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Conformal Cosmology with a Complex Scalar Field and a Gauge-Mediated Supersymmetric Breaking Potential

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Abstract: We discuss the implications of a complex scalar field in conformal cosmology in the presence of a gauge-mediated supersymmetric (SUSY) breaking potential. A connection between the cosmological constant, the SUSY mass of the field and the Hubble mass is obtained. The transition from the complex to the real scalar field reveals a number of results in FRW cosmology, which are in agreement with data provided by recent astronomical observations, in particular, the pulsating white dwarf G116-B15A concerning the estimates of the probable range of the gravitational constant.

Keywords: Accelerated Universe; Complex Scalar Field; Conformal Cosmology; Gauge-Mediated Supersymmetric Breaking Potential.

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1 Introduction

Scalar-tensor theories belong to the class of gravitational models where a scalar field is added to the tensor field of general relativity. The simplest scalar field theory is the inclusion of a non-minimal coupling between the scalar field $\phi$ and the scalar curvature $R$ of the form $-\xi R \phi^2$ [1], $\xi$ being the non-minimal coupling parameter. This coupling is motivated from the superstring theory [2] and induced gravity [3]. Besides, this particular form of non-minimal coupling appears in quantum corrections to the scalar field in curved spacetime [4, 5]. It modifies the field equations and leads to several interesting insights in physics of the early universe (mainly the inflationary epoch) and physics of the late universe (description of the current accelerated expansion) (see [6, 7] and references therein). In the standard model of elementary particle physics, the non-minimal coupling plays a critical role through the Higgs field, and the recent detection of the Higgs particle corroborates that scalar fields exist in nature [8]. Connecting theories and observations of the early universe, the late universe and particle physics is a dazzling problem in theoretical physics. From the point of view of quantum field theory, the scalar field may be real or complex, and complexified scalar fields are indeed important during inflation [9], the early universe [10–12] and may also account for the accelerated expansion of the universe [13–16]. The particular value $\xi = 1/6$ corresponds to the conformal coupling and is interesting as it leads to the physically interesting and appealing evolution of the universe [17–21]. From the theoretical point of view, the value of $\xi$ should be estimated from observations or should be obtained from a more fundamental theory. In this paper, we wish to point out the possibility of using a complex scalar field to discuss the evolution of the universe for a special class of potential arising in a supersymmetric (SUSY) breaking scenario. The SUSY is, in fact, a well-motivated framework for the physics of the early universe. Phenomenological theories with SUSY breaking sector contains new interactions and auxiliary fields not found in the standard model. A broken SUSY occurs when one (or more) auxiliary field acquires a non-zero vev, which is then detected through the mediating interactions [22]. The dynamics of SUSY breaking is important in cosmology as it reduces, largely, the fine tuning in the cosmological constant and contributes to the vacuum energy density, i.e. the dark energy. Therefore, we are motivated to investigate the evolution of the universe by means of a gauge-mediated SUSY potential. The paper is organized as follows: in Section 2, we introduce the gauge-mediated SUSY breaking potential with a complex scalar field, and we discuss a number of its features; in Section 3, we discuss its implications in Friedmann-Robertson-Walker (FRW) cosmology and finally, conclusions are given in Section 4. It is noteworthy that all figures within the manuscript are plotted using an explicit range for all physical quantities for illustration purposes.

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2 Gauge-Mediated SUSY Breaking Potential with a Complex Scalar Field

In this work, we choose the following SUSY breaking potential:

\[ V(\phi\phi^*) = m_{\text{SUSY}}^2 \ln \left( 1 + \frac{1}{m_{\text{SUSY}}^2} \phi^* \phi - cH^2 (\phi^* \phi)^2 + \frac{\lambda^2}{M^2} (\phi^* \phi)^4 \right) , \quad (1) \]

where \( m_{\text{SUSY}} \) is the SUSY mass of the field, \( H = \chi / t \) is the Hubble parameter with \( \chi \) a real constant, \( \lambda \) is a dimensionless coupling constant and \( c = 1/2 \) and \( M = 2.4 \times 10^{18} \) GeV \( \gg \lambda \) is the reduced Planck mass. We plot, in Figure 1, the shape of the SUSY potential after setting \( m_{\text{SUSY}} = H = c = \lambda = M = 1 \) for illustration purposes and \( \phi = x + iy \), whereas in Figure 2, we plot the shape of the SUSY potential for very large time, i.e. \( H = \chi / t \rightarrow 0 \).

We observe that the potential behaves parabolically around the origin, which is one of the main features of the SUSY gauge breaking potentials with low-energy gauge mediation. This form of the potential is similar to the potential arising in the gauge-mediated SUSY breaking scenario in minimal supersymmetric standard model (MSSM) with flat directions [23, 24] in connection with baryogenesis mainly for \( m_{\text{SUSY}} \ll M \) and considered in [25] to study Q-ball formation through the Affleck–Dine mechanism, which explains successfully the baryon number of the universe based on complex scalar field. In principle, in the gauge-mediated SUSY breaking model, the potential of a flat potential is parabolic at the origin. The mass term, which is proportional to the Hubble parameter (\( H = m_{\chi} \) is the Hubble mass), appears from a Kahler potential and characterizes the SUSY breaking by a finite energy density that dominates the universe [26]. In fact, the shape of this potential differs from the shape of the standard Higgs potential in the unitary gauge \( V(\phi\phi^*) = -\frac{1}{2} \phi^* \phi + \frac{1}{4} (\phi^* \phi)^2 \) (plotted in Fig. 3) or from the modified Higgs potential of the higher-order (here of the 6th degree) \( V(\phi\phi^*) = -\frac{1}{2} \phi^* \phi + \frac{1}{6} (\phi^* \phi)^3 \) (plotted in Fig. 4).
obtained in the effective field theory, which includes higher-dimensional operators in a power-counting expansion, which results in a modified Higgs self-coupling opening the door towards a new physics beyond the standard model [27–29]. Furthermore, the presence of the logarithmic part in (1) leads to an unusual behaviour of the Higgs field as the field is, at present, nonpolynomial. In fact, the nonpolynomial Higgs potential holds a number of remarkable features as it reduces the lower Higgs mass bound without introducing metastability in the Higgs effective potential [30]. The presence of the logarithmic coupling terms, which can be traced back to the coupling in the effective superpotential and Kahler potential opens novel opportunities for electroweak breaking. Hence, we have a departure from the usual standard Higgs scenario [31].

We start from the total action of the theory in the absence of matter (the metric signature is (−, +, +, +)):

\[ S = \int \sqrt{-g} d^4x \left( \frac{R + 2\Lambda}{16\pi G} - \frac{1}{6} \phi \phi' R - \frac{1}{2} \phi' \phi \phi'' + \frac{g}{6} \phi \phi' \phi' - \frac{1}{2} \phi' \phi \phi'' - V(\phi^3) \right), \]  

where \( G \) is the Newton’s coupling constant, \( g \) is the determinant of the metric tensor \( g_{\mu\nu} \), and \( \Lambda \) is the cosmological constant. Here, \( \phi' \) is the complex conjugate of \( \phi \). Obviously, this action contains a conformally invariant scalar field, and one naturally expects that the resulting energy-momentum tensor holds special properties, which are quite different from those obtained using the ordinary scalar field or using the minimal coupling principle [32]. The case of conformal coupling has many motivating features, in particular, when treating the universe at low energy scales. The dynamics of the FRW model with dark energy in the form of phantom energy gives rise, under some restrictions, to a fractal structure in the phase space of initial conditions [32]. It was also proven that a flat universe with a conformally coupled phantom field results on a chaotic behaviour and that the accelerated expansion of the universe is a general feature without SUSY [33]. Additional features were obtained in [34] by extending the Bekenstein’s approach for massless conformal scalar fields, in particular, the emergence of an asymptotically power law inflationary behaviour and a bouncing cosmology. It was also argued in [35] that a singular-free closed universe may be obtained using a conformally coupled scalar field. More recently, it was argued in [36] that a scalar field with a quartic self-interaction potential conformally coupled to gravity yields a repulsive gravity, which implies a bouncing radiation-dominated universe. It is notable that a number of observational constraints on the conformal coupling between the scalar field and the scalar curvature can be found from astrophysical observation.

The variations of the action with respect to the scalar field and the metric tensor yield correspondingly the Klein–Gordon and the Einstein field equations:

\[ \Box \phi - \frac{R}{6} \phi - \frac{m_{\text{SUSY}}^2}{\phi} + \frac{g}{6} \phi \phi' + \frac{3\lambda^2}{M^2} \phi^3 \phi' = 0, \]  
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}^S, \]  

where

\[ T_{\mu\nu}^S = \delta^{\nu}_{\alpha} \delta^{\beta}_{\nu} \partial_{\alpha} \phi \partial_{\beta} \phi - \frac{1}{2} g_{\mu\nu} \left( g^{\alpha\beta} \partial_{\alpha} \phi \partial_{\beta} \phi + m_{\text{SUSY}}^4 \right) \]

\[ + \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} g^{\mu\nu} R - \frac{1}{6} \nabla_{\alpha} \nabla_{\nu} (\phi \phi') + \frac{1}{6} g^{\mu\nu} \Box (\phi' \phi), \]  

\( \nabla_{\nu} \) being the covariant derivative. In particular, for a constant scalar field \( \phi \neq 0 \), the trace of (4) gives:

\[ R = \frac{16\pi G}{1 - \frac{4\pi G}{3} \phi \phi} \left( m_{\text{SUSY}}^4 \ln \left( 1 + \frac{1}{m_{\text{SUSY}}^2} \phi \phi' \right) - \phi' R - \frac{1}{2} g^{\mu\nu} R \right) + 4\Lambda. \]  

We discuss, at first, the following two independent cases:

2.1: For large \( |\phi| \), we can neglect the logarithmic term in the scalar potential and therefore inserting (6) into (3) gives a constant scalar field:
\[(\phi \phi')^2 = \frac{M^2}{5\lambda} \left(3c^2 - \frac{2\Lambda}{3}\right), \quad (7)\]

and hence, physical solutions occur if \(4\Lambda < 9c^2\). As the Hubble mass is too small, the cosmological constant is tiny as well. In fact, the Hubble mass belongs to the class of ultralight scalars, which arise in supergravity theories [37]. Although it is small, it may have interesting consequences in the physics of the late universe and its present accelerated expansion [38, 39].

2.2: For small \(|\phi| = m_{\text{SUSY}}\), the logarithmic term in the scalar potential dominates, and hence, 
\[V(\phi \phi') = (m_{\text{SUSY}} - m_{\text{SUSY}}^2) \phi \phi'.\]
For a constant scalar field, we obtain accordingly:
\[\phi \phi' = \frac{2\Lambda + 3(m_{\text{SUSY}}^2 - m_{\text{SUSY}}^2)}{8\pi G (m_{\text{SUSY}}^2 - m_{\text{SUSY}}^2)}, \quad (8)\]

constrained by \(2\Lambda > 3(m_{\text{SUSY}}^2 - m_{\text{SUSY}}^2) > 0\). Using the previous constraint \(4\Lambda < 9c^2\), we find, consequently, the following inequality between cosmological masses: 
\[9c^2 / 2 > 2\Lambda > 3(m_{\text{SUSY}}^2 - m_{\text{SUSY}}^2) > 0\] in agreement with the prediction of a weak-scale SUSY [40]. Assuming that the cosmological constant varies as \(\Lambda - m_{\text{SUSY}}^2\), one can verify that the matching between (7) and (8) is obtained if:
\[c_{\text{SUSY}}^2 - m_{\text{SUSY}}^2 \approx M^2, \quad (9)\]
where \(M \approx G^{-1/2}\) is the reduced Planck mass as stated previously. Remarks A and B show that the cosmological constant is connected to two different mass scales at different epochs of time and that there exists a period of time where the SUSY mass is of the order of the Hubble parameter, i.e. ultralight SUSY mass. Such an ultralight SUSY mass can represent the dark matter content of the universe [41, 42]. This naturally suggests that the cosmological constant varies with time as it is naturally expected [43] as a constant value is powerless to give explanation about the enormous difference between the observed value and the numerical value obtained from the SUSY and quantum field theories.

3 FRW Cosmology with SUSY Breaking Potential

To discuss at present the evolution of the universe in the presence of the SUSY potential (1), we set \(\phi = \phi(t)e^{i\omega t}\) (transition from complex to real scalar field) [13]. In particular, in the case of a FRW metric spacetime described by the following flat metric with scale factor \(a(t)\):
\[
ds^2 = -dt^2 + a^2(t)(dr^2 + r^2(d\theta^2 + \sin^2 \theta d\psi^2)), \quad (10)\]
and filled with perfect fluid with density \(\rho\) and pressure \(p\), the equations of motion in the presence of matter are:
\[H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \left(\frac{1}{2} \phi^2 + \frac{1}{2} \phi^2 \theta^2 + V(\phi) + H \phi \phi + \rho\right) + \frac{\Lambda}{3}, \quad (11)\]
\[\dot{H} + H^2 = -\frac{4\pi G}{3} \left(\frac{1}{2} \phi^2 + 2 \phi^2 \theta^2 - 2V(\phi) - H \phi \phi + \rho + 3p\right) + \frac{\Lambda}{3}, \quad (12)\]
\[\dot{\phi} + 3H \phi - \phi \theta^2 - \frac{R}{6} \frac{dV}{d\phi} = 0, \quad (13)\]
\[\dot{\theta} + \left(2 \frac{\phi}{\phi'} + 3H\right) \dot{\theta} = 0, \quad (14)\]
where \(R = 6H + 12H^2\) with \(H = \dot{a}/a\) as mentioned previously, i.e. \(a(t) \propto t\). The last equation gives already after simple integration \(\phi \phi' \theta = \omega t\), \(\omega\) being an integration constant interpreted as angular velocity. We neglect its presence in the equations of motion as the factor \(\phi \phi'\) make all contributions from the angular motion decrease so fast (faster than the matter density) along with the expansion of the universe provided that the scalar field does not decrease as fast as \(a^{-3}\) [13]. As from (11) (or (4)) an effective gravitational constant \(G_{\text{eff}} = G \left(1 - 4\pi G / 3 \rho \right)^{-1}\) arises in the theory and depends obviously on the dynamical scalar field, we presume that the gravitational constant varies with time as well. A dynamical \(G\) is supported by a number of observational results coming up from Lunar Laser Ranging and distant Type Ia supernova observation (see [44] and references therein). In the absence of particle creation, the continuity equation \(\nabla \cdot T^{\mu \nu} = 0\) yields \(\rho + 3H(p + \rho) = 0\) and \(\Lambda + 8\pi G \rho = 0\). Assuming the equation of state \(p = (\gamma - 1)\rho\), \(\gamma\) being a real parameter, we obtain after simple integration \(\rho = k a^{-\gamma}\); \(k\) is an integration constant, which is set equal to unity for simplicity. We discuss again two cosmological stages.

3.1 Large Scalar Field

As for large \(\phi\) we must have \(4\Lambda < 9c^2\) and as \(m_{\text{SUSY}}^2 \propto H^2\), we set \(\Lambda = \beta H^2\), where \(\beta\) is a real constant. As in our
arguments $H = \chi / t$, then, the relation $\Lambda = \beta H^2$ shows that the cosmological constant decays in time. In fact, cosmological scenarios with a time-dependent cosmological constant were proposed and investigated largely in literature as they can solve the discrepancy between the theoretical and observational limits (they differ for 120 orders of magnitude). It is then natural to assume that there is a dynamically decaying cosmological constant. As the universe evolves from the high-energy regime (early epoch) to the low-energy regime (present epoch), the cosmological constant decreases to its present value. It is notable that the ansatz $\Lambda = \beta H^2$ was obtained in literature based on different arguments [45–49]. The potential is approximated by $V(\phi) = -\frac{H^2}{6} \phi^2 / 2$ as $M = 2.4 \times 10^{18}$ Gev $> \lambda$. Negative potentials appear in supergravity, brane and cyclic cosmologies and are explored in decaying dark energy models [50–59]. They hold a number of striking consequences and are not forbidden in cosmology. Equations (11) and (13) are, respectively, simplified to:

$$\frac{\beta \chi^2}{6 t^2} = \frac{1}{2} \phi^2 + \frac{\chi_0}{t} + \frac{t^{-\gamma \chi}}{\phi},$$

$$\frac{t^2}{t} \phi + 3 \phi = \frac{t(1-\chi)}{t^2} \chi^2 = 0.$$  

The effective potential is, at present, $V(\phi) = (\chi - 2) H^2 \phi^2 / \chi$. One plausible solution of (16) is given by $\phi(t) = t^m$ provided that $m(m - 1) + 3 m + \chi (1 - \chi) = 0$. The differential equation $\Lambda + 8 \pi G \rho = 0$ in its turn gives $G(t) = t^{-\gamma \chi - 2}$ and from (15) follows the following useful constraints $\beta \chi^2 = 3 m^2 + 6 m + 2 = 3 \gamma \chi + 2 m$. It is obvious that for $\gamma = 0$ (vacuum-dominated energy), the scalar field increases linearly in time, the gravitational constant decreases and the scale factor increases as $a(t) \propto t^\beta$. It is effortless to check that for $\beta = 3$, we find $\chi = 1.7$, $m = 0.3$ and $\gamma = 0.27$. The universe is, therefore, accelerated in time and dominated by dark energy with the equation of the state parameter $w = \gamma - 1 = 0.73$ in agreement with observations. The scalar field increases slowly in time, and the gravitational constant decays as $G(t) \propto t^{-\alpha}$. However, for $\beta = 5/2$, we find $\chi = 2.27$, $m = 0.46$, $\gamma = 0.15$, $w = \gamma - 1 = 0.85$ and $G(t) \propto t^{-1.12}$. Finally, for $\beta = 2$, we find $\chi = 5.6$, $m = 1.48$, $\gamma = 0.03$ and $G(t) \propto t^{-2.9}$. We observe that the behaviour of the gravitational constant is very sensitive to the value of the parameter $\beta$. It should be noted that real solutions exist for $\beta \geq 2$. Coming to the amount of variation of the gravitational constant, several estimates of the probable range of $G$ were obtained from numerous sources, e.g., data provided by the binary pulsar PSR 1913+16, Type Ia Supernova, lunar occultation, etc. (see [50] and references therein). Using the date provided by the pulsating white dwarf G116-B15A, the following estimates have been obtained: $|\dot{G} / G| < 4.1 \times 10^{-11}$ year$^{-1}$ in [60] and $-2.5 \times 10^{-10}$ year$^{-1} < \dot{G} / G < 4.1 \times 10^{-10}$ year$^{-1}$ in [61]. Assuming $t_0 = 14$ Gyr, we get $|\dot{G} / G| = 4.28 \times 10^{-11}$ year$^{-1}$ if $G(t) \propto t^{-0.5}$, $|\dot{G} / G| = 8 \times 10^{-11}$ year$^{-1}$ if $G(t) \propto t^{-1.12}$ and $|\dot{G} / G| = 2 \times 10^{-10}$ year$^{-1}$ if $G(t) \propto t^{-2.9}$. However, as in our approach $G(t) \propto t^{-1/2} = t^{-2n}$, we can use this date to fix an estimate range of the parameter $w$. We, thus, obtain $-0.29 < m < 0.29$, and therefore, for $\beta = 3$, we find $0.27 < \gamma < 0.5$, whereas for $\beta = 5/2$, we find $0.2 < \gamma < 0.37$ and finally, for $\beta = 2$, we obtain $0.048 < \gamma < 0.16$, which represents the best fit with the results obtained [62]. For large $\phi$ and, in particular, for $\beta = 2$, the universe is expanding accelerately, dominated by a decaying cosmological constant and by dark energy, a decreasing gravitational constant, an increasing scalar field and a quadratic time-dependent potential of the form $V(\phi) = -H^2 \phi^2 = -t^2 \phi^2$ plotted in Figure 5 (unit of time is the second for illustration purposes).

Observe that for a large $\phi$, the universe is dominated by power laws in agreement with observations provided from WMAP7 and WMAP7 + BAO + $H_0$ [63, 64]. It should be emphasized that power law cosmology has been discussed in literature and used in different contexts [65–72] and does not face the horizon and the flatness problems. Besides, power law cosmology may solve dynamically the cosmological constant problem. In cosmic history, the universe passes by periods dominated by power laws, in particular, the radiation ($a(t) \propto t^{2/3}$) and dust ($a(t) \propto t^{2/3}$) epochs so one logically expects to dominate the late-time dynamics. Besides, the power law expansion regime at small redshifts is as good fit to the SN1a data. Therefore, the universe, nowadays, can be modelled during a brief history of time by a power law expansion of the scale factor. It is notable that the variations of the scale factor characterise the whole history of the universe. In addition, the
sign of the deceleration parameter $q = -\ddot{a}a/\dot{a}^2$ is important as the accelerated increase in the scale factor occurs at $q < 0$, whereas the accelerated growth of the expansion velocity ($H > 0$) corresponds to $q < -1$ [73]. A universe with $q > 0$ describes a universe that is decelerating in time. In our approach, $q = (1 + \chi)/\chi$, and therefore, $q = -0.41$ for $(\beta, \chi) = (3, 1.7)$, $q = -0.56$ for $(\beta, \chi) = (2.5, 2.27)$ and $q = -0.82$ for $(\beta, \chi) = (2, 5.6)$. These results are in agreement with the range obtained from the SNeIa data and from the observational Hubble data and their combination, respectively [74, 75]. In fact, the approximate value of the deceleration parameter at the present epoch is $q = -0.70_{-0.2}^{+0.2}$ and consequently, the results obtained in this subsection are encouraging. It is notable that the deceleration parameter varies slowly with time ($q = 0.5$ during the matter-dominated epoch, whereas $q = -1$ during the dark energy/cosmological constant-dominated epoch).

### 3.2 Small Scalar Field

For small $\phi$, the potential $V(\phi) = m_{\text{SUSY}}^2 \phi^2 - cH^2 \phi^2 = -cH^2 \phi^2$ as $m_{\text{SUSY}}^2 < m_\phi^2$. Therefore, (11) and (13) are, respectively, simplified to:

$$
\left(1 - \frac{\beta}{3} \frac{\chi^2}{t^2}\right) \dot{a} = t^{3/2} \left(\frac{1}{2} \dot{\phi}^2 + \frac{\chi}{t} \ddot{\phi} \dot{\phi} - \frac{1}{2} \frac{\chi^2}{t^2} \phi^2 + t^{3/2} \right),
$$

(17)

$$
\ddot{\phi} + \frac{3\chi}{t} \dot{\phi} + \frac{\chi(1 - \chi)}{t^2} \phi = 0.
$$

(18)

A plausible solution of (18) is given by $\phi(t) \propto t^n$ provided that $n(n - 3) + 2\gamma n + \chi(1 - \chi) = 0$. Equation (17) gives the constraints $(9 - 2\beta)\chi^2 = 3(n^2 + 2\chi^2 + 2)$ and $2 = 3\chi + 2n$. For $\beta = 2$, we find $\chi = 1.13$, $n = 0.06$ and $\gamma = 0.55$. Therefore, the universe is expanding acceleratedly in time, whereas the scalar field is roughly constant in time and is dominated by dark energy. The gravitational constant increases as $G(t) \propto t^{-19}$ and $G/G_0 = 1.35 \times 10^{11}\text{year}^{-1}$ in agreement with the results obtained in [76]. If we choose $\beta = 2.5$, we get $\chi = 1.34$, $n = 0.14$ and $\gamma = 0.42$. For a low scalar field, we find $q = -0.11$ for $(\beta, \chi) = (3, 1.13)$ and $q = -0.25$ for $(\beta, \chi) = (2.5, 1.34)$.

To recapitulate, we observe that for $\beta = 2$, the scalar field varies as $\phi(t) \propto t^{0.06}$ for a small $\phi$ and as $\phi(t) \propto t^{1.68}$ for a large $\phi$, whereas the scale factor varies as $a(t) \propto t^{13}$ for a small $\phi$ and $a(t) \propto t^{5.6}$ for a large $\phi$. The cosmological constant decays as $\Lambda = 2H^2$, and the gravitational constant varies as $G(t) \propto t^{-19}$ for a small $\phi$ and $G(t) \propto t^{-2.9}$ for a large $\phi$. The EoS parameter varies from $w = -0.45$ (small $\phi$) to $w = -1$ (large $\phi$).

### 4 Conclusions

To conclude, we have discussed the cosmological implications of a complex scalar field in the presence of a gauge-mediated SUSY breaking complex scalar potential motivated from MSSM and Affleck–Dine mechanism. We have analysed its connection with geometry by means of the spacetime curvature, and it was observed that a physical solution is obtained if $9c m^2_{\text{SUSY}} / 2 > 2\Lambda > 3(c m^2_{\text{SUSY}} - m^2_{\text{SUSY}}) > 0$ and $c m^2_{\text{SUSY}} - m^2_{\text{SUSY}} = M^2$. A transition from a complex to a real scalar field is done in order to discuss the implications of the gauge-mediated SUSY breaking potential in FRW cosmology. We have considered power law cosmology, which, in general, emerges when classical fields are coupled to a spacetime curvature. For both large and small scalar fields, the gauge-mediated SUSY potential is reduced to a negative potential, and its implications in conformal coupling FRW cosmology has offered a number of motivating features, which are in agreement with recent observations, in particular, to what concerns the probable range of the gravitational coupling constant provided by the pulsating white dwarf G116-B15A. The universe is acceleratedly expanding with time and is dominated by vacuum energy, and we can conclude that this physical effect is due to the variations of the gravitational constant from slowly increasing to decreasing in time. In the cases of large and small scalar fields, the universe is characterised by a negative deceleration parameter, and therefore, both scenarios describe a certain period of time where the universe is acceleratedly expanding with time. It is worth mentioning that the model constructed in this work describes the dynamics of the universe after the inflationary era, which is characterised by an exponential increase in the scale factor, i.e. a de Sitter-type metric. The gauge-mediated SUSY breaking potential is, therefore, practical not only for dark matter but also for dark energy. Further analyses in addition to confrontation with astrophysical observations are required. Work in this direction is in progress.

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