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Selection and Enhancement of the Single Harmonic Emission Event in the Water Window Region

Abstract: The control of the high-order harmonic generation in the half-cycle region has been investigated by using the improved polarization gating (PG) technology. It is found that by properly controlling the delay time of the PG pulse, the contribution of the harmonic plateau is nearly coming from the single harmonic emission event, which is much better for producing the single attosecond pulses (SAPs). Further, by properly adding an ultraviolet pulse and a half-cycle pulse in the driven laser polarization direction, the harmonic yield can be enhanced and the harmonic cutoff can be extended, showing a high-intensity harmonic plateau covering the whole water window region. Finally, through the Fourier transformation of some selected harmonics, a 35 as SAP in the water window region can be obtained.

Keywords: High-Order Harmonic Generation; Polarization Gating Technology; Single Attosecond Pulse in Water Window Region; Single Harmonic Emission Event.

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1 Introduction

High-frequency laser source as the potential tool to probe the ultrafast dynamics in physics, chemistry, and materials science attracted much attention in the laser technology community [1–3]. High-order harmonic generation (HHG), which is created when atoms and molecules are driven by the intense laser field, as one of the most important method to obtain the high-frequency laser source has been widely investigated in the last two decades [4–8].

On the basis of the three-step model (STM) reported by Corkum [9] in 1993, the emission of the harmonic can be achieved in half cycle of the laser field with the processes of ionization, acceleration, and recombination. Further, a harmonic plateau with the cutoff frequency (energy) of $I_p + 3.17 \times (I/4\omega^2)$ can be obtained, where $I_p$ is the ionization potential and $I$ and $\omega$ are the laser intensity and the laser frequency, respectively. Finally, by superposing some selected harmonics on the harmonic plateau, the attosecond pulses with very high frequency (photon energy) can be obtained. According to the STM, two messages about the harmonic spectra can be found. First, the harmonic emission happens in each half cycle of the laser field. Thus, the contribution of the harmonic spectrum is coming from the multiple harmonic emission events (HEEs). Second, the harmonic cutoff is sensitive to the laser intensity and the laser frequency.

Although it is well known that the high-intensity single attosecond pulses (SAPs) covering the extreme ultraviolet (UV) and the X-ray regions (i.e. the water window region from 2.3 to 4.4 nm) are much better for practical applications, because of the effect of the multi-HEEs during the harmonic emission process, the harmonic emission time and phase for a given harmonic order are not the same. As a result, only the attosecond pulse trains with the multi-peaks can be obtained. Thus, how to select the single HEE and produce the high-intensity SAPs covering the whole water window X-ray region become an important issue in the attosecond science community. For instance, (i) by using the single-cycle 3.3 fs pulse [10], a SAP with a duration of 160 as can be obtained. (ii) With the combination of the multi-color field [11–15], some sub-50 as SAPs can be obtained. (iii) By properly modulating the laser field in time [16–18] and space [19–22], some SAPs shorter than 30 as can be produced. (iv) By using the polarization gating (PG) technology [23–29], not only the single HEE can be selected but also the SAPs can be generated from the multi-cycle laser field.

Among the above schemes, the PG technology is the most important and practical method to produce the SAPs in experiments. This is because in the PG scheme, the multi-cycle laser field can be used, which is much easier to obtain in experiments. However, because of the gating...
effect, the harmonic yield and the harmonic cutoff from the PG scheme are decreased in comparison with those from the linearly polarized laser field. Therefore, how to extend the harmonic cutoff and enhance the harmonic yield when using the PG scheme are very important in this community. Thus, in this paper, we propose an improved PG scheme to extend the harmonic cutoff and to enhance the harmonic yield from a He atom. The scheme can be separated into three parts. First, by properly choosing the delay time, the laser wavelength, and the laser intensity of the PG field, the single HEE with the proper cutoff and intensity can be selected during the harmonic emission process. Second, with the help of the UV pulse, the harmonic yield can be enhanced. Moreover, because of the effect of UV resonance ionization, the harmonic enhancement has two peaks around $\lambda_{UV} = 62.5 \text{ nm}$ and $\lambda_{UV} = 125 \text{ nm}$ ($\lambda_{UV}$ is the wavelength of the UV pulse), corresponding to the one-photon and the two-photon resonance ionization between the ground state and the first excited state of the He atom, respectively. Third, by properly adding a half-cycle pulse (HCP), the harmonic cutoff from the selected single HEE can be extended to the whole water window region. Finally, a 35 as water window SAP with the intensity enhancement of 300 times (larger than two orders of magnitude) can be obtained.

2 Methods

The HHG driven by the PG laser field can be investigated through the two-dimensional time-dependent Schrödinger equation (2D-TDSE) [30–34]. Atomic units (a.u.) are used throughout this paper unless stated otherwise.

$$i \frac{\partial \phi(x, y, t)}{\partial t} = \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2} - \frac{1}{2} \frac{\partial^2}{\partial y^2} + V(x, y) + xE_x(t) + yE_y(t) \right] \phi(x, y, t).$$

(1)

Here, $x$ and $y$ are the 2D electronic coordinates, which are defined by $-250 \text{ a.u.} < x$ and $y < 250 \text{ a.u.}$ with $\Delta x = \Delta y = 0.5 \text{ a.u.}$ The absorbing regions on $x$ and $y$ extend over the last 50 grid points. $V(x, y) = -a/\sqrt{x^2 + y^2 + B}$ is the soft Coulomb potential with $a = 1.0$ and $b = 0.07$ to match the ionization potential of 24.6 eV for the ground state of the He atom. $E_x(t) = E_{\text{driven}}(t)$ and $E_y(t) = E_{\text{gating}}(t)$ are the combined laser field in the driven and gating polarization directions, respectively, which can be expressed as follows:

$$E_x(t) = E_{\text{driven}}(t) = E_{1f}(t + \tau_1/2) \cos(\omega_1 t) + E_{2f}(t - \tau_{\text{delay}}/2) \cos(\omega_2 t),$$

(2)

$$E_y(t) = E_{\text{gating}}(t) = E_{1f}(t + \tau_1/2) \cos(\omega_1 t) - E_{2f}(t - \tau_{\text{delay}}/2) \cos(\omega_2 t),$$

(3)

$$f(t) = \exp(-4 \ln(2) t^2/r^2), \quad i = 1, 2.$$  

(4)

Here, $E_{1}$, $\omega_1$, $\tau_1$ ($i = 1, 2$), and $T$ are the laser amplitude, the laser frequency, the pulse duration, and the optical cycle of the co-rotating and the counter-rotating circularly polarized laser fields, respectively. $\tau_{\text{delay}}$ is the delay time between the two circularly polarized pulses. $\phi(x, y, t)$ is the time-dependent wave function, which can be propagated by using the standard second-order split-operator method [35]. The time space is chosen to be $\Delta t = 0.1 \text{ a.u.}$

According to the Ehrenfest theorem [36], the time-dependent dipole acceleration can be written as follows:

$$a(t) = -\langle \phi(x, y, t) | \nabla_x \psi(x, y, t) + \vec{E}(t) \phi(x, y, t) \rangle$$

$$= -\langle \phi(x, y, t) \left( \frac{\partial V(x, y)}{\partial x} + E_x(t) \right) \vec{e}_x \rangle$$

$$+ \left( \frac{\partial V(x, y)}{\partial y} + E_y(t) \right) \vec{e}_y \langle \phi(x, y, t) \rangle - a_x(t) \vec{e}_x - a_y(t) \vec{e}_y.$$  

(5)

Further, the radiated HHG power spectrum $S(\omega)$ can be obtained by Fourier transforming the time-dependent dipole acceleration:

$$S(\omega) \sim \left| \int \exp(-i\omega t)a(t)dt \right|^2$$

$$= \left| \int \exp(-i\omega t)(a_x(t) \vec{e}_x + a_y(t) \vec{e}_y)dt \right|^2$$

$$= S_x(\omega) + S_y(\omega).$$

(6)

As can be seen, the total HHG spectrum $S(\omega)$ is coming from the superposition of two components in the driven $S_x(\omega)$ and gating $S_y(\omega)$ directions. The ellipticity of the harmonics is given by the following [37]:

$$\varepsilon = \frac{1 + P - \sqrt{1 + 2P \cos(2\delta) + P^2}}{\sqrt{1 + 1 + 2P \cos(2\delta) + P^2}},$$

(7)

where $I = \sqrt{S_x(\omega)/S_y(\omega)}$ and $\delta = \arg[\sqrt{S_x(\omega)}] - \arg[\sqrt{S_y(\omega)}]$ are the intensity ratio and the phase difference of the two components, respectively. The linearly, elliptically, and circularly polarized harmonics correspond to $\varepsilon = 0, 0 < \varepsilon < 1$, and $\varepsilon = 1$, respectively.

Finally, the temporal profiles of the SAPs can be obtained by the inverse Fourier transformation of some selected harmonics:

$$I_{\text{SAP}}(t) = \left| \sum_q \left( \int a(t)e^{-i\omega qt}dt \right)e^{i\omega qt} \right|^2,$$

(8)

where $q$ are the selected harmonics orders.

It should be noted that although solving the three-dimensional TDSE is the most accurate method for studying the electron motion in the HHG process, the present 2D-TDSE in the driven and gating laser polarization directions is also a common method to investigate the HHG process in PG technology [27–29, 34].
3 Results and Discussion

3.1 Selection of Single HEE from PG Scheme

As it is well known [23–26] that the reduction of the HEEs from the PG scheme is because the temporal laser polarization of the combined field can be changed from circularly polarized to linearly polarized and back to circularly polarized, thus, only the HEEs from the linearly polarized part can be found. For instance, Figure 1a–e show the laser profiles, the time-frequency analyses of the harmonics [38], and the harmonic spectra from the PG scheme with \( t_{\text{delay}} = 0 \) and \( t_{\text{delay}} = 2.0T \).

The two circularly polarized laser fields are chosen to be 10T-800 nm with \( I_{1,2} = 1.0 \times 10^{14} \) W/cm\(^2\). As can be seen, for the case of \( t_{\text{delay}} = 0 \) (corresponding to the linearly polarized single-color field), because of the multi-cycle effect, there are multi-HEEs during the harmonic emission process, which is absolutely unbeneficial to produce the SAPs, as shown in Figure 1a and b. For the case of \( t_{\text{delay}} = 2.0T \), because of the gating effect, the intensities of the HEEs from the rising and the falling parts of the laser field are decreased, and the intense HEE during the laser amplitude part can be obtained (i.e. the HEE from \( t = 0.25T \) to \( t = 0.75T \)), which is beneficial to reduce the sub-peaks of the attosecond pulses, as shown in Figure 1c and d. However, through analyzing Figure 1e, we see that the harmonic cutoff and the harmonic yield from \( t_{\text{delay}} = 2.0T \) are both decreased compared with the \( t_{\text{delay}} = 0 \) case. Here, if the delay time of the PG field is further increased, the harmonic cutoff and the harmonic yield can be further decreased (the HHG spectra from the larger \( t_{\text{delay}} \) are not shown in this paper), which is unfavorable to obtain the SAPs with the higher efficiency and the higher photon energy. Thus, in this paper, the delay time of the two circularly polarized laser field is chosen to be \( t_{\text{delay}} = 2.0T \) as a proper value.

Figure 2a shows the HHG spectra from the driven and the gating components for the case of \( t_{\text{delay}} = 2.0T \). As seen, for the case of the lower harmonics, the two components have the similar contributions to the HHG spectrum, while for the case of the higher harmonics, the intensities of the harmonics from the driven component are higher than those from the gating component. Thus, according to (7), the nearly linearly polarized harmonics can be obtained in the higher harmonics region, as shown in Figure 2b, which is much better for producing the linearly polarized SAPs.

Figure 1: The laser profiles and the time-frequency analyses of the HHG from the PG scheme with (a, b) \( t_{\text{delay}} = 0 \) and (c, d) \( t_{\text{delay}} = 2.0T \). The two circularly polarized laser fields are chosen to be 10T-800 nm with \( I_{1,2} = 1.0 \times 10^{14} \) W/cm\(^2\). (e) The HHG spectra for the cases of the PG fields with \( t_{\text{delay}} = 0 \) and \( t_{\text{delay}} = 2.0T \).

Figure 2: (a) The HHG spectra from the driven and the gating components for the case of the PG field with \( t_{\text{delay}} = 2.0T \). (b) The ellipticities \( \varepsilon \) of the harmonics for the case of the PG field with \( t_{\text{delay}} = 2.0T \).
Through the above analyses, we see that the PG scheme is beneficial to reduce the HEEs and to produce the linearly polarized SAPs in the multi-cycle pulse region. However, the harmonic cutoff and the harmonic yield will be decreased, which is unfavorable to produce the high-intensity and the high-photon energy SAPs. On the basis of the STM, the harmonic cutoff is proportional to \( \lambda^2 \), where \( \lambda \) means the wavelength of the conventional laser field. Thus, recent investigations on the extension of the harmonic cutoff by using the mid-infrared laser field attracted much attention in HHG community [39-42].

Therefore, in Figure 3a, we present the wavelength scaling of the harmonic cutoff energy driven by the PG scheme. The pulse durations of the different wavelength lasers are all 10\( \tau \), and the delay times are all chosen to be 2.0\( T \). It should be noted that the unit of the harmonic cutoff is electron volt (eV). Thus, the wavelength scaling of the harmonic cutoff energy presents the nearly linear structure as the laser wavelength changes. But if the unit of the harmonic cutoff is chosen to be the harmonic order, then the harmonic cutoff is nearly proportional to \( \lambda^2 \), which agrees with the STM. Clearly, the harmonic cutoff energy can be extended as the laser wavelength increases; however, the harmonic yield from the longer wavelength pulse (i.e. 1600 nm pulse) is decreased in comparison with that from the shorter wavelength pulse (i.e. 800 nm pulse), as shown in Figure 3b. Of course, the further extension of the harmonic cutoff can be achieved when the laser wavelength is larger than 1600 nm; however, the remarkable decrease of the harmonic yield is not good for producing the practical SAPs in experiments. Therefore, we choose \( \lambda = 1600 \) nm as the fundamental laser wavelength in the following discussion. Moreover, the 1600 nm pulse can be easily produced from the 800 nm pulse in experiments. It should be noted that in the former PG investigations, the pulse intensities of the two circularly polarized pulses are the same. However, in this paper, we set them as different to see what happens to the HHG spectra. Clearly, we see that as \( I_1 \) increases, only the smaller extension of the harmonic cutoff can be found, as shown in Figure 3c. However, as \( I_2 \) increases, the harmonic cutoff can be extended obviously, as shown in Figure 3d.

To better see the harmonic emission process in the asymmetric PG field, in Figure 4, we present the laser profiles and the time-frequency analyses of the HHG driven by the PG field with the different laser intensities. For the case of \( I_{1,2} = 1.0 \times 10^{14} \) W/cm\(^2\) (Fig. 4a and b), as discussed in the 800 nm pulse case, the intensities of the driven field and the gating field are almost the same on the rising and the falling parts of the laser field, as shown in Figure 4a. Thus, the intensities of the HEEs from these two parts are decreased, as shown in Figure 4b. However, when \( 0 < t < 0.75T \), the intensity of the gating part is very small, and a nearly linearly polarized part from the driven field can be found in this region. As a result, only one intense HEE with the

![Figure 3](image-url)

**Figure 3**: (a) The wavelength scaling of the harmonic cutoff energy driven by the PG scheme. The pulse durations of the above given wavelengths are all 10\( \tau \), and the delay times are all chosen to be 2.0\( T \). (b) The HHG spectra driven by the 800 and 1600 nm PG fields. (c, d) \( I_1 \) and \( I_2 \) effects on the HHG spectra. The wavelength of the PG field is chosen to be 1600 nm in the following discussion.
nearly linear polarization can be found from $t = 0.25T$ to $t = 0.75T$, as shown in Figure 4b. For the cases of $I_1 = 2.5 \times 10^{14} \text{ W/cm}^2$, $I_2 = 1.0 \times 10^{15} \text{ W/cm}^2$ (Fig. 4c and d) and $I_1 = 1.0 \times 10^{14} \text{ W/cm}^2$, $I_2 = 2.5 \times 10^{14} \text{ W/cm}^2$ (Fig. 4e and f), the nearly linearly polarized parts of the laser field are coming from $t = 0.5T$ to $t = 1.25T$ and from $t = -0.5T$ to $t = 0.25T$, respectively, as shown in Figure 4c and e. As a consequence, the intense single linearly polarized HEEs from $t = 0.75T$ to $t = 1.25T$ and from $t = -0.25T$ to $t = 0.25T$ can be obtained, respectively, as shown in Figure 4d and f. Moreover, because of the enhancement of the laser intensity, the emitted photon energies from these two HEEs are both extended. However, for the case of $I_1 = 2.5 \times 10^{14} \text{ W/cm}^2$ and $I_2 = 1.0 \times 10^{14} \text{ W/cm}^2$, the single HEE is coming from the sub-peak of the amplitude region (i.e. from $t = 0.75T$ to $t = 1.25T$), whereas for the case of $I_1 = 1.0 \times 10^{14} \text{ W/cm}^2$ and $I_2 = 2.5 \times 10^{14} \text{ W/cm}^2$, the single HEE is coming from the max-peak of the amplitude region (i.e. from $t = -0.25T$ to $t = 0.25T$). This is the reason behind the smaller and the larger extensions of the harmonic cutoffs from these two cases, respectively.

### 3.2 Enhancement of the Selected Single HEE by Using UV-Resonance Ionization

Through the above analyses, we see that the single HEE can be selected and extended when using the asymmetric mid-infrared PG scheme. However, the lower harmonic efficiency becomes a severe problem on the generations of the SAPs. Thus, in this part, we focus on the enhancement of the harmonic yield with the help of the UV-resonance ionization [43–45]. First, the ultrashort UV pulse is chosen to be 1.0 fs/125 nm with a pulse intensity of $I_3 = 0.5I_1$ ($I_3$ means the pulse intensity of the UV pulse). The reason for choosing the 125 nm UV pulse is because its photon energy is meeting the two-photon resonance ionization between the 1s and the 2p states of the He atom, where the ionization probability can be remarkably enhanced [43–45]. Here, because the higher harmonics are mainly coming from the driven laser component, thus, only the linearly polarized UV pulse is adopted and introduced into the driven laser polarization direction. Figure 5a shows the enhanced ratios of the harmonic yield compared with the PG + UV case and the single PG case [$S_{PG+UV}(\omega)/S_{PG}(\omega)$]. Here, the efficiency of the harmonic spectrum is calculated by superposing the harmonic order from $100\omega_0$ to the cut-off region. As can be seen, an optimal enhancement of the harmonic yield can be achieved around $t_{\text{delay}} = -0.4T$ ($t_{\text{delay}}$ is the delay time of the UV pulse), and the harmonic yield from the PG + UV case is enhanced by 80 times in comparison with the single PG case. Figure 5b–d show the HHG spectra driven by the PG + UV field with $t_{\text{delay}} = -0.4T$, the laser profiles in the driven laser component, and the time-frequency analysis of the harmonics driven by the PG + UV field. On the basis of the STM and through the above analyses, we know that the ionization process of the above-selected HEE is around $t = -0.5T$, and the acceleration-recombination process is coming from $t = -0.25T$ to $t = 0.25T$. Now, through analyzing Figure 5c, we see that when the delay time of the

![Figure 4: The laser profiles and the time-frequency analyses of the HHG from the 1600 nm PG fields with (a, b) $I_{2,3} = 1.0 \times 10^{14} \text{ W/cm}^2$; (c, d) $I_1 = 2.5 \times 10^{14} \text{ W/cm}^2$, $I_2 = 1.0 \times 10^{15} \text{ W/cm}^2$; and (e, f) $I_1 = 1.0 \times 10^{14} \text{ W/cm}^2$, $I_2 = 2.5 \times 10^{14} \text{ W/cm}^2$.](image-url)
UV pulse is chosen to be \(-0.4T\), the amplitude part of the UV pulse mainly covers the ionization process of the above HEE, where the ionization probability can be enhanced because of the UV-resonance ionization [43–45]. Therefore, the intensity of the HEE from \(t = -0.25T\) to \(t = 0.25T\) is enhanced, as shown in Figure 5d, and is responsible for the enhancement of the harmonic yield, as shown in Figure 5b. Here, we only analyze the harmonic emission process when \(\tau_{\text{delay}} = -0.4T\). This is because when the other delay time of the UV pulse is chosen, the main body of the UV pulse is not covering the ionization process of the selected HEE. Thus, the enhancement of the harmonic cannot be achieved.

Figure 6a shows the UV phase effect on the harmonic enhancement. As can be seen, the enhanced ratios of the harmonics are almost not changed as the UV phase changes, which is much better for experimental design. Figure 6b shows the HHG spectra driven by the PG + UV field with \(\tau_3 = 1.0\) fs and \(\tau_3 = 3.0\) fs (\(\tau_3\) is the pulse duration of UV pulse). Clearly, the enhancement of the harmonic yield is insensitive to the pulse duration of the UV pulse. This point is different to references [44] and [45], where the pulse duration of the UV pulse is very short. This is because the fundamental fields in references [44] and [45] are linearly polarized laser field. Therefore, when the UV-resonance ionization happens during the UV pulse covering region, the enhancement of the multi-HEEs can be achieved, which is unfavorable to generate the SAPs. So, only the very short UV pulse can be used to the linearly polarized UV-resonance enhancement scheme [44, 45]. However, in this paper, because of the gating effect, the intensities of the HEEs from the rising and the falling parts of the laser field are decreased. Thus, when adding the UV pulse with the longer pulse duration, although the UV-resonance ionization can also be achieved during the UV pulse covering region, only the HEEs from the laser amplitude region can be remarkably enhanced. Figure 6c shows the enhanced ratios of the harmonic yield as a function of the intensity ratio of the UV pulse. It shows that the harmonic efficiency can be further enhanced as \(I_3\) increases, and the enhanced ratio of 342 times can be found when \(I_3 = 1.0I_1\). It should be noted that the further enhancement of the harmonic yield can also be found as the UV intensity further increases. However, in this paper, we want to use the low-intensity combined field to obtain the high-intensity SAPs in the water window region. So, the maximum intensity of the UV pulse is set as \(I_3 = 1.0I_1\), and the maximum harmonic enhancement of 342 times can be found. Figure 6d shows the wavelength scaling of the UV pulse on the harmonic yield. The delay times and the intensities of the UV pulses are all \(\tau_{\text{delay}} = -0.4T\) and \(I_3 = 1.0I_1\). As seen in this figure, the enhanced ratios of the harmonics can be separated into three parts. For instance, (i) when \(50\) nm < \(\lambda_{\text{UV}}\) < \(140\) nm (\(\lambda_{\text{UV}}\) means
the wavelength of the UV pulse), the enhanced ratios of the harmonics are higher than 250 times, and two enhanced peaks can be found around \( \lambda_{UV} = 62.5 \, \text{nm} \) and \( \lambda_{UV} = 125 \, \text{nm} \), which is corresponding to the one-photon and two-photon resonance ionization between the 1s and the 2p states of the He atom [44, 45]. (ii) When 140 nm < \( \lambda_{UV} < 180 \, \text{nm} \), the enhanced ratios of the harmonics can also be found, but the enhanced ratios are only from 100 times to 200 times. (iii) When 180 nm < \( \lambda_{UV} < 220 \, \text{nm} \), the enhanced ratios of the harmonics are remarkably decreased.

### 3.3 Extension of the Single HEE in the Water Window Region by Using HCP

Through analyzing Section 3.2, we see that with the help of the UV-resonance ionization, the harmonic yield can be enhanced by 342 times in comparison with the single PG case, and only one HEE is contributed to the harmonic spectrum, which is much better for producing the high-intensity SAPs. However, the harmonic cutoff is only 335\( \omega_0 \), which is not covering the whole water window X-ray region. Thus, in this section, we will focus on the extension of the harmonic cutoff. Through analyzing Figure 5d, we see that the selected high-intensity HEE is coming from \( t = -0.25T \) to \( t = 0.25T \); thus, the best scheme for the harmonic extension is to only extend this HEE and present the others no change. Recently, Orlando et al. [46] and Feng et al. [47] theoretically found that the control of the harmonic emission process in the half-cycle time can be achieved by using the HCP, which can be obtained from the resonant propagation of a few-cycle pulse through asymmetrical media with periodic sub-wavelength structure [48]. Thus, in this section, we use the HCP to further control the HEE in the half-cycle region. The HCP \( [E_3(t)] \) can be expressed as follows:

\[
E_4(t) = kE_1\theta(t - t_{\text{delay}3}) \left[ \frac{400(t - t_{\text{delay}3})^3 \exp\left[ -8(t - t_{\text{delay}3})/\tau_4 \right]}{\tau_4^4} - \frac{0.004(t - t_{\text{delay}3})^5 \exp\left[ -(t - t_{\text{delay}3})/\tau_4 \right]}{\tau_4^5} \right], \tag{9}
\]

where \( k \), \( \tau_4 \), and \( t_{\text{delay}3} \) are the amplitude ratio, the pulse duration, and the delay time of the HCP.

Figure 7a shows the harmonic cutoff as a function of \( t_{\text{delay}3} \). The amplitude ratio and the pulse duration of the HCP are chosen to be \( k = 0.3 \) and \( \tau_4 = 3.0 \, \text{fs} \), respectively. As seen, with the introduction of the HCP, the extension of the harmonic cutoff can be achieved, and a maximum extension of the harmonic cutoff can be found around \( t_{\text{delay}3} = -0.2T \). Figure 7b–d show the laser profiles in the driven laser polarization direction, the time-frequency analysis of the harmonics, and the HHG spectrum driven by the combined field with \( t_{\text{delay}3} = -0.2T \). Clearly, when \( t_{\text{delay}3} = -0.2T \), the main unipolar peak of the HCP is from \( t = -0.25T \) to \( t = 0.25T \); thus, only the temporal laser
profile from this region is enhanced, and the others are almost not changed, as shown in Figure 7b. As a result, the accelerated electron in this region will obtain the additional energy, thus leading to the extension of the single HEE from $t = -0.25T$ to $t = 0.25T$ and is responsible for the harmonic cutoff extension, as shown in Figure 7c and d, respectively. Here, we only analyze the harmonic emission process when $t_{\text{delay}} = -0.2T$. This is because when the other delay times are adopted, the unipolar peak of the HCP is not covering the temporal laser profile from $t = -0.25T$ to $t = 0.25T$. Thus, the free electron cannot receive the optimal energy during its accelerated process.

Figure 8a shows the harmonic cutoff as a function of the amplitude ratio of the HCP. The other parameters of the laser fields are the same as those in Figure 7b–d. We see that as the intensity of the HCP increases, the harmonic cutoff can be further extended, and a harmonic cutoff of $710\omega_1$ covering the whole water window region can be found when $k = 0.6$. Of course, the harmonic cutoff can be further extended when $k > 0.6$. However, the purpose of this paper is to obtain the harmonic plateau covering the water window region. Thus, $k = 0.6$ is enough. Figure 8b shows the HHG spectra for the cases of the single PG field and the PG + UV + HCP combined field with $k = 0.6$. As can be seen, with the help of the present UV-HCP beam, not only the harmonic yield can be enhanced by 342 times but also the harmonic cutoff can be extended to the water window region. Moreover, through analyzing the time-frequency analysis of the harmonic spectrum shown in Figure 8c, we see that the harmonic spectrum is mainly...
Figure 9: The temporal profile of SAP from the PG + UV + HCP field. The inset of Figure 9 shows the temporal profile of SAP from the PG field.

contributed by the single HEE, which is much better for producing the high-intensity SAPs in the water window region.

3.4 Generation of High-Intensity SAPs in the Water Window Region

As discussed in the introduction, the single HEE is beneficial for the generation of the SAPs. Now, through the above analyses, we see that with the help of the present PG + UV + HCP laser beam, the selection and the enhancement of the single HEE covering the whole water window region can be achieved. Therefore, by properly selecting the filtering window covering the water window region from $360\omega_1$ to $710\omega_1$, a 35 as SAP in the water window region can be directly obtained, as shown in Figure 9. Moreover, the signal intensity of the produced SAP is enhanced by at least 300 times (larger than two orders of magnitude) compared with the single PG field case, as shown in the inset of Figure 9.

4 Conclusion

In conclusion, an efficient scheme to select and enhance the single HEE during the harmonic emission process has been proposed and studied by using the improved PG technology. The results can be separated into three parts.

First, by properly choosing the delay time of the PG field, the single HEE can be selected. Moreover, as the laser wavelength increases as well as the asymmetric laser intensity is adopted, the harmonic cutoff from the single HEE can be extended.

Second, by properly adding a UV pulse, the ionization process of the harmonic emission process can be controlled by the UV-resonance ionization, thus leading to the enhancement of the harmonic yield. Moreover, the harmonic yield can be further enhanced as the UV intensity increases and a maximum harmonic enhanced ratio of 342 times can be found.

Third, with the help of the HCP, the acceleration-recombination process of the harmonic emission process can be controlled in the half-cycle region. Therefore, the emitted photon energies from the selected single HEE can be further extended to the water window region.

All in all, through the present scheme, the selection and the enhancement of the single HEE covering the whole water window region can be achieved, and a 35 as water window SAP with an intensity enhancement larger than 300 times (larger than two orders of magnitude) can be obtained.

It should be emphasized that there are some advantages in the present PG + UV + HCP scheme. First, the PG technology is a common and a viable technology for producing the SAPs in experiments, especially when using the multi-cycle pulse. Second, the enhancement of the harmonic yield, caused by the UV-resonance ionization, is insensitive to the phase and the pulse duration of the UV pulse, which is better for experiment design. Third, the produced high-intensity SAP in this paper is coming from the low-intensity combined driven laser field ($<10^{16}$ W/cm$^2$), which is also easier to obtain in laboratories. Thus, the present scheme would provide an efficient route to generate the high-intensity water window SAPs in experiments.

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