Chaotic Solitons for the (2+1)-Dimensional Modified Dispersive Water-Wave System

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With an improved mapping approach, a series of excitations of the (2+1)-dimensional modified dispersive water-wave (MDWW) system is derived. Based on the derived solitary wave excitation, we obtain some special chaotic solitons. – PACS numbers: 05.45.Yv, 03.65.Ge

Key words: Improved Mapping Approach; MDWW System; Chaotic Soliton.

1. Introduction

Solitons and chaos are two most important notions of nonlinear science [1]. They are widely applied in many natural sciences [2 – 5] such as chemistry, biology, mathematics, communication, and particularly in almost all branches of physics like fluid dynamics, plasma physics, field theory, optics, and condensed matter physics [6 – 8]. Usually, these two features are treated independently since one often believes that solitons are the basic excitations of the integrable models while chaos is the elementary behaviour of the non-integrable systems. However, conclusion may not be complete especially in higher dimensions. In recent studies of soliton systems, we have found that some characteristic lower-dimensional arbitrary functions exist in the excitations of some two-dimensional integrable models. This means that with these functions one can introduce chaotic behaviour into solutions of these integable models, which implies that any exotic behaviour may propagate along their characteristics. To verify the above viewpoint, we take the (2+1)-dimensional modified dispersive water-wave (MDWW) system [9] as a concrete example, which reads

$$\begin{aligned} &u_{ty} + u_{xxy} - 2v_{xx} - (u^2)_{xy} = 0, \\ &v_t - v_{xx} - 2(uv)_x = 0. \end{aligned} \tag{1}$$

The MDWW system is used to model nonlinear and dispersive long gravity waves travelling in two horizontal directions in shallow water with uniform depth, and can also be derived from the celebrated Kadomtsev-Petviashvili (KP) equation by a symmetry constraint [10].

2. New Exact Solutions of the (2+1)-Dimensional MDWW System

In the usual extended mapping method, the basic idea of the algorithm is as follows: Consider a given nonlinear partial differential equation (NPDE) with independent variables $x(= x_0 = t, x_1, x_2, \cdots, x_m)$, and one dependent variable u, in the form

$$P(u, u_t, u_{x_i}, u_{x_i x_j}, \cdots) = 0, \tag{2}$$

where P is in general a polynomial function of its arguments, and the subscripts denote the partial derivatives. One assumes its solution in the form

$$u = A(x) + \sum_{i=1}^{n} B_i(x)\phi^i q(x), \tag{3}$$

with

$$\phi' = \sigma + \phi^2, \tag{4}$$

where σ is a constant and the prime denotes differentiation with respect to q. To determine u explicitly, one may substitute (3) and (4) into the given NPDE, collect the coefficients of the polynomials in ϕ, then eliminate each coefficient to derive a set of partial differential equations of A, B_i, and q, solve this system of partial differential equations to obtain A, B_i, and q. Finally, as (4) possesses the general solution

$$\phi=\begin{cases}-\sqrt{-\sigma}\tanh(\sqrt{-\sigma}q), & \sigma<0,\\ -\sqrt{-\sigma}\coth(\sqrt{-\sigma}q), & \sigma<0,\\ \sqrt{\sigma}\tan(\sqrt{\sigma}q), & \sigma>0,\\ -\sqrt{\sigma}\cot(\sqrt{\sigma}q), & \sigma>0,\\ -1/q, & \sigma=0,\end{cases} \quad (5)$$

one substitutes A, B_i, q and (5) into (3), and obtains exact solutions of the given NPDE in concern.

In order to find some new compound solutions of the MDWW system, we try to improve the above method. We rewrite the ansatz (3) as

$$u=A(x)+\sum_{i=1}^{n}B_i(x)\phi^i q(x) + C_i(x)\phi^{i-1}q(x)\sqrt{\sigma+\phi^2 q(x)}. \quad (6)$$

First, let us make a transformation of (1): $v=u_y$. Substituting this transformation into (1), yields

$$u_{yt}-u_{xxy}-(u^2)_{xy}=0. \quad (7)$$

Now we apply the improved mapping approach to (7). By the balancing procedure, ansatz (6) becomes

$$u=f+g\phi(q)+h\sqrt{d+\phi^2}, \quad (8)$$

where f, g, h and q are functions of (x,y,t) to be determined. Substituting (8) and (4) into (7) and collecting the coefficients of the polynomials of ϕ, then setting each coefficient to zero, we have

$$f=-\frac{1}{2}\frac{q_t-q_{xx}}{q_x},\quad g=-\frac{1}{2}q_x,\quad h=\frac{1}{2}q_x, \quad (9)$$

with

$$q=\chi(x,t)+\varphi(y), \quad (10)$$

where $\chi\equiv\chi(x,t)$, $\varphi\equiv\varphi(y)$ are two arbitrary variable separation functions of (x,t) and of y, respectively. Based on the solutions of (4), one thus obtains an explicit solution of (1).

Case 1. For $\sigma=-1$, we can derive the following solitary wave solutions of (1):

$$u_1=\frac{1}{2}\frac{-\chi_{xx}+\chi_t+\chi_x^2(\tanh(\chi+\varphi)+\mathrm{i}\,\mathrm{sech}(\chi+\varphi))}{\chi_x}, \quad (11)$$

$$v_1=-\frac{1}{2}\frac{\chi_x\varphi_y(\mathrm{i}\sinh(\chi+\varphi)-1)}{\cosh(\chi+\varphi)^2}, \quad (12)$$

$$u_2=\frac{1}{2}\frac{-\chi_{xx}+\chi_t+\chi_x^2(\coth(\chi+\varphi)+\mathrm{csch}(\chi+\varphi))}{\chi_x}, \quad (13)$$

$$v_2=-\frac{1}{2}\frac{\chi_x\varphi_y}{\cosh(\chi+\varphi)-1}. \quad (14)$$

Case 2. For $\sigma=1$, we obtain the following periodic wave solutions of (1):

$$u_3=\frac{1}{2}\frac{-\chi_{xx}+\chi_t-\chi_x^2(\tan(\chi+\varphi)-\sec(\chi+\varphi))}{\chi_x}, \quad (15)$$

$$v_3=\frac{1}{2}\frac{\chi_x\varphi_y(\sin(\chi+\varphi)-1)}{\cos(\chi+\varphi)^2}, \quad (16)$$

$$u_4=\frac{1}{2}\frac{-\chi_{xx}+\chi_t+\chi_x^2(\cot(\chi+\varphi)+\csc(\chi+\varphi))}{\chi_x}, \quad (17)$$

$$v_4=\frac{1}{2}\frac{\chi_x\varphi_y}{\cos(\chi+\varphi)-1}. \quad (18)$$

Case 3. For $\sigma=0$, we find the following variable separated solution of (1):

$$u_5=\frac{1}{2}\frac{\chi_t-\chi_{xx}}{\chi_x}+\frac{\chi_x}{\chi+\varphi}, \quad (19)$$

$$v_5=-\frac{\varphi_y\chi_x}{(\chi+\varphi)^2}. \quad (20)$$

3. Some Localized Excitations with Chaotic Behaviors in the (2+1)-Dimensional MDWW System

In this section, we mainly discuss some localized coherent excitations with chaotic behavior in the (2+1)-dimensional MDWW system. For simplification, we only discuss the field v_2 of (14), namely

$$V=v_2=-\frac{1}{2}\frac{\chi_x\varphi_y}{\cosh(\chi+\varphi)-1}. \quad (21)$$

In (2+1) dimensions, one of the most important nonlinear solutions is the dromion excitation, which is localized in all directions exponentially. For instance, if we choose χ and φ as

$$\chi=1+\exp(x+ct),\quad \varphi=1+\exp(y), \quad (22)$$

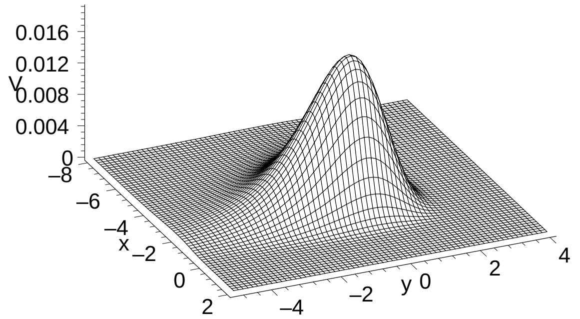

Fig. 1. A plot of a single dromion structure for the physical quantity V given by the solution (21) with the choice (22) and $t = 2, c = 1$, $\sigma = -1$.

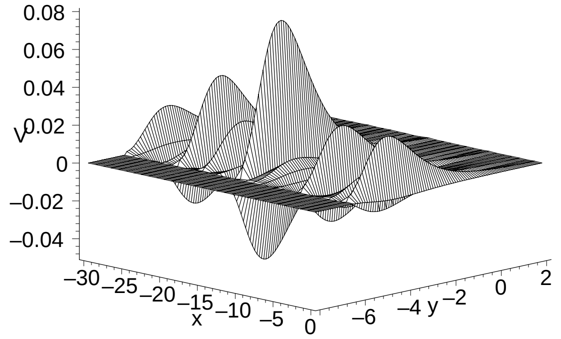

Fig. 3. A plot of the chaotic line soliton for the field V determined by (21) with condition (25) and $t = 10, k = 1$, $\sigma = -1$.

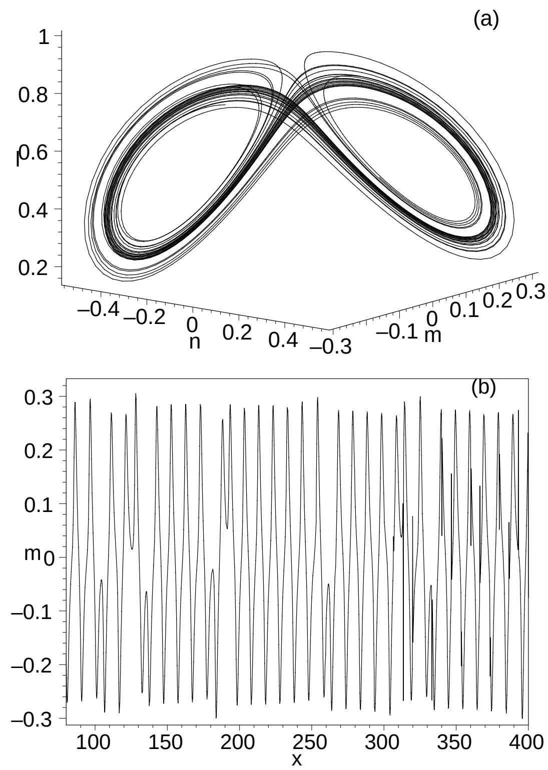

Fig. 2. (a) A typical attractor plot of the chaotic NSG system (23) with the initial condition (24). (b) A typical plot of the chaotic solution m of (23) related to (a).

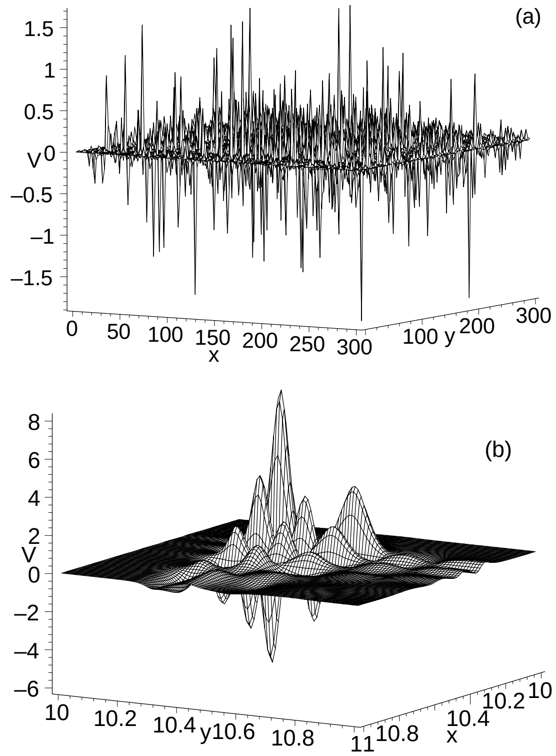

Fig. 4. (a) A plot of the chaotic pattern for the field V expressed by (21) with condition (26) at $t = 0$ and $\sigma = -1$. (b) An enlargement of the center area of (a).

we can obtain a simple dromion structure for the physical quantity V of (21) presented in Fig. 1 with fixed parameters $\sigma = -1$, $t = 2$, and $c = 1$.

3.1. Chaotic Line Solitons

In addition, if the functions χ and φ are assumed to be solutions of a chaotic dynamical system, we can derive some localized excitations with chaotic behavior. For example, χ is defined to be a solution of the following nuclear spin generator (NSG) system [11, 12]:

$$m_\xi = -1.3m + n, \quad n_\xi = -m - 1.3n(1 - 3l), \qquad (23)$$
$$l_\xi = 0.15 - 0.52l - 3.9n^2,$$

where m, n, and l are functions of ξ ($\xi = x + kt$ or $\xi = y$). The NSG system is a high frequency oscillator which generates and controls the oscillations of a nuclear magnetization vector in a magnetic field. One of typical chaotic attractors for the NSG equation (23) system is depicted in Fig. 2 when

$$m(0) = 1, \quad n(0) = 2, \quad l(0) = 0. \tag{24}$$

Now we take

$$\chi(x,t) = 1 + m(x + kt), \quad \varphi(y) = 1 + \exp(y), \tag{25}$$

where $m(x+kt)$ is the solution of the NSG system (23) with the initial conditions (24). By this choice, the dromion localized in all directions is changed into a chaotic line soliton, which presents chaotic behavior in the x-direction though still localized in y-direction. Figure 3 shows the corresponding plot of the chaotic line soliton for the field V of (21) at fixed time $t = 10$ with parameter $k = 1$.

3.2. Chaotic Soliton Patterns

Furthermore, if χ and φ are all selected as chaotic solutions of some lower-dimensional non-integrable model, the field V of (21) will behave chaotically in all directions and will yield a chaotic pattern. For example, χ and φ may be chosen as

$$\chi(x,t) = 1 + m(x + kt), \quad \varphi(y) = 1 + m(y), \tag{26}$$

where $m(x+kt)$ and $m(y)$ are the solutions of the NSG system (23). Figure 4a shows a plot of the special chaotic pattern for field V expressed by (21) with condition (26) at time $t = 0$. In order to show that the chaotic behavior is due to the peak value of solitons, we enlarge a small region in Fig. 4a, and the result is shown in Fig. 4b, which presents clearly a kind of dromion with chaotic structure.

4. Summary and Discussion

In this paper, via the improved mapping approach, we have found new exact solutions of the MDWW system, which have not been described in the previous literature. Additionally, using the NSG system, Fang recently obtained some chaotic solitons of the (2+1)-dimensional generalized Broer-Kaup system [13]. Along with the above line, we used the NSG system to get some new chaotic solitons of the (2+1)-dimensional MDWW system, which are different from the ones of the previous work.

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