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Impact of Relativistic Electron Beam on Hole Acoustic Instability in Quantum Semiconductor Plasmas

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Abstract: We studied the influence of the classical relativistic beam of electrons on the hole acoustic wave (HAW) instability exciting in the semiconductor quantum plasmas. We conducted this study by using the quantum-hydrodynamic model of dense plasmas, incorporating the quantum effects of semiconductor plasma species which include degeneracy pressure, exchange-correlation potential and Bohm potential. Analysis of the quantum characteristics of semiconductor plasma species along with relativistic effect of beam electrons on the dispersion relation of the HAW is given in detail qualitatively and quantitatively by plotting them numerically. It is worth mentioning that the relativistic electron beam (REB) stabilises the HAWs exciting in semiconductor (GaAs) degenerate plasma.

Keywords: Degenerate Plasma; Hole Acoustic Wave Instability; Relativistic Electron Beam; Semiconductor Plasma.

1 Introduction

Plasma waves are electrostatic and electromagnetic in linear and non-linear regimes. The electrostatic waves propagating with the dynamics of ions are low frequency waves such as lower hybrid, ion acoustic wave (IAW), hole acoustic wave (HAW) and ion cyclotron wave. The acoustic mode is one of the fundamental electrostatic modes that was extensively studied by the researchers for their vast applications in laboratory and space plasmas [1–5]. For instance, these waves play a significant role in the study of the naturally occurring plasmas in space such as corona, solar wind, comets, earth’s ionosphere and chromospheres etc. [6–12]. An example of electrostatic low-frequency compressional waves in electron-ion plasmas is the IAWs. In these waves ions play a similar role as that of the neutral atoms in the excitation of sound waves in neutral fluids [13]. The interaction among ions in plasmas extends over the large distances for their electric fields, due to which these waves can travel through the collisionless plasmas. The semiconductor materials consist of holes and electrons exhibiting the properties of solid-state plasmas. The oscillating electric field is established by the separation of charges due to mass difference between semiconductor plasma species [14]. This oscillatory electric field generates HAWs in semiconductor quantum plasmas. The experimental studies of the acoustic modes were carried out in a variety of materials like GaAs, MgO, Si, Al₂O₃ and SiO₂. These low-frequency modes can be studied in quantum semiconductor plasmas at a nanoscale by using the quantum-hydrodynamic model (QHD) of plasmas. The quantum effects become prominent and cannot be ignored when the average inter-particle distance is less than the de-Broglie wavelength associated with each of the plasma particle (holes and electrons). The quantum effects appear through Bohm potential, Fermi pressure, exchange and correlation potentials. The exchange and correlation potentials are due to the spin effects in the plasma that play a significant role in the ultra-small electronic devices made up of semiconductors [15–19]. The QHD for each of the plasma species provides complete description of the nano-sized semiconductor devices [20–24] and lasers [25–28]. For example, nanowires, nanophotonics [29–31], resonant tunneling diodes [32–34], super-luminescence which occurs for picoseconds [35–37] and high gain photoconductive current filament [38]. Recently the excitation of low-frequency HAW was studied in quantum semiconductor plasmas [20].

In this manuscript we studied the excitation of low frequency, long wavelength electrostatic HAW in semiconductor plasmas. The HAWs are generated through the relativistic beam of electrons and then the effects of plasma parameters such as beam velocity, concentration of plasma species, thermal velocity of electrons and multiple quantum effects on them were investigated. There
is an extensive amount of literature describing both the production and the application of the electron beam. Prono et al. discussed the production of intense relativistic electron beam (REB) with large power outputs (≥10^{12} W) and high efficiency (>50 %) with possible application of plasma heating in mirror machines, tokamaks and laser pumping. Several authors have studied the effects of REB interaction with plasmas. The REB can impart the strong electrostatic fields or hydromagnetic waves in plasmas [1]. Furthermore, the structure of the wake-field by using an ultra-REB pulse propagating through plasma, was studied extensively by Rosenzweig et al., Amatuni et al. and Ruth et al. The applications of the electron beam pumped into semiconductor quantum plasmas were elaborated in [20, 39]. On contrary to the optical pumping, the significant feature of the electron beam into the bulk plasmas is the independence of matching with the band gap energy. In our article we are studying the dispersion properties of the electrostatic waves exciting in semiconductor plasmas whenever the semiconductor is exposed to an REB. The point to be noted is that our electron beam is thermally relativistic whereas the electron beam speed or streaming is non-relativistic. Therefore, the macroscopic structure of the semiconductor might not be affected on exposing it to the REB. The study of electron beam effects on the electrostatic waves exciting in semiconductors is our quest. Recently, we investigated on the impacts of the non-REB on the HAWs, upper hybrid and lower hybrid waves exciting in the semiconductor plasmas. We found that the HAWs instability reduces on increasing the thermal temperature of beam electrons. In other words, the beam electron temperature tends to stabilise the HAWs. Therefore, the motivations behind the current study are:

(i) Firstly to check if the thermally REB stabilises the HAWs or not.

(ii) The possible generation of electrons beam in laboratory persuades us to analyse the effects of the relativistic beam on the HAWs in semiconductor plasmas.

The present study provides information and feedback as to what can happen if a chip made of semiconductor or a simple semiconductor is exposed to the environment of relativistic beam in a lab or an astrophysical environment such as solar wind etc. On employing the QHD equations in Section 2, we have solved the fluid equations of semiconductor plasmas for the study of linear dispersion relation. The dispersion relation is used to obtain the growth rate of the electrostatic HAWs. A numerical analysis of the growth rate of the HAW instability is done with experimental parameters at nanoscale with a particular example of GaAs in Section 3. A summary of the results is presented in Section 4.

2 Derivation of Linear Dispersion Relation

A homogeneous dense semiconductor quantum plasma consisting of the electrons holes and beam electrons is considered. The REBs are externally pumped into the semiconductor plasma system as a source of energy with streaming velocity \(v_{bo}\). The subscript notation “e,” “h” and “b” are used for electrons, holes and beam electrons, respectively. In the absence of any perturbation the plasma charge quasi-neutrality condition reads as, \(n_{e0} = n_{h0} \), where \(n_{e0}, n_{h0}\) and \(n_{bo}\) are the equilibrium number densities of semiconductor electrons, holes and beam electrons, respectively. In the study of the HAWs in semiconductor plasmas, the role of holes is considered similar to the ions in gaseous plasmas. The linearised set of equations for the semiconductor electrons, holes and the beam electrons with thermal effects as well as degenerate effects [arising through degenerate pressure due to the high number density of the electrons (holes), quantum recoil force due to the electrons (holes) tunneling through a potential barrier known as the Bohm potential, and electrons (hole) exchange and correlation effects due to their spins] are

\[
\frac{\partial \tilde{\Psi}_{i(e,h)}(e,h)}{\partial t} + \frac{q_{(e,h)}\nabla \Phi}{m_{(e,h)}} + \frac{\nabla V_{xc(e,h)}}{m_{(e,h)}} + \frac{\nabla p_{i(e,h)}}{m_{(e,h)}n_{b0}} - \frac{\hbar^2}{4m_{(e,h)}^2n_{b0}} \nabla^2 n_{i(e,h)} = 0
\]

(1)

\[
\frac{\partial n_{(e,h,b)}}{\partial t} + n_{(e,h,b)} \nabla \cdot \tilde{\Psi}_{i(e,h,b)} = 0
\]

(2)

\[
\left( \frac{\partial}{\partial t} + \frac{\tilde{\Psi}_{b0}}{v_{b0}} \nabla \right) \frac{\tilde{\Psi}_{b1}}{m_{b}^\ast} \frac{e}{m_{b}} \nabla \Phi + \frac{1}{m_{b}^\ast} \nabla \tilde{\Psi}_{b1} = 0
\]

(3)

where \(\hbar = h/2\pi\) (Planck’s constant), \(q_{i}, m_{i}, n_{i}\) are the charges, effective masses and number densities of semiconductor electrons and holes, respectively. The \(V_{xc}\) is the exchange-correlation potential of semiconductor species and is defined as

\[
V_{xc(e,h)} = \left[ \frac{0.985e^2}{\epsilon} \frac{1}{\nabla^2 n_{b0}} \left( 1 + \frac{0.034}{\alpha_{b,0}^\ast} \ln \left( 1 + 18.37 \alpha_{b,0}^\ast n_{b0}^3 \right) \right) \right]
\]

where \(\alpha_{b,0}^\ast = \frac{e^2}{\epsilon^2 m_{b0}^\ast}\) is the Bohr atomic radius, \(\epsilon\) is the relative dielectric constant of material and “e” is the magnitude of electronic charges [40, 41].
The equation of state for relativistic beam of electrons is:

\[ P_b = \frac{mc^2}{e} \left( \frac{K_{(a)}}{K_{(a)}} - \frac{1}{a} \right) n_b \]  

(4)

This state equation is obtained from the well-known thermodynamic expression \( \frac{P_b}{V} = -\frac{\partial E_b}{\partial V} \), where \( V \) and \( E_b \) are volume of the plasma system and total energy of the relativistic beam of electrons, respectively. The total energy \( E_b \) is obtained by solving the integral relation

\[ E_b = \frac{n_b a}{4\pi(m_c)^2} V \int (e-m_c^2)f(p)\sin\theta p^2 dp d\Phi d\Phi, \]

is the modified Maxwellian distribution function describing the REBs. After performing the integration and some straightforward calculations, the total energy expression for the beam electrons appears to be as

\[ E_b = n_b \frac{m_c^2}{K_{(a)}} - 1 \]

where, \( K_{(a)} \) is the MacDonald function. The general expression for MacDonald function is

\[ \int_0^\infty \cos n \theta \exp(-acosh\theta) d\theta = K_{(a)} \]

in which \( a \) is the relativistic factor that is the ratio of rest mass energy to thermal energy of beam electrons, i.e. \( a = \frac{m_c^2}{K_{(a)}} \). For very large value of relativistic parameter \( (a >> 1) \), the state equation (4) of the REBs is converted to state equation \( (P_b = nkT_b) \) of non-REBs.

It is better to mention that thermal and streaming velocities of beam electrons make a different sense. Thermal velocity defining the thermal temperature of the beam electrons is associated with the random motion whereas streaming speed independent of temperature is concerned with the drifting of the electron beam. As far as the presence of relativistic factor in equation of continuity of beam is concerned, we ignore the inertial effects of electron beam on exciting the HAW effects. So, we do not use practically in our calculations the equation of continuity for beam electrons. As our equation of continuity is general and applicable to all the species of the plasma, we have hidden the relativistic factor in the definition of density and velocity for electron beam in equation of continuity. Our beam electrons are thermally relativistic whereas streaming speed is non-relativistic; therefore, we did not write the relativistic factor with beam streaming velocity. The degenerate Fermi semiconductor species (electrons, holes) obey statistical pressure law, \( P_{(e,h)} = \frac{m_{(e,h)} v_{(e,h)}^2 n_{(e,h)}^2}{3n_{(e,h)}^2} \), for which

\[ \frac{v_{(e,h)}^2}{2} = \frac{3}{5} \frac{v_{(e,h)}^2}{2} \left[ 1 + \frac{5}{12} \frac{\Theta_{(e,h)}}{T_{(e,h)}} \right] \]  

and

\[ \frac{v_{(e,h)}^2}{2} = \frac{2k_b T_{(e,h)}}{m_{(e,h)}} \]

where \( T_{(e,h)} = \frac{\hbar^2 (3\pi^2 n_{(e,h)})^{2/3}}{2m_{(e,h)}} \) is the Fermi temperature of semiconductor electrons and holes, \( T_{(e,h)} \) is the non-zero thermal temperature, \( n_{(e,h)} = n_{(e,h)}^{0} + n_{(e,h)} \) number density with its equilibrium value \( n_{(e,h)}^{0} \) and perturbed number density \( n_{(e,h)} \), respectively. In \( (\omega - k) \) space, the velocity components of the semiconductor electrons, holes, and the REBs can be obtained, from (2) and (3), respectively

\[ \tilde{v}_{(e,h)} = \frac{q_{(e,h)}}{\omega mc_{(e,h)}} \Psi_{(e,h)} \tilde{k} \]  

(5)

\[ \tilde{v}_{(e,h)} = \frac{e}{m_{(e,h)}(\omega - k \cdot \tilde{k})} \Psi_{(e,h)} \tilde{k} \]  

(6)

where \( \omega - k \cdot \tilde{k} \) is the Doppler shifted frequency, \( \omega_{(e,h)} \) is the streaming speed of beam electrons, and

\[ \Psi_{(e,h)} = \frac{\Phi - m_{(e,h)}c^2}{e} \left[ \frac{K_{(a)}}{K_{(a)}} - 1 \right] n_{(e,h)} \]

is the effective beam electrons potential, and

\[ \Psi_{(e,h)} = \frac{\Phi + m_{(e,h)}v_{(e,h)}^2}{e} \left[ v_{(e,h)}^2 + v_{(e,h)}^2 + k^2 h^2 \right] n_{(e,h)} \]

such that

\[ H_{(e,h)} = \frac{\hbar}{2m_{(e,h)}} \]  

is the effective potential of semiconductor species (electrons, holes) due to electrostatic potential \( \Phi \) and quantum effects arising through Fermi pressure, Bohm potential, exchange and correlation potential of semiconductor species (electrons, holes) with square of exchange speed given as

\[ v_{(e,h)}^2 = \left( 0.328n_{(e,h)}^{3/2}e^2 + 0.011616e + 18.37a_{(e,h)}n_{(e,h)} \right) \]

\[ \left( 1 + 18.37a_{(e,h)}n_{(e,h)} \right) \]

(7)

The simultaneous equations (1)–(6) provide us the perturbed number densities of the beam electrons and the semiconductor species (electrons and holes) as

\[ n_{(e,h)}^2 = \frac{en_{(e,h)} \Phi}{\omega^2} \left( v_{(e,h)}^2 + v_{(e,h)}^2 + k^2 H_{(e,h)} \right) \]
The dispersion relation of the HAWs in the semiconductor quantum plasmas becomes
\[
\omega^2 = v_{s,h}^2 k^2 + \frac{C^2_{s,h}}{\lambda_{s,De}^2 k^2 + 1} \left[ v_{s,h}^2 - \frac{m_e c^2}{m_e} \left( \frac{K_{(a)}}{K_{(a)}} - \frac{1}{a} \right) \right] k^2 - \omega_{pe}^2 \lambda_{s,pe}^2 k^2
\]

Equation (16) is the modified linear dispersion relation of HAWs in electron-hole-electron beam plasma including the effects of exchange-correlation, thermal temperature, quantum statistical pressure and quantum recoil effects of the degenerate electrons and holes. Here \( C^2_{s,h} = \frac{\omega_{pe}^2 v_{s,h}^2}{\omega_{pe}^2} \) is the square of sound speed containing the thermal effects and quantum characteristics such as degenerate pressure, Bohm potential due to tunneling and exchange-correlation potential, and the \( \lambda_{s,De}^2 = \frac{v_{s,h}^2}{\omega_{pe}^2} \) is modified Debye length having all the quantum features of semiconductor plasmas. The absence of electron beam, \( \omega_{pe} = 0 \), drags the modified dispersion relation (16) to the standard dispersion relation of HAW propagating in semiconductor quantum plasmas.

\[
\omega^2 = v_{s,h}^2 k^2 + C^2_{s,h} k^2 \left[ v_{s,h}^2 - \frac{m_e c^2}{m_e} \left( \frac{K_{(a)}}{K_{(a)}} - \frac{1}{a} \right) \right] k^2 - \omega_{pe}^2 \lambda_{s,pe}^2 k^2 \]

Equation (17), elaborates the propagation of HAW in semiconductor quantum plasmas without relativistic beam of electrons. The first term on right hand side describes the contribution of holes, which comes from their thermal equivalent effects, whereas the second term indicates the contribution of the electrostatic field interactions exciting the acoustic waves in the semiconductor plasmas.

### 3 Numerical Solution and Graphical Description

The frequency of the wave is usually the superposition of real and imaginary frequencies of the waves, i.e. \( \omega = \omega_r + \text{Im} \omega \) where \( \omega_r \) is the real frequency and \( \text{Im} \omega \) is the imaginary frequency whose sign indicates either the wave will grow or propagate through the plasma. The experimental observations of the acoustic modes were carried
out for a number of materials and semiconductors such as Al₂O₃, Si, MgO, SiO₂, and GaAs. As our analytical plasma model is developed for semiconductor plasmas pumped by REB, considering one of the semiconductor plasma like GaAs. However, our model can also be applied equally to the other semiconductors such as GaSb, GaN and InP. It was observed in our previous article, that HAW instability arises in the semiconductor (GaAs) degenerate plasma when non-relativistic external beam of electrons is used as an energy source [41]. The normalisation of the imaginary part of (16) is given as

\[ \text{Im} \omega^2 = \frac{\text{Im} \omega^2}{\omega_{pe}^2}, \quad \omega_{p(b,h)}^2 = \frac{\omega_{pe}^2}{\omega_{pe}^2} = 1, \quad k^2 = \frac{k^2 v_e^2}{\omega_{pe}^2}, \]

The HAW spectrum is observed in Figures 1–4 that describe multiple features under different conditions and parameters like

\[ n = 4.7 \times 10^{20} \text{cm}^{-3}, \quad \frac{m_e}{m_i} = 0.067, \quad \frac{m_e}{m_i} = 0.5, \quad \epsilon = 12.8. \]

Figure 1 shows the effect of relativistic factor associated with externally injected electron beam. The negligible relativistic effects come into play when the relativistic parameter “\( a \)” has higher value. It is noted from different curves shown in Figure 1 that the spectrum of the HAWs becomes unstable at very large values of relativistic factor. It is worth mentioning that for all values of \( a \), the semiconductor plasma species are taken non-relativistic, however quantised. The smaller values of relativistic parameter that impart comparable thermal energy with the rest mass energy of beam electrons reduce the spectrum of acoustic modes as well as the growth rate. It is confirmed that at values of \( a \) which corresponds to REB, there is no longer instability in the acoustic waves for all the possible values of plasma parameters. Such trend was also observed in our previous article, that is, the temperature of non-REB tends to stabilise the HAW instability in semiconductor quantum plasmas. So in the current study the observed stabilising character of REB to the HAW instability arising in semiconductor dense plasmas is as expected in the previous study of these waves when energy source electron beam was non-relativistic. It is clear from Figure 2, an increment in the value of thermal energy of external beam electrons tends to less resonate with the field energy of acoustic modes and henceforth, reduces the growth rate.

It is also clear from Figures 1 and 2 that the instability curves gradually become narrow with increasing temperature of semiconductor electrons and external relativistic beam of electrons. It is also clear from the mathematical relation, as temperature of beam electrons increases the value of relativistic factor decreases. When thermal energy of beam electrons is compared to the rest mass energy of electrons, the beam becomes relativistic and relativistic factor approaches to unity. Hence it is confirmed that the instability exists only when the beam electrons are non-relativistic whereas instability disappears when electron beam becomes relativistic. To find out what is happening here we plotted the state equation of beam electrons in relativistic and non-relativistic regimes. Shortly different values of the relativistic parameter \( a \) define either the plasma is relativistic, ultra relativistic or non-relativistic.
Figure 1 elaborates the instability of the HAWs for three different values of relativistic parameter $a = 235,000$, 232,000 and 229,000. It is noticed that HAW instability reduces on reducing the relativistic parameter and leads to no instability at even smaller values of relativistic factor (the small values of $a$ that correspond to relativistic beam). Figure 2 shows the graphical retrieval of HAW instability due to non-relativistic electron beam at larger values of relativistic factor. The plot (2) authenticates the literature that the larger values of $a = 229,000$ direct the electron beam to non-relativistic. These results once again confirm our previous work with non-relativistic electron beam. The plot (2), therefore, is nothing except only to verify that our beam becomes non-relativistic for large values of relativistic factor.

In Figure 3a–c, we have plotted the normalised state equations of both the relativistic and non-relativistic beam electrons vs. beam electron density divided by the hole number density $\sigma = n_{b0} / n_{h0}$ at equilibrium in the semi-conductor plasma. It can be noted that at large value of relativistic factor the pressure values of relativistic and non-relativistic beam electrons are very close to each other. In other words the value of relativistic factor $a$ in the order of $3 \times 10^5$ make the beam electrons to be non-relativistic.

Figure 4: (a) Pressure variations of beam electrons with respect to scaled beam electron density $\sigma = n_{b0} / n_{h0}$ for a fixed value of relativistic factor $a = 0.1$. (b) Pressure variations of beam electrons with respect to scaled beam electron density $\sigma = n_{b0} / n_{h0}$ for a fixed value of relativistic factor $a = 1$. (c) Pressure variations of beam electrons with respect to scaled beam electron density $\sigma = n_{b0} / n_{h0}$ for a fixed value of relativistic factor $a = 235,000$. 
4 Summary

We derived the dispersion relation for the HAWs in semiconductor quantum plasmas under the influence of external relativistic beam of electrons. We found that there is no instability arising in the HAW profile under the influence of relativistic beam of electrons i.e. relativistic factor ($\alpha \ll 1$). The variation of different parameters does not play any role to cause the instability such as, electrons to hole number density ratio, propagation angle, streaming speed of beam electrons and thermal effects of semiconductor species. The instability appears on the other hand when the external beam of electrons is non-relativistic. We also checked that the growth rate increases on increasing the streaming speed of beam electrons, electrons to hole number density ratio and relativistic factor (having more non-relativistic plasma). The growth rate decreases with increasing normalised semiconductor species temperature. This shows that our dispersion relation is truly valid for the case when REB with relativistic factor $\alpha \ll 1$ suppressed the instability in semiconductor quantum plasmas. It is showing full agreement of qualitative and quantitative form of equation of state for REBs.

References