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Intense Isolated Short Attosecond Pulse Generation from a Coherent Superposition State in a Spatially Inhomogeneous Field

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Abstract: We theoretically present an efficient method of generating an intense isolated short attosecond (as) pulse in a spatially inhomogeneous field. It is shown that this spatiotemporally combined field can significantly extend the harmonic cut-off and enhance the harmonic efficiency when the initial state is a coherent superposition of the ground and excited states. Then, a highly efficient continuum spectrum with an extremely wide bandwidth is directly generated. Due to the introduction of the spatial inhomogeneity, the short path is selected and the long one is removed; as a result, an intense isolated 17.3-as pulse is obtained straightforwardly. In addition, we also investigate the influences of the parameters including the population of the excited state, spatial inhomogeneity, carrier–envelope phase, pulse duration, and intensity on the harmonic spectrum.

Keywords: High-Order Harmonic Generation; Isolated Attosecond Pulse; Spatially Inhomogeneous Field; Supercontinuum.

1 Introduction

The appearance and development of isolated attosecond (as) pulses pave a way to study and control the basic ultrafast electronic processes in atoms and molecules with unprecedented time resolutions [1–3]. Up to now, high-order harmonic generation (HHG) is the most promising method to produce a single attosecond pulse in experiments. The physical mechanism of HHG can be well understood based on a semi-classical three-step model [4, 5]. In detail, the electron first tunnels through the barrier formed by the atomic potential, then it oscillates almost freely in the laser field, and finally it may return the ground state by recombining with the parent ion and emit a harmonic photon. The highest-energy harmonic photon obeys the cut-off law of \( I_p + 3.17U_p \), where \( I_p \) is the ionisation potential and \( U_p = \frac{E_0^2}{\hbar^2} = \frac{1}{4\omega^2} \) is the ponderomotive energy of the electron in the laser field. This process is repeated at each half of an optical cycle of the fundamental laser pulse, which results in the generation of an attosecond pulse train with a periodicity of half an optical cycle.

In practical application, the straightforward attosecond metrology prefers an isolated attosecond pulse; thus, lots of effort has been paid to obtain an isolated pulse. Presently, there are two techniques to produce such a single attosecond pulse. One is HHG from a few-cycle driving pulse [1, 2] and the other is HHG from a temporal confined pulse by using the polarisation gating technique [3, 6, 7]. For the former, an isolated 80-as pulse of extreme ultraviolet radiation was experimentally obtained by a 3.3-fs/720-nm laser pulse with a stabilised carrier-envelope phase (CEP) [8]. For the latter, an isolated single-cycle attosecond pulse as short as 130 as was successfully generated by phase-stabilised 5-fs driving pulse with a modulated polarisation state [3]. Recently, a single 67-as extreme ultraviolet (XUV) pulse is produced by using the double optical gating technique [9]. With these techniques above, scientists can generate a single attosecond pulse by filtering the harmonics in the cut-off or in the plateau region. However, as we know, in the cut-off region, the bandwidth of continuous harmonics is narrow and the harmonic efficiency is low, which is adverse to the generation of an intense and short isolated attosecond pulse. To solve the problems, a series of methods have been proposed to broaden the bandwidth of continuous harmonics for the aim of compressing the duration of the attosecond pulse or enhancing the yield of the attosecond pulse, such as the two-colour field [10, 11], static electric field [12–15], chirped field [16], ionisation gating [17], three-colour field [18], two-colour circular polarised pulses [19], and so on. It has been reported that the two-colour controlling scheme can broaden the bandwidth of continuous harmonics and reduce the duration of the attosecond pulse.

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By adopting the controlling scheme, some authors have successfully produced an isolated attosecond pulse with a shorter duration [20–24]. However, according to previous studies [10, 11, 20, 22–24], in the two-colour scheme, the continuous harmonics originate from the electron ionised in the half cycle with the second-highest electric field peak. Because the strength of the second-highest electric field peak is much weaker than that of the highest electric field peak, the ionisation rate of the electron in this half cycle is lower, resulting in continuous harmonics with a low yield. Recently, two new researches showed that the two colour fields consisting of the 2000-nm fundamental pulse and the 400-nm controlling pulse can simultaneously enhance the harmonic yields and extend the spectral cut-off [25, 26]. Alternatively, a realistic strategy of enhancing the harmonic efficiency is to prepare the initial state as a coherent superposition of two bound states, which was first proposed by Gauthey et al. [27]. In this method, a coherent superposition state can induce dipole transitions between the continuum and the ground state via the excited state responsible for the ionisation, which has been demonstrated theoretically by Watson et al. [28]. Wang et al. [29] further verified the outstanding advantage of the coherent superposition state to gain high conversion efficiency. Previously, many researches also showed that the HHG efficiency and the intensities of isolated attosecond pulses could be significantly improved with a coherent superposition state scheme [30–35].

Over the past few years, plasmonic field enhancement has attracted more attention in theory and experiments. Such field enhancement can be achieved by local nanoplasmonic fields generated in metallic nanoantennas [36, 37], metal nanotips [38, 39], metal waveguides [40, 41], etc. On account of surface plasmon resonances, the incident laser intensity can be increased by up to several orders of magnitude, which substantially reduces the required intensity of the incident laser pulse for HHG and permits XUV emission with a repetition rate achieving scores of megahertz (MHz), and this enhancement exhibits spatial dependence; namely, the local electric field is spatially inhomogeneous. So far, the HHG based on plasmonic nanostructures has been reported in two experiments [37, 41]. Theoretical investigations about HHG have been published by the spatially inhomogeneous fields [42, 43]. Recently, several studies also demonstrated that the spatially inhomogeneous field not only has the ability to spatiotemporally control the electron dynamics in the HHG process but also generate the broadband supercontinuum and isolated attosecond pulses [44–48]. Very recently, we obtained a single ultrashort attosecond pulse with the duration of 12.3 as by the two-colour spatially inhomogeneous field [49].

Although isolated attosecond pulses generated by single- and two-colour spatially inhomogeneous fields have been investigated [44–47, 50–52], the produced isolated attosecond pulses with these schemes are still significantly larger than 1 atomic unit of time (24 as), which is the time scale of electron motion in atoms. Therefore, these isolated attosecond pulses are limited in application. In this work, we further study the HHG and the generation of isolated attosecond pulses for helium ion (He⁺) in a spatially inhomogeneous field. Our goal is to obtain an intense single ultrashort attosecond pulse with a duration of <24 as. The calculated results show that, when the initial state is a coherent superposition state, the enhancement of both the spectral cut-off and harmonic efficiency can be simultaneously achieved by the spatially inhomogeneous field. In addition, this method can remove the long quantum path and enhance the short one, so a highly efficient and ultra-wide supercontinuum with a single quantum path contribution is directly generated. By filtering some well-chosen continuous harmonics, an intense isolated attosecond pulse as short as 17.3 as is obtained straightforwardly. Finally, we systematically discuss the effects of the population of the excited state and parameters of the laser field on the harmonic spectrum. All these results are further analysed by the three-step model and time-frequency analysis method.

2 Theoretical Model

The dynamics of an atomic electron in a strong laser field is primarily along the direction of the linearly polarised laser field, so the HHG process can be studied by numerically solving the one-dimensional time-dependent Schrödinger equation in the single-active-electron approximation. In the dipole approximation, it can be expressed by (atomic units are used throughout)

\[ i\frac{\partial}{\partial t} \psi(x, t) = \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x) - xE(x, t) \right] \psi(x, t), \]

where \( V(x) \) is the soft Coulomb potential and it can be described as \( V(x) = -z / \sqrt{x^2 + a} \).

Here, we use \( z=2 \) and \( a=0.5 \), and the binding energies of the 1s ground and 2s excited states for the He⁺ ion are 54.4 and 13.6 eV, respectively. The spatially inhomogeneous electric field \( E(x, t) \) is given by

\[ E(x, t) = E_0 f(t) \cos[\omega(t - \frac{t}{2})] \cos[\phi](1 + \epsilon x), \]

where \( E_0 \) and \( \omega \) are the peak amplitude and frequency of the laser electric field. \( \phi \) is the CEP of the laser electric
field. \( f(t) = \sin^2\left(\frac{\pi}{T}t\right) \) presents the temporal envelope of the laser electric field, and \( T = 5T_0 \) (\( T_0 \) is the optical cycle of the 800-nm laser pulse) is the pulse duration, roughly corresponding to 5-fs full width at half-maximum. The parameter \( \epsilon \) defines the spatial inhomogeneity of the laser electric field and its unit is in the reciprocal length. In our simulations, the peak intensity of the driving laser is \( I = 1.0 \times 10^{15} \) W/cm\(^2\). It is also important to note that the quoted intensity is actually the local field intensity, not the initial input laser intensity. The latter is several orders of magnitude smaller than the former and enables the nanoplasmonic target to withstand the thermal damage in an experiment. Additionally, in such interaction conditions, the field enhancement for the HHG occurs on an interaction length of \( \leq 1 \) \( \mu \)m; therefore, the phase matching condition is not required.

Equation (1) can be accurately solved in view of the split-operator method [53]. In our calculations, the length of the integration grid is 26576, the spatial step is 0.15, and the time step is 0.02. To prevent the reflection of the wave function from the boundary, after each time step the wave function is multiplied by a \( \cos^{1/8} \) mask function that of the integration grid is 2457.6, the spatial step is 0.15, \( \mu \)m; therefore, the phase matching condition is not required.

\[
\mathbf{\partial} = \mathbf{\partial}_x \frac{\partial}{\partial x} + \mathbf{E}(x, t) + x \frac{\partial \mathbf{E}(x, t)}{\partial x} \mathbf{\psi}(x, t),
\]

and the HHG spectrum is obtained by

\[
P_q(\omega) = \left| \frac{1}{2\pi} \int_0^T \mathbf{a}(t) e^{-i\omega t} dt \right|^2,
\]

where \( q \) shows the harmonic order. By superposing several orders of the harmonics, the temporal profile of the attosecond pulse can be obtained by

\[
I(t) = \left| \sum_q a_q e^{i\omega_q t} \right|^2,
\]

where \( a_q = \int \mathbf{a}(t) e^{-i\omega t} dt \).

In our calculations, the initial state is chosen to be a coherent superposition of the 1s ground state and 2s excited state, and it can be written as

\[
\mathbf{\psi}(x, t) = a|1s\rangle + b|2s\rangle,
\]

where \( a \) and \( b \) are the amplitudes of the 1s ground and 2s excited states, respectively, which should satisfy the condition of \( a^2 + b^2 = 1 \). The initial population of the 2s excited state can be determined by \( p = b^2 \). Based on our calculations, when the initial population of the 2s excited state is \( p = 0.5 \), the harmonic efficiency reached its maximum, as seen from the subsequent result. Therefore, the parameters \( a \) and \( b \) are all set to be 0.5, which also shows that the initial state is prepared as the coherent superposition of the 1s ground and 2s excited states with an equal population (hereafter called the coherent superposition state, unless otherwise specified). Note that such a coherent superposition state can be realised by a one-photon resonant excitation process [55] or adiabatic transition from a single ground state to a superposition of excited states such as in [56, 57] or a multiphoton excitation [28, 58, 59].

The time-dependent ionisation probability is defined by

\[
P_{\text{ion}}(t) = 1 - \sum_n \left| n(t) \mathbf{\psi}(x, t) \right|^2,
\]

where the summation runs over all the bound states \( n \).

3 Results and Discussion

First, we investigate the HHG spectra of the He\(^+\) ion from the 1s ground state in the 800-nm homogeneous field and in the 800-nm inhomogeneous field with \( \epsilon = 0.004 \), respectively. As shown by the solid black curve in Figure 1,
the harmonic spectrum of the homogeneous field presents a two-plateau structure with the highest cut-off of the 167th order, and the harmonics in the second plateau are continuous, which creates a 29-eV supercontinuum. For the case of the inhomogeneous field, the harmonic spectrum is significantly extended but has no obvious cut-off, which is due to the short pulse duration and the spatial inhomogeneity of the local electric field. In addition, the HHG spectra for the above two cases are inefficient and exhibit a strong modulated structure, which is attributed to the low ionisation yield and the interference of the long and short quantum paths. We also consider the harmonic spectrum of the He+ ion from the coherent superposition state in the homogeneous field (see the dashed blue curve in Figure 1), which shows that the spectral profile and cut-off position has no change and only the harmonic efficiency is substantially increased compared with the case of the 1s ground state in the homogeneous field. By the combination of the spatially inhomogeneous field and the coherent superposition state, the harmonic spectrum shows a different structure, which is in striking contrast to the case of the coherent superposition state in the homogeneous field (the spectral cut-off is at the 167th order, and only several harmonics near the cut-off are smooth), as shown by the solid red curve in Figure 1. The whole harmonic spectrum is elevated, and the spectral cut-off is greatly expanded to the 970th-order harmonic, which is larger than the result for the 1s ground state in the homogeneous field. Meanwhile, the harmonics from the 140th order to the cut-off are regular and continuous, which result in an ultrabroad supercontinuum with a bandwidth of about 1287 eV. Compared with the case of the ground state using the homogeneous field, the harmonic efficiency in the plateau region is enhanced by 4–9 orders of magnitude. It can also be found that the modulation of the harmonic spectrum for this case is much more diminished, and then the spectrum becomes more regular and smoother, indicating the selection of the single quantum path.

Moreover, we also study the effect of the initial population of the 2s excited state \( p \) on the harmonic spectrum. Here, except for the parameter \( p \), other parameters are the same as those in Figure 1. Figure 2a plots the harmonic spectra with different initial populations of the 2s excited state of \( p = 0.01 \) (black curve), \( p = 0.1 \) (green curve), \( p = 0.3 \) (blue curve), \( p = 0.5 \) (red curve), \( p = 0.7 \) (cyan curve), \( p = 0.9 \) (magenta curve), and \( p = 1 \) (orange curve), respectively. From this figure, it can be seen that \( p \) does not vary the cut-off position and the spectral profile of the harmonics, but only affects the harmonic efficiency, even in the very low initial population of \( p = 0.01 \). In addition, for all values of \( p \), we also find that the high-efficiency supercontinuum can still be acquired in comparison with the 1s ground state in the homogeneous field, especially for the low-energy part of the supercontinuum. To further understand the details between the harmonic efficiency and the initial population of the 2s excited state, we study the emitted efficiency of the supercontinuum with different initial populations of the excited state. It should be noted that the emitted efficiency of the supercontinuum can be achieved by summing the harmonic intensities from the 140th to 1160th order. The corresponding result is presented in Figure 2b. Clearly, for the cases of \( p < 0.4 \), the emitted efficiency nearly linearly increases with the increase of \( p \). For the cases of \( p > 0.6 \), the emitted efficiency almost

![Figure 2](image-url)
linearly decreases with the enhancement of $p$. As $p$ varies from 0.4 to 0.6, the emitted efficiency has little change. Thereby, in our scheme, the population of the 1s excited state is not stringent, and a little more or less population than 0.5 will not generate obvious changes of the HHG spectrum, which is favourable for manipulating our scheme in practical experimental implementations.

A deeper interpretation of the spectrum can be obtained by investigating the harmonic emission times in terms of the time–frequency analysis method. The time–frequency distribution of the HHG are achieved by means of the wavelet transform of the induced dipole acceleration $a(t)$ [60],

$$A(t_0, \omega) = \int a(t) \sqrt{\omega} W(\omega(t-t_0)) \, dt,$$

where $W(\omega(t-t_0))$ is the mother wavelet and it is expressed as

$$W(x) = \left( \frac{1}{\sqrt{\pi}} \right)^3 e^{\omega x^2} e^{-\omega x^2},$$

where $\omega = 30$ in our calculations. As a contrast, the time–frequency distribution of the HHG spectrum for the 1s ground state in the homogeneous field is also given in Figure 3a. As shown in this figure, there are four main peaks contributing to the harmonics with the maximal harmonic orders of 102, 148, 167, and 141, which are labelled as $P_1$, $P_2$, $P_3$, and $P_4$, respectively. As the harmonics higher than the 148th order largely originate from peak $P_3$, this leads to a supercontinuum with a 29-eV bandwidth. For the supercontinuum, there are two quantum paths (so-called long and short paths) with different emission times for the same harmonic order; therefore, the interference of these two paths generates an evident modulation in the harmonic spectrum shown by the solid black curve in Figure 1, which will go against the isolated attosecond pulse generation. Figure 3b illustrates the time–frequency distribution of the HHG from the coherent superposition state in the inhomogeneous field. It can be clearly seen from this figure that there are still four emission peaks marked as $P'_1$, $P'_2$, $P'_3$, and $P'_4$ contributing to the harmonic generation. Among these four peaks, the intensities of $P'_2$ and $P'_3$ are almost comparable but much weaker than those for $P'_2$ and $P'_4$, so their contributions to the HHG can be ignored. Compared with the 1s ground state in the homogeneous field, as the highest harmonic order is significantly enlarged to 970 and the second-highest one is compressed to 140, the energy difference between them is remarkably increased, forming a 1287-eV ultrabroad supercontinuum. It can also be found from Figure 3b that the intensity of peak $P'_2$ is much stronger than that of peak $P_4$ in Figure 3a, reflecting the efficient enhancement of the supercontinuum in this case. Interestingly, for peak $P'_2$, there is only a short path contributing to the same harmonic, namely the short-path harmonics are effectively selected, which is the principal reason for the generation of the broadband and smooth supercontinuum. Considering all the results above, we conclude that a highly efficient and ultrabroad supercontinuum with the contribution of a single quantum path can be obtained by the spatially inhomogeneous field combined with the coherent superposition state, which is highly possible to generate an intense isolated and narrow attosecond pulse.

When a single quantum path is only selected to contribute to the HHG, an ultrashort isolated attosecond pulse can be directly obtained via superposing several continuous harmonics, which are emitted in phase, and the corresponding results are given in Figure 4. For comparison, the produced attosecond pulse for the 1s ground state is not stringent, and a little more or less population than 0.5 will not generate obvious changes of the HHG can be ignored. Compared with the 1s ground state in the homogeneous field, as the highest harmonic order is significantly enlarged to 970 and the second-highest one is compressed to 140, the energy difference between them is remarkably increased, forming a 1287-eV ultrabroad supercontinuum. It can also be found from Figure 3b that the intensity of peak $P'_2$ is much stronger than that of peak $P_4$ in Figure 3a, reflecting the efficient enhancement of the supercontinuum in this case. Interestingly, for peak $P'_2$, there is only a short path contributing to the same harmonic, namely the short-path harmonics are effectively selected, which is the principal reason for the generation of the broadband and smooth supercontinuum. Considering all the results above, we conclude that a highly efficient and ultrabroad supercontinuum with the contribution of a single quantum path can be obtained by the spatially inhomogeneous field combined with the coherent superposition state, which is highly possible to generate an intense isolated and narrow attosecond pulse.

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![Figure 3](image-url)  
**Figure 3**: The time-frequency distributions for the HHG spectra corresponding to the solid black (a) and red (b) curves in Figure 1. The abbreviation o.c. represents the optical cycle at 800 nm throughout this paper.
state in the homogeneous field is also considered. As seen from Figure 4a, by filtering the harmonics from the 150th to 170th order, 2-as pulses are generated. This is because there are two electronic paths with different emission times corresponding to each harmonic, and the harmonics in the supercontinuum region are not excited synchronously. Among them, the weak pulse comes from the short path and the strong one derives from the long path. Figure 4b–d present the temporal profiles of the attosecond pulses for the coherent superposition state by superposing different harmonic orders in the inhomogeneous field.

To more clearly understand the quantum results above, we perform the classical electron trajectory simulation by using the three-step model [4, 5] and calculate the ionisation probability. The Newton equation that describes the classical behaviour of the electron in the inhomogeneous field can be expressed by

$$\ddot{x} = \nabla_x E(x, t) + [1 + 2c x(t)] E(t).$$

By solving the Newton equation, we can extract the classical electron trajectory in the spatially inhomogeneous field, and the calculated results are presented...
in Figure 5b. For the sake of contrast, we also show the classical electron trajectory in the homogeneous field. Figure 5a shows the dependence of the harmonic order on the ionisation and emission times in the homogeneous field. From this figure, the electrons are majorly ionised near the peaks of the laser field and forms four main emission peaks labelled by \( R_2, R_3, R_4 \), and \( R_5 \) on the returning energy map, which are responsible for the electrons ionised near 1.0 o.c., 1.52 o.c., 2.04 o.c., and 2.55 o.c., respectively. For each peak, there exist two kinds of electron trajectories corresponding to the same harmonic order in every half optical cycle. A trajectory with earlier ionisation but later emission times is called the long trajectory, and a trajectory with later ionisation but earlier emission times is called the short trajectory. The maximal values of these four peaks are the 92nd-, 138th-, and 131st-order harmonics, respectively. Therefore, the HHG emissions are primarily generated by the electrons ionised near 1.75 o.c. and 3.0 o.c., which form two notable emission peaks labelled as \( R_2 \) and \( R_4 \) on the returning energy map. The peak values of \( R_2 \) and \( R_4 \) are the 930th and 140th orders, respectively. Note that the peak value of \( R_4 \) is much higher than that of \( R_1 \) in Figure 5a. As a result, the harmonic cut-off for this case can be significantly extended due to the field inhomogeneity, which also results in the considerable enlargement of the supercontinuum. Moreover, the supercontinuum in our scheme is mainly caused by the ionised electrons in the range from 1.47 o.c. to 2.22 o.c., while the ionisation probability of the electron in the time range is much larger than that in the range from 1.98 o.c. to 2.23 o.c. for the case of the 1s ground state in the homogeneous field. This is the reason why the yields of the supercontinuum are efficiently enhanced in the inhomogeneous field with the coherent superposition state. More striking, in the range of peak \( R_2 \), though there are two electron trajectories with different ionisation times corresponding to the same harmonic order, the emission times of the electrons with the long trajectory are nearly the same as that of the electrons with the short trajectory. Thus, the two electron trajectories are merged into one and the short one dominates the harmonic emission. Once the single electron trajectory is selected, an ultrashort isolated attosecond pulse can be obtained directly by superposing a large range of continuous harmonics. All the results are in general agreement with those derived from the time-frequency analysis.

**Figure 5:** (a) The dependence of the harmonic order on the ionization (green circles) and emission (red diamond) times and the corresponding ionization probability (dotted blue curve) in the homogeneous field. (b) The same as (a) but for the inhomogeneous field. All parameters are the same as those in Figure 1.
In this study, by introducing the spatially inhomogeneous parameter $\epsilon$ to the temporal electric field, we expand the spectral cut-off and broaden the supercontinuum width. However, one wants to know what the effect of this parameter on the HHG is. To clarify better, Figure 6a presents the harmonic spectra with different values of $\epsilon = 0$, $\epsilon = 0.001$, $\epsilon = 0.002$, $\epsilon = 0.003$, and $\epsilon = 0.004$, respectively. As shown in this figure, the harmonic spectrum is very sensitive to the change of the parameter $\epsilon$. When $\epsilon$ varies from 0 to 0.004, the spectral cut-off is extended from the 157th order for $\epsilon = 0$ to the 970th order for $\epsilon = 0.004$. The underlying reason is as follows: the electrons contributing the highest harmonic photon are further reaccelerated in the continuum and acquire extra kinetic energy from the inhomogeneous field before returning to the parent ion. In addition, as $\epsilon$ increases, the supercontinuum becomes wider and the modulation in the harmonic spectrum is much removed, implying the selection of the single quantum path. To explore in detail the effect of the parameter $\epsilon$ on the harmonic spectrum and supercontinuum, we investigate the HHG process in the inhomogeneous field with different values of $\epsilon$ according to the three-step model. Figure 6b shows the dependence of the first and second cut-offs of the HHG on the parameter $\epsilon$ in the inhomogeneous field. As shown in this figure, the first cut-off of the harmonic spectrum has a little change in the whole range of $\epsilon$, and the maximal value and the minimum one are the 147th order for $\epsilon = 0.002$ and the 113th order for $\epsilon = 0.003$, respectively. However, for the case of the second cut-off, as the change of $\epsilon$ is in the range of 0–0.004, the second cut-off of the spectrum slowly increases from 0 to 0.0025 and then steeply increases up to 0.004. Thus, the frequency difference between the highest and second-highest cut-offs is enlarged, which is also the reason that the supercontinuum width becomes broader for the large value of $\epsilon$. In addition, when the parameter $\epsilon$ is further increased to 0.005, though the spectral cut-off can be markedly extended, the modulation of the low-energy part of the harmonic spectrum will become stronger and at the same time the harmonics of the high-energy part in the supercontinuum will become lower in conversion efficiency. However, from the point of view of generating ultrashort isolated attosecond pulses, as $\epsilon$ is in the range of 0.004–0.005, isolated sub-20-as pulses can be directed by superposing some properly selected harmonics. Overall, the enhancement of the harmonic cut-off and supercontinuum width as well as the modulation reduction favour the relatively larger $\epsilon$ value.

For short laser pulses, the corresponding durations are only a few femtoseconds; thus, the CEP of the laser pulse is extremely important in the HHG process. Because of this, we study the CEP effect of the inhomogeneous field on the harmonic spectrum. Figure 7a presents the harmonic spectrum as a function of the CEP of the driving field, which shows that the spectral cut-off has a certain degree of dependence on the CEP of the driving field, especially in the range from 0.4$\pi$ to 1.2$\pi$. However, for other CEP values, the dependence between the harmonic cut-off and CEP is not so sensitive. This implies that, for the extension of the harmonic cut-off gained by the inhomogeneous field, the CEP of the driving field should be considered, but not all. A deep insight about the CEP effect can be investigated by calculating the harmonic spectra with
different CEP values of $\varphi$. Figure 7b gives the harmonic spectra with seven different CEP values of $\varphi = 0$, $\varphi = 0.4\pi$, $\varphi = 0.6\pi$, $\varphi = 0.9\pi$, $\varphi = 1.2\pi$, $\varphi = 1.5\pi$, and $\varphi = 1.8\pi$, respectively. As seen from this figure, for the cases of $\varphi = 0.6\pi$, $\varphi = 0.9\pi$, and $\varphi = 1.2\pi$, the spectral cut-off decreases and the supercontinuum becomes short, and the harmonics in the plateau region reveal strong modulation, particularly in the cases of $\varphi = 0.9\pi$ and $\varphi = 1.2\pi$, whereas, for other values of the CEP, the harmonic cut-off and spectral profile are nearly unchanged, indicating that the ultrabroad supercontinuum with smooth structure can be produced in a wide CEP range.

Finally, we investigate the effects of the duration and intensity of the driving field on the harmonic spectrum. Figure 8a shows the harmonic spectra generated in the inhomogeneous field with three different durations. From this figure, as the duration of the driving field changes from 4 to 6 fs, the harmonic spectra for the cases of three durations display similar double-plateau structure with two cut-offs, and the spectral cut-off is greatly extended with the augmentation of the duration. In addition, as the duration increases, the modulation of the harmonics in the second plateau, from which the supercontinuum derives, becomes strong, especially in the low-frequency part of the harmonic spectrum. This typical behaviour can be clearly reflected by the green curve in Figure 8a, where the modulation in the low-frequency part of the harmonic spectrum is too strong to generate a single attosecond pulse. However, the harmonics in the high-frequency part is still regular and smooth, which can be used to synthesise an ultrashort isolated attosecond pulse. It should be noted that, when the duration of the driving field is controlled in the range of 4–6 fs, the broadband supercontinuum can be achieved. If the duration is decreased (<4 fs), the harmonic efficiency and spectral cut-off as well as the supercontinuum width will be reduced. If the duration is further increased (>6 fs), the harmonics in the plateau region will exhibit an irregular structure, which does not generate the broadband supercontinuum and isolated short attosecond pulse. In Figure 8b, we present the harmonic spectra with three different laser intensities in the inhomogeneous field. Evidently, the harmonic spectra for the cases of three intensities still reveal the same two-plateau structure, and the harmonics in the second plateau are almost regular and smooth, which correspond to the supercontinuum. When the intensity of the driving field increases, the first cut-off of the spectrum has little change, whereas the second one is significantly extended. Thus, the frequency difference between the first and second cut-offs distinctly becomes large, and the enlarged frequency difference also results in the further extension of the supercontinuum. This also indicates that, by properly increasing the laser intensity, both the spectral cut-off and supercontinuum can be further enhanced. This unique character can be used for generating the broadband supercontinuum, which is in favour of experimentally implementing the production of an isolated ultrashort attosecond pulse. We also find that from our simulated results, in the above selected parameter range, increasing the pulse duration or the intensity of the driving field will lead to the enhancement of the conversion efficiency of the harmonics, as shown in Figure 8c and d, which indicates that the high-efficiency harmonic emissions can be obtained for the relatively large duration or intensity of the laser field and also helps to produce an intense isolate short attosecond pulse.
Conclusion

In summary, we have presented an efficient method for obtaining an intense isolated ultrashort attosecond pulse from the He\(^+\) ion with the coherent superposition state in the spatially inhomogeneous field. It is shown that the combination of the spatially inhomogeneous field and the coherent superposition state can not only significantly extend the harmonic cut-off but also efficiently enhance the harmonic yields. Then, the high-efficiency and ultra-broad supercontinuum with a bandwidth of 1287 eV is generated directly. Moreover, with the introduction of the spatial inhomogeneity, the long path is eliminated completely; only the short path is selected to effectively contribute to the supercontinuum. As a result, an isolated ultrashort attosecond pulse with a duration of 17.3 as is obtained. Such a short single attosecond pulse will enable the detection and control of the electronic dynamics in atoms and molecules. Finally, we would like to point out several features of the scheme. (i) The population of the 1s excited state is not stringent, and a little more or less population than 0.5 will not generate obvious change of the HHG spectrum. (ii) In this scheme, the inhomogeneous parameter \(\varepsilon = 0.004\) is chosen to obtain the broadband supercontinuum with a single quantum path contribution and an isolated sub-20-as pulse, whereas our simulations also show that similar results can be achieved when the parameter \(\varepsilon\) is kept in the range of 0.004–0.005. (iii) For the broadband supercontinuum production, the CEP of the driving field should be considered but not all of it. (iv) When the duration of the driving field is limited in the range from 4 to 6 fs, the high-efficiency supercontinuum with a smooth structure can be obtained, and the conversion efficiency of the harmonics tends to increase for the relatively large duration. (v) By properly improving the intensity of the driving field, the harmonic cut-off and supercontinuum width as well as the harmonic efficiency can be further enhanced. These fascinating features can...
make one more easily manipulate our scheme in practical experimental implementations.

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