 1.14$  we found is quite close to what is found in [7], where  $\alpha = 1$  was reported. The results obtained with theoretical models we know about are more dispersive. The macroscopic model using the Burger's equation [1] yields  $\alpha = 1.4$ . Microscopic models display a variety of values, e. g. in [12]  $\alpha = 1.8$

is reported, while the cellular automaton model [3] yields  $\alpha = 1$ , especially when used in a multi-lane context [6]. In this article a dependency of the exponent  $\alpha$  on the total flow in the system has been reported, however it remains to be proven, whether this dependency has the same origin as the dependency observed on  $\zeta_2$  in our empirical data.

The small exponent found for  $\zeta_2$  makes sense when interpreted as a random walk with “anti-persistence”. In such walks, an increase in the past is countered by a tendency to decrease in the future, which makes sense for traffic flow data, where one has a limited range of velocities.

Additionally, it can be seen that the analysis of traffic flow done here will open up the possibility of a different characterization of traffic flow. Finally we want to point out that a model for traffic flow should be able to reproduce the statistics characterized here in order to show its validity.

### Acknowledgements

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