Influence of Compliant Walls and Heat Transfer on the Peristaltic Transport of a Rabinowitsch Fluid in an Inclined Channel

Abstract: In this paper, we investigate the peristaltic pumping of a Rabinowitsch fluid in an inclined channel under the effects of heat transfer and flexible compliant walls. The expressions for the velocity, the temperature, and the coefficient of the heat transfer are obtained. The influence of emerging parameters on the velocity, the temperature, the coefficient of heat transfer and the trapping phenomenon of the Newtonian, dilatant and pseudoplastic fluid models are also analyzed graphically. We find that the velocity and the temperature fields decrease for shear thickening fluid; but the velocity and temperature fields of the shear thinning, and Newtonian fluids increase with an increase in the angle of inclination. Furthermore, there were more trapping boluses occurring for the Newtonian fluid case as compared to the pseudoplastic and dilatant fluids cases. However, as the angle of inclination increases, the size of trapping bolus decreases.

Keywords: Flexible Compliant Walls; Heat Transfer; Inclined Channel Flow; Peristaltic Transport; Rabinowitsch Fluid.

1 Introduction

The peristaltic mechanism of propelling physiological fluids occurs naturally in living body biological systems, such as the esophagus, small blood vessels, gastrointestinal tract, stomach, fallopian tube, ureter and the gut. The reliable action of peristaltic pumping has significant applications to chemical and medicine equipment because of several advantages, such as contamination-free pumping, ability to reduce the mechanical strain and accurate dosing in the biomechanical instruments like dialysis machines, open-heart bypass pump machines and artificial lungs and tissues. Past studies have discussed the physiological and mechanical situations of peristaltic pumping under the theory of lubrication approach [1–8].

During the past few decades, the classical behavior of the non-Newtonian fluid flows has significance in biological, medical and industrial applications. The complex rheological properties of the non-Newtonian model can be described by Rabinowitsch fluid, because of the cubic stress model exhibits the characteristics of pseudoplastic, dilatant, and Newtonian nature. This provides a better understanding of shear thinning (e.g. polymer solutions, blood plasma, syrup and latex paint), shear thickening (e.g. corn starch, sand in water) and Newtonian (e.g. water, air) rheology. The recent investigations on the Rabinowitsch fluid have been reported in the literature [9–15].

The bioheat transfer process in biomedical engineering includes the physiological thermoregulation; blood perfusion in biological tissues during the clinical hyperthermia treatment; temperature distribution before, during and after the exercise; and the impact of fever and exposure to sunlight. Fetecau et al. [16] investigated the flow of viscous fluid and the temperature distribution over a heated infinite plane with arbitrary shear stress to the fluid by employing the Laplace transforms. Dumitru Vieru et al. [17] studied the unsteady MHD natural convection flow with Newtonian heating and constant mass diffusion over an infinite vertical plate, which applies an arbitrary time-dependent shear stress to a viscous optically thick fluid. Very recently, Hayat et al. [18] addressed the impact of the Darcy-Forchheimer relation and radial magnetic field on the peristalsis of the Eyring-Powell nanomaterial in a curved channel.

The displacement of compliant walls controls the wave deformation, and researchers have focused on the influence of wall properties on the peristaltic flow with heat transfer. For example, Abd Elnaby and Haroun [19]
reported that the driving mechanism of the muscles are governed by a set of equations in terms of variables, which are related to the displacement of the compliant wall. They assumed that the driving mechanism of the muscle is of the form of a sinusoidal wave of moderate amplitude imposed on the compliant walls of the channel. Vajravelu et al. [20] investigated the combined effects of heat and mass transfer on the peristaltic flow of the Sisko nanoliquid with motile gyrotactic microorganism and compliant wall constraints. The authors [20, 21] noticed that the velocity increases with increasing wall tension and mass characterizing parameters, whereas the velocity decreases with damping nature of the wall. Other past studies have also investigated this topic [22–28].

To the best of our knowledge, no investigation has focused on studying the peristaltic pumping of a Rabinowitsch fluid in a symmetric inclined channel with compliant walls. Therefore, in the current study, we examine the effects of wall elasticity on the peristaltic flow of the Rabinowitsch fluid with heat transfer.

## 2 Mathematical Formulation

We consider a Cartesian coordinate system \((x, y)\) for the peristaltic flow of an incompressible Rabinowitsch fluid through an inclined uniform symmetrical channel in the laboratory frame of reference. The channel walls are flexible and are assumed to be a stretched membrane. The channel walls are maintained at constant temperature \(T_0\) (see Figure 1).

![Figure 1: The physical model.](image)

The wall deformation \(\eta(x, t)\) due to the infinite train of the peristaltic wave travelling along the channel is given by

\[
y = \eta(x, t) = a + b \sin \frac{2\pi}{\lambda}(x - ct),
\]

where \(a, b, c\) and \(t\) represent the channel mean width, amplitude, wave length, wave speed and time, respectively.

The two-dimensional unsteady governing equations for the flow field are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho g \sin \beta,
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \rho g \cos \beta,
\]

\[
\zeta \rho \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau_{xx} \frac{\partial u}{\partial x} + \tau_{yy} \frac{\partial v}{\partial y} + \tau_{xy} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right),
\]

where \(u\) and \(v\) denote the velocity components in the \(x\) and \(y\) directions, respectively; \(\rho\) denotes the density of the fluid; \(g\) denotes the acceleration due to gravity; \(p\) is the pressure; \(\tau_{xx}, \tau_{xy}, \tau_{yy}\) are the extra stress tensor components; and \(T, \zeta\) and \(\kappa\) denote the temperature, specific heat at constant volume and thermal conductivity of the fluid, respectively.

The constitutive equation relating to shear stress and the strain rate of the Rabinowitsch fluid model [14, 15] is given by

\[
\tau_{xy} + \mu_0 \tau_{xy}^2 = \mu \frac{\partial u}{\partial y},
\]

where \(\mu_0\) represents the pseudoplasticity coefficient, which acts as non-linear factor in the non-Newtonian fluid model, and \(\mu\) represents the fluid viscosity. The model is reduced to pseudoplastic when \(\mu_0 > 0\), the dilatant when \(\mu_0 < 0\), and Newtonian when \(\mu_0 = 0\).

The equation of the flexible compliant wall motion [22, 27] is expressed as

\[
L(\eta) = p - p_0.
\]
The operator \( L \) represents the motion of the stretched membrane, such that

\[
L = -\tau \frac{\partial^2}{\partial x^2} + m \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} + B \frac{\partial^4}{\partial x^4} + H, \tag{8}
\]

in which \( \tau, m, C, B \) and \( H \) denote the longitudinal tension in the membrane per unit width, the mass per unit area, the coefficient of the wall damping forces, the flexural rigidity of the plate and the spring stiffness respectively. In addition, \( p_0 = 0 \) is the pressure on the outside surface of the wall and we assume that \( p_0 = 0 \).

Due to the symmetric motion of the flexible walls and for the sake of simplicity, we analyze the flow in the half-width of the channel. The appropriate boundary conditions in dimensional form are given by

\[
\begin{align*}
  u & = 0 \quad \text{at} \quad y = \eta, \quad \tag{9} \\
  \frac{\partial u}{\partial y} & = 0 \quad \text{at} \quad y = 0, \quad \tag{10} \\
  T & = T_0 \quad \text{at} \quad y = \eta, \quad \tag{11} \\
  \frac{\partial T}{\partial y} & = 0 \quad \text{at} \quad y = 0. \quad \tag{12}
\end{align*}
\]

Eq. (9) represents the no-slip boundary condition at the wall \( y = \eta \). Eq. (10) and Eq. (12) imply that the velocity and the temperature attain maximum values on the axis of the channel. Eq. (11) indicates that the temperature at the wall is \( T_0 \).

The continuity of stress implies that at the interface of the wall and the fluid, the pressure must be the same as that which acts on the fluid at \( y = \eta \). By employing the \( x \)-component of momentum, the equation leads to

\[
\frac{\partial}{\partial x} L(\eta) = \frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho g \sin \beta + \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right). \tag{13}
\]

The dimensionless parameters are given below.

\[
\begin{align*}
  x' &= \frac{x}{\lambda}, \quad y' = \frac{y}{\lambda}, \quad u' = \frac{u}{a}; \quad v' = \frac{v}{c}, \quad \eta' = \frac{\eta}{\lambda}, \\
  \delta &= \frac{\alpha}{\lambda}, \quad \rho' = \frac{\rho a^2}{\mu \lambda c}, \quad Re = \frac{\alpha c}{\nu}, \quad \nu = \frac{\mu}{\rho}, \\
  \tau_{xx}' &= \frac{\alpha}{\mu \lambda} \tau_{xx}, \quad \tau_{xy}' = \frac{\alpha}{\mu \lambda} \tau_{xy}, \quad \tau_{yy}' = \frac{\alpha}{\mu \lambda} \tau_{yy}, \\
  \phi &= \frac{\phi}{ac}, \quad \phi = \frac{b}{a}, \quad \alpha = \frac{\nu^2 c^2}{a^2 \mu_0}, \quad \tau' = \frac{tc}{\lambda}, \\
  E_1 &= -\frac{\tau a^3}{\lambda^3 \mu c}, \quad E_2 = \frac{mca^3}{\lambda^3 \mu}, \quad E_3 = \frac{Cc^3}{\lambda^2 \mu}, \quad E_4 = \frac{Ba^3}{\lambda^5 \mu c}, \quad E_5 = \frac{Ha^3}{\lambda^5 \mu c}, \quad F = \frac{vc}{ga\tau}, \\
  \theta &= \frac{T - T_0}{T_0}, \quad Ec = \frac{c^2}{\zeta T_0}, \quad Pr = \frac{\mu \zeta}{k} \tag{14}
\end{align*}
\]

Using the above dimensionless quantities in (1)–(6) and in boundary conditions (9)–(13), along with the assumptions of long wavelength and low Reynolds number, and by dropping the primes, we obtain

\[
\begin{align*}
  & \tau_{xy} + a \tau_{xy} = \frac{\partial u}{\partial y}, \tag{15} \\
  & \frac{\partial \tau_{xy}}{\partial y} + \sin \beta = \frac{\partial p}{\partial x}, \quad \tag{16} \\
  & \frac{\partial p}{\partial y} = 0, \quad \tag{17} \\
  & \frac{\partial^2 \theta}{\partial y^2} + Br \tau_{xy} \left( \frac{\partial u}{\partial y} \right) = 0. \quad \tag{18}
\end{align*}
\]

The boundary conditions in the non-dimensional form are given by

\[
\begin{align*}
  u &= 0 \quad \text{at} \quad y = \eta + 1 + \phi \sin 2\pi (x - t), \quad \tag{19} \\
  \frac{\partial u}{\partial y} & = 0 \quad \text{at} \quad y = 0, \quad \tag{20} \\
  \theta & = 0 \quad \text{at} \quad y = \eta, \quad \tag{21} \\
  \frac{\partial \theta}{\partial y} & = 0 \quad \text{at} \quad y = 0, \quad \tag{22} \\
  \frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \sin \beta = \left( E_1 \frac{\partial^3 \theta}{\partial x^3} + E_2 \frac{\partial^3 \theta}{\partial y \partial x^2} + E_3 \frac{\partial^3 \theta}{\partial y \partial x} + E_4 \frac{\partial^3 \theta}{\partial y^3} + E_5 \frac{\partial \theta}{\partial x} \right) \quad \text{at} \quad y = \eta. \quad \tag{23}
\end{align*}
\]

where \( Br = Ec \cdot Pr \) (Brinkman number).

### 3 Analytical Solution

Solving (15)–(18) with the help of (19)–(23), we obtain

\[
\begin{align*}
  u &= \left( \frac{y^2 - \eta^2}{2} \right) \left( E - \sin \beta \right) \tag{24} \\
  & \quad + a \left( E - \sin \beta \right)^3 \left( \frac{y^4 - \eta^4}{4} \right),
\end{align*}
\]
\[ \theta = Br \left( E - \frac{\sin \beta}{F} \right)^2 \left( \frac{\eta^4 - y^4}{12} \right) + a \left( E - \frac{\sin \beta}{F} \right)^2 \left( \frac{\eta^6 - y^6}{30} \right) \]  (25)

The coefficient of heat transfer at the wall is given by

\[ Z = \eta_x \frac{\partial \theta}{\partial y} \bigg|_{y=\eta} = -\eta_x Br \left( E - \frac{\sin \beta}{F} \right)^2 \left( \frac{\eta^3}{3} + a \left( E - \frac{\sin \beta}{F} \right)^2 \frac{\eta^5}{5} \right), \]  (26)

where

\[ E = 8\phi \pi^3 \left( \frac{E_1}{2\pi} \sin 2\pi (x - t) - \left( E_1 + E_2 - 4\pi^2 E_4 - \frac{E_5}{4\pi^2} \right) \cos 2\pi (x - t) \right). \]

The expression for the stream function can be calculated with the help of the relation expressed as

\[ u = \frac{\partial \psi}{\partial y}, \]  (27)

### 4 Results and Discussion

The influences of the dimensionless pseudoplasticity parameter \( \alpha \), the body force parameter \( F \), angle of inclination \( \beta \), the wall tension parameter \( E_1 \), the mass characterizing parameter \( E_2 \), the wall damping parameter \( E_3 \), the wall rigidity parameter \( E_4 \) and the wall elastic parameter \( E_5 \) on the temperature, heat transfer coefficient and the streamline patterns are presented through graphs. The expression for the velocity is given in Eq. (24). The effects of the abovementioned parameters on the flow characteristics of the dilatant (\( \alpha < 0 \)), Newtonian (\( \alpha = 0 \)) and pseudoplastic (\( \alpha > 0 \)) fluids are illustrated graphically in Figures 2–4, respectively.

Figure 2 depicts the variation of velocity for different values of the body force parameter \( F \). We notice that the velocity of the dilatant fluid increases with an increase in the body force parameter, whereas the velocity of pseudoplastic fluid and Newtonian fluid decreases with increasing body force parameter. Figure 3 shows the effect of the angle of inclination on the velocity. From the figure, we observe a decrease in the velocity for shear thickening fluid. However, the velocities of shear thinning and Newtonian fluids increase with an increase in the inclination parameter angle. If the angle of inclination parameter is zero, then the results are reduced to the case of horizontal channel.

Figure 4a–c represent the velocity profiles for several sets of values of the parameters \( E_1, E_2, E_3, E_4 \) and \( E_5 \) for the dilatant (\( \alpha = -0.1 \)), Newtonian (\( \alpha = 0 \)) and pseudoplastic (\( \alpha = 0.1 \)) fluids. From Figure 4a, we see that the velocity increases with increasing wall tension parameter, whereas the velocity decreases with increasing \( E_2 \). This is true with the parameters \( E_3, E_4 \) and \( E_5 \) for the dilatant fluid. From Figure 4b–c, in the cases of the viscous and pseudoplastic fluids, we notice that the velocity increases with increasing \( E_1 \) and \( E_2 \), but quite the opposite is true with increasing \( E_3, E_4 \) and \( E_5 \). Further, the results from the velocity profiles
Figure 4: The velocity profiles of \( E_1, E_2, E_3, E_4 \) and \( E_5 \) with \( \phi = 0.6, F = 2, \beta = \frac{\pi}{6}, x = 0.2, t = 0.1 \) for (a) \( \alpha = -0.1 \) (b) \( \alpha = 0.0 \) (c) \( \alpha = 0.1 \).

Figure 5: The temperature profiles for \( Br \) with \( E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01, \phi = 0.6, F = 2, \beta = \frac{\pi}{6}, x = 0.2, t = 0.1, \alpha = -0.5, 0.0, 0.5. \)

Figure 6: The temperature profiles for \( F \) with \( E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01, \phi = 0.6, Br = 0.02, \beta = \frac{\pi}{6}, x = 0.2, t = 0.1, \alpha = -0.5, 0.0, 0.5. \)

Figure 7: The temperature profiles for \( \beta \) with \( E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01, \phi = 0.6, Br = 0.02, F = 2, x = 0.2, t = 0.1, \alpha = -0.5, 0.0, 0.5. \)
The temperature profiles of 

\[ E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01, \phi = 0.6, \beta = 0.2, \]  

\[ F = 2, x = 0.2, t = 0.1 \]  

for (a) \( \alpha = -0.005 \) (b) \( \alpha = 0.0 \) (c) \( \alpha = 0.1 \).

![Figure 8](image_url)

**Figure 8:** The temperature profiles of \( E_1, E_2, E_3, E_4, \) and \( E_5 \) with \( \phi = 0.6, F = 2, \beta = 0.2, x = 0.2, t = 0.1 \) for (a) \( \alpha = -0.1 \) (b) \( \alpha = 0.0 \) (c) \( \alpha = 0.1 \).

The expression for fluid temperature is given in Eq. (25) and its variations with the emerging parameters are sketched in Figures 5–8. The temperature distribution in terms of Brinkman number \( Br \) is depicted in Figure 5. From this figure, we see that the temperature of the dilatant fluid decreases with the increase of \( Br \), but the temperatures of the pseudoplastic and Newtonian fluids increase.
with the increase of $Br$. Figure 6 illustrates the effect of body force parameter on the temperature. We observe that the temperature of the dilatant fluid increases with the increase in the body force parameter, whereas the temperatures of the pseudoplastic and Newtonian fluids decrease with the increase in the body force parameter. Figure 7 is drawn to study the influence of the inclination angle on the temperature. From the plots, we observe that, with the increase in the angle of inclination, the temperature of the shear thickening fluid decreases, whereas those of the shear thinning and Newtonian fluids increase. The temperature variations of the dilatant, viscous and pseudoplastic fluids with $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$ are shown in Figure 8a–c. As can be seen, the temperature increases
with increasing $E_1$ and $E_2$ and decreases with increasing $E_3$, $E_4$, and $E_5$. Further, the results from the temperature graphs indicate that the heat transfer is higher in the pseudoplastic fluid model as compared with those in the dilatant and viscous models.

The heat transfer coefficient graphs for the different parameters are plotted in Figures 9–12. Figure 9a–c reveal that, with increasing $Br$, the magnitude of the heat transfer coefficient increases for all three types of fluids. Figure 10a–c show that an increase in the body force parameter decreases the heat transfer coefficient in the dilatant fluid but increases it in the Newtonian and pseudoplastic fluids. Figure 11a–c demonstrate that an increase in $\beta$ leads to a heat transfer coefficient decrease in the shear thickening fluid and an increase in the Newtonian and shear thinning fluids. Figure 12a–c exhibit that the thermal transfer coefficient decreases with the increase of $E_1$, $E_2$, and $E_3$, and increases with the increase of $E_4$ and $E_5$ for the dilatant fluid. Moreover, the thermal transfer coefficient increases with increasing $E_1$, $E_2$, and $E_3$, and decreases with increasing $E_4$ and $E_5$ for the viscous and shear thinning fluids.

The streamline patterns of the shear thickening, Newtonian, and shear thinning fluids for $\beta = 0$ and inclination angle $\beta = \frac{\pi}{6}$ are presented in Figure 13a–c and in Figure 14a–c, respectively. From these figures, we can see that the volume of the trapped bolus decreases when the fluid changes from dilatant to pseudoplastic. There is also a greater number of trapped boluses occur for the Newtonian fluid as compared with the pseudoplastic and dilatant fluids. Further, as the angle of inclination increases, the size of the trapped bolus decreases. Figure 15a–e demonstrates how the size of the trapped bolus decreases with an increase in $E_1$, $E_2$, and $E_4$ but increases for $E_3$ and $E_5$.

**5 Conclusions**

We examine the effects of the wall elasticity parameter on the peristaltic flow of the Rabinowitsch fluid in an inclined channel with heat transfer. Some of the interesting findings are presented below.

1. With an increase in the inclination parameter angle, the velocity and the temperature fields of the shear thickening fluid decrease, but the velocities of the shear thinning and Newtonian fluids increase.
2. The velocity is low in the dilatant fluid model as compared with those in the viscous and pseudoplastic models.
3. The heat transfer is higher in the pseudoplastic fluid model as compared with those in the dilatant and viscous models.
4. The velocity and temperature of the viscous and pseudoplastic fluids increase with increasing $E_1$ and $E_2$ and decrease with increasing $E_3$, $E_4$, and $E_5$.
5. As $Br$ increases, the magnitude of the heat transfer coefficient increases for all three types of fluids.
6. There are more trapped bolus in the Newtonian fluid as compared with those in the pseudoplastic and dilatant fluids.
Figure 13: The streamlines of $\alpha$ with $E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01, \phi = 0.6, \beta = 0, F = 2, x = 0.2, t = 0.1$ for (a) $\alpha = -0.1$, (b) $\alpha = 0.0$ (c) $\alpha = 0.1$.

Figure 14: The streamlines of $\alpha$ with $E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01, \phi = 0.6, \beta = \frac{\pi}{6}, F = 2, x = 0.2, t = 0.1$ for (a) $\alpha = -0.1$, (b) $\alpha = 0.0$ (c) $\alpha = 0.1$. 
Figure 15: The streamlines of $E_1, E_2, E_3, E_4$ and $E_5$ with $\beta = \frac{\pi}{6}$, $\phi = 0.6, F = 2, x = 0.2, t = 0.1, \alpha = 0.1$ for (a) $E_1 = 0.1, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01$ (b) $E_1 = 0.6, E_2 = 0.04, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01$ (c) $E_1 = 0.6, E_2 = 0.1, E_3 = 0.4, E_4 = 0.002, E_5 = 0.01$ (d) $E_1 = 0.6, E_2 = 0.1, E_3 = 0.8, E_4 = 0.002, E_5 = 0.01$ (e) $E_1 = 0.6, E_2 = 0.1, E_3 = 0.8, E_4 = 0.008, E_5 = 0.01$ (f) $E_1 = 0.6, E_2 = 0.1, E_3 = 0.8, E_4 = 0.008, E_5 = 0.08$.

References


