Amplified Tunnelling from a Pair of Balanced Loss and Gain Cavities

Abstract: An exceptional point (EP) or pole is an extraordinary physical phenomenon of a parity-time (PT)-symmetric system. In this article, we design a compact pseudo-PT-symmetric system in which a gain resonator and a loss resonator are connected by a metal film. An amplified tunnelling is found with the coupling of the two resonators induced by the plasmonic resonance. Both EP and the pole effects can result in a jump in the transmitance and reflectance with nearly equal, large values. The pole effect can be achieved by adjusting either the gain coefficient or the incidence angle.

Keywords: Cavity; Coupling; Pole; PT-symmetry.

1 Introduction

Recently, optical analogues of parity-time (PT)-symmetric systems have been studied in optical structures [1–20]. For a one-dimensional structure, the states of PT-symmetric structure can be described by the eigenvalues of the scattering (S) matrix. The eigenvalues of the S matrix either form pairs with reciprocal moduli or are all unimodular. The former corresponds to the PT-symmetry-broken phase, so the two corresponding eigenstates exhibit amplification and dissipation, respectively. The latter corresponds to the PT-symmetric phase, so the eigenstates exhibit neither a net amplification nor dissipation. At the PT-symmetry-breaking threshold, i.e. the exceptional point (EP), the two eigenstates of the S matrix coalesce, and the S matrix has only one eigenvector and eigenvalue [1, 2]. Unique phenomena with the structure usually occur at EP, such as loss-induced transparency [3], unidirectional invisibility [4], band-merging [5, 6], topological chirality [7, 8], laser mode selectivity [9, 10], and one-way-enhanced reflectance [11]. A review article has specially dealt with such effects [12]. Apart from EP, another interesting behaviour of the PT-symmetric structure is the pole effect occurring within the PT-symmetry-broken phase [1, 2]. The poles are just the coherent perfect absorber (CPA)-laser solutions [1]. Some PT-symmetric structures with nonlinear media have been proposed and studied [13–15]. However, most PT-symmetric structures are based on the waveguide structure or layered structure with infinite or many period units. In fact, the simple finite resonator coupling system can also form the PT-symmetric structure [16, 17]. In [16], an optical isolator was achieved, and in [17] an enhanced sensitivity at high-order EPs was found. In [16], the resonator coupling system was composed of coupled active-passive microtoroids fabricated at the edges of silicon chips. The two toroids were coupled to tapered optical fibres. To accurately tune the coupling strengths of the toroid-toroid and tapered fibre-toroid structures, the microcavities and fibre tapers have to be all mounted on nanopositioning translation stages for precisely controlling their separation. In fact, to satisfy the exact PT-symmetric condition, the structural parameters of the two microtoroids have to be controlled to be absolutely identical, which leads to enormous experimental difficulty. Therefore, a simpler and practical experimental platform of the PT-symmetric resonator coupling system is much needed. In this article, we present the design of such a coupling system. The two resonators are composed of layered gain and loss resonators that are coupled through a metal film. Such a system may achieve a unique pole effect. The structure has the advantage over the system composed of microtoroid resonators because of its layered structure, which can be more easily achieved in an experiment.

2 Model and Theoretical Analysis

As shown in Figure 1, the designed system has a pair of coupling prisms with base angles θ. Each prism is composed of the medium with permittivity εp. The two prisms are fixed face to face with a precise interval at the optical experimental platform. The gain (G) layer and loss (L) layer are connected with a metal (M) film and placed between the two prisms. For the realisation of the gain and loss media, refer to [15]. Linearly polarised light from a super-continuum source is normally incident on one of
Prism index distribution is a conjugate symmetric function, i.e. the two resonators can also be coupled through the layer M. The length, the two layers become two resonators. The two resonators and the mode coupling, the structural parameters must be carefully optimised. For the structure shown in Figure 1, the structural parameters are taken as follows: \( n_g = 3.48 \), \( n_a = 1 \), \( n_g = 3.025 - i \gamma \), \( n_l = 3.025 + i \gamma \), \( d_a = 400 \text{ nm} \), \( d_g = d_l = 1500 \text{ nm} \), and \( d_m = 50 \text{ nm} \). Based on the Drude model, \( n_m \) is given by

\[
n_m = \sqrt{1 - \frac{\omega_{\text{ep}}^2}{\omega^2}}, \tag{5}\n\]

where \( \omega \) is the frequency of the incident wave and \( \omega_{\text{ep}} \) is the frequency of electronic plasma, which is equal to \( 1.2 \times 10^{16} \text{ s}^{-1} \) [21]. The transmittance and reflectance of the structure can be calculated through the scatter matrix (see Appendix). To excite the plasmonic resonance in the metal film, we choose the incidence of the TM waves (the

**Figure 1:** Schematic of the designed system. Based on the two symmetric coupling prisms and the metal film (black layer), the gain layer and the loss layer form the coupled resonators.

where \( \omega \) is the eigenfrequency, we have

\[
\frac{\mathbf{d}}{\mathbf{d}t} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = -i \omega \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}. \tag{2}\n\]

Combining (1) and (2) leads to the eigen matrix equation

\[
\begin{pmatrix} (\omega_0 + ig) & \kappa \\ \kappa & (\omega_0 + iy) \end{pmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \omega \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}. \tag{3}\n\]

In the PT-symmetric system, \( \gamma = -g \). By solving the algebraic equation, we obtain

\[
\omega = \omega_0 \pm \sqrt{\kappa^2 - g^2}. \tag{4}\n\]

Equation (4) determines the state of the system. If \( \kappa > g \), \( \omega \) has two real values denoting the PT-symmetric state and there is mode splitting. If \( \kappa < g \), \( \omega \) has two complex values denoting the PT-symmetry-broken state. \( \kappa = g \) means the EP at which the two eigenstates just coalesce.

If \( n_m \) has an imaginary value, the condition \( n(z) = n(\bar{z}) \) in Figure 1 is not satisfied. In this case, the metal film induces an additional perturbation and the system becomes quasi-PT-symmetric. Because the metal film is very thin, the perturbation is very small. In fact, at the condition of the quasi-PT symmetry, the structure can show a similar EP effect or pole effect. In this case, we can still study the structure with the help of PT-symmetry theory.

### 3 Numerical Results and Analysis

In this section we perform a numerical study on the optical properties of the system based on the analysis in the previous section. To excite the resonance modes of the resonators and the mode coupling, the structural parameters must be carefully optimised. For the structure shown in Figure 1, the structural parameters are taken as follows: \( n_g = 3.48 \), \( n_a = 1 \), \( n_g = 3.025 - i \gamma \), \( n_l = 3.025 + i \gamma \), \( d_a = 400 \text{ nm} \), \( d_g = d_l = 1500 \text{ nm} \), and \( d_m = 50 \text{ nm} \). Based on the Drude model, \( n_m \) is given by
electromagnetic field has only the components of $H_y$, $E_x$, and $E_z$). The system state can be verified through the conservation relations of PT-symmetric structure [2]:

$$\frac{R_f + R_b}{2} - T = 1,$$  \hspace{1cm} (6)

where $R_f$ and $R_b$ denote the reflectance for forward incidence ($+z$ direction) and backward incidence ($-z$ direction), respectively.

We first take $\tau = 0.011$ and incident angle $\theta = 30^\circ$. $\tau$ is the gain and loss efficient of layers G and L, respectively. Its increase leads to the increase of $g$ in (3). The results are shown in Figure 2. For two incidence directions, there are two transmittance and reflectance peaks with a large interval, meaning a tunnelling effect. The two peaks denote the mode-splitting. In Figure 2c, we plot the values of $\frac{R_f + R_b}{2} - T$ and find that they are all less than 1. Especially, just at the two peak positions, the curve has two dips. Thus, the current structure is on the PT-symmetric state, though all the $R$ and $T$ peak values are larger than unity. The peak over unity is called the supermode [16]. The doublet feature of each PT supermode in the spectra is a result of the mode splitting caused by the mode coupling [16]. The right peak is slightly higher than the left one. The peak values for both $R$ and $T$ are direction-independent, but the reflectance peaks have a small difference at their bases for the two opposite incidence states. The larger interval is because of the small $\tau$, which leads to a small $g$ and large positive $\kappa^2 - g^2$. Thus the difference between the two real eigenfrequencies in (4) is very large.

Interesting phenomena occur when the value of $\tau$ is increased. With $\kappa$ invariant, an increase of $\tau$ leads to a decrease of $\kappa^2 - g^2$. The 3D plots are shown in Figure 3. With the increase of $\tau$, both the transmittance and reflectance peaks become closer and higher. Before the two peaks meet, a slowly varying process occurs, and the system is always on the PT-symmetric state. However, when the two peaks meet at $\tau = 0.011799$, the two peaks merge into one with a jump of up to $10^6$. The jump in the peak value should be due to the EP effect. At the EP, the two eigenmodes of the system just coalesce. The mode of the loss resonator is totally absorbed by the gain resonator, which leads to a total resonance state of the coupling system. The total resonance state is such that the large external pumping energy is transformed into electromagnetic energy. As a result, a jump in enhancement of the peak occurs. Because EP is sensitive to the structural parameters, the $\tau$ value has to be precisely adjusted to reach the state. In order to observe the jumping process in detail, only the fourth root of $T$ and $R$ are shown in Figure 3. Furthermore, the reflectance for two opposite incidence directions has little difference. These results make the system clearly different from other PT-symmetric structures in which the reflection is totally unidirectional [4, 11].

In order to demonstrate the EP effect, we plot the values of $\frac{R_f + R_b}{2} - T$ from $\tau = 0.011795$ to $\tau = 0.01180$ in Figure 4a and the detailed reflectance peak values from $\tau = 0.011790$ to $\tau = 0.011799$ in Figure 4b. In Figure 4a, when $\tau$ is below 0.011799, all the curves of $\frac{R_f + R_b}{2} - T$ are below 1 and have an oscillation in the coupling area. Thus
\( \tau < 0.011799 \) corresponds to the PT-symmetric state. However, when \( \tau \) is up to 0.011799, the curve of \( \frac{R_f + R_b}{2} - T \) is just over 1 at the position of the right reflectance peak. Thus the conservation condition of (A.2) is broken. When \( \tau \) is increased from 0.011799 to 0.0118, the peak value of \( \frac{R_f + R_b}{2} - T \) quickly increases to 4. \( \tau = 0.011799 \) is just the threshold at which \( \frac{R_f + R_b}{2} - T \) is >1. From Figure 4b we find that, when \( \tau \) is increased from 0.0117980 to 0.0117993, the merged peak shows a jump and goes up to the maximum. Thus we deduce that the EP occurs near \( \tau = 0.0117993 \). The peak is at \( \omega = 1469.803 \) THz. To give a further demonstration, we plot the two eigenvalues \( q_1 \) and \( q_2 \) of the scatter matrix of the system in Figure 4c. First, the two eigenvalues form both unit values, denoting the PT-symmetric state. It is just at \( \omega = 1469.803 \) THz, where the two eigenvalues begin to diverge and form pairs with reciprocal moduli. The threshold frequency is just the highest peak frequency of reflectance. Thus the threshold point in Figure 4c can also be called the EP in another form through the eigen equation of the scatter matrix. In Section 2, EP was defined through the eigen matrix equation of the coupled modes. We note that the two reciprocal peaks of \( q_1 \) and \( q_2 \)
and $q_2$ are close to the EP. The two reciprocal peaks of $q_1$ and $q_2$ are defined as the poles corresponding to the CPA solutions [1]. Thus Figure 4a–c is in agreement with each other. The mode asymmetry becomes stronger near EP. The right and left peaks correspond to the gain resonator and loss resonator, respectively. During the coupling process, the left peak is absorbed by the right peak, which leads to a nonsymmetrical shape at the EP. As a result, a jump in the peak value occurs. However, when $\tau$ is $>0.011799$, e.g. it is 0.0118, the merged peak falls and recovers the symmetric shape. In this case, the system leaves the merged EP and pole.

Besides changing the gain $g$, we can also adjust the value of $\kappa$ to study the properties of the system. Here we fix $\tau = 0.0116$ and increase the incidence angle from $\theta = 31.3^\circ$. The results are shown in Figure 5. It shows that with the increase of $\theta$, the two split modes also merge. Thus, the increase of $\theta$ will lead to the decrease of $\kappa$ and $\kappa^2 - g^2$. EP occurs at $\theta = 31.353^\circ$, at which the two split modes coalesce with the maximum mode peak value. The corresponding reflectance spectrum and the two eigenvalues of the scatter matrix are shown in Figure 6. An important behaviour can be found, that is, the poles just occur at the threshold value and the peak value just occurs at the merging point of the EP and the poles.

Although the configuration in this study is different from the study in [16], both studies deal with the coupling of gain and loss resonators. Thus we find a similar evolution process in which the system changes from the unbroken state to the broken state, as well as mode splitting and merging. However, the jump in the transmittance and reflectance at the EP is not found in [16]. The unique EP effect in our study may be due to perturbation by the metal film. In the light of physics, the imaginary value of $n_{\text{in}}$ induces a plasmonic resonance, which leads to a large coupling of the two resonators in the EP state. The plasmonic-resonance-induced coupling leads to a strong energy transformation. Away from the EP, the perfect coupling condition disappears and the mode gain decreases. The pole phenomenon has been studied previously [1, 2]. The merging of the EP and pole is a unique behaviour in this structure. Such a result is also induced by the plasmonic resonance effect of the metal layer and is different from that in [1], in which the pole occurs in the PT-symmetry-breaking range. Thus the unique merging of the EP and pole in the current system makes the EP to have an extraordinary amplification tunnelling effect.

As is known, the plasmonic resonance results from the TM waves. In order to study the coupling effect without the plasmonic resonance, we have carried out the same study under the same conditions but with the incidence of TE wave (the electromagnetic field has only the components of $E_y$, $H_x$, and $H_z$). Using all the previous values of $\tau$, we only obtain a single reflectance peak with values just larger than unity and near-zero transmittance. Such results indicate that without the plasmonic resonance the coupling efficient $\kappa$ is very small so that no mode splitting occurs. Thus we choose $\tau = 0.0011$ and $\theta = 30^\circ$ to calculate the forward reflectance and transmittance. The value of $\tau$ has been decreased to 1/10 of its previous value. The results are shown in Figure 7. As a result, $\kappa^2 - g^2$ in (6) is larger than zero, which leads to the mode splitting in Figure 7. We find only at the position of dips or peaks $T + R$ is a little larger than 1, and at other frequencies $T = 0$, $R = 1$. As $\tau$ increases, we find similar mode-merging behaviours and large peak values. The forward reflectances with different $\tau$ values are shown in Figure 8. We find that the EP should occur at $\tau = 0.00853$, at which the two split modes just merge into one. But the maximum

Figure 5: Forward reflectance with different $\theta$ values and $\tau = 0.0116$.

Figure 6: Two eigenvalues and forward reflectance spectrum with $\theta = 31.353^\circ$ and $\tau = 0.0116$.
special structure and incidence conditions, the system can excite unique merging and splitting of the EP and pole. At the merged point or the pole, the system can achieve the same large transmittance and reflectance. The unique EP or pole effect in this study may find important applications in many fields. Because of the jump at the merged point or the pole, the system will be extraordinarily sensitive to the perturbation of the environmental factor, e.g. the refractive index of air layer. Thus it can be used as sensor for poisonous gases in an atmospheric monitor. In addition, the large transmittance or reflectance may excite a large electric field, which may be used in nonlinear optics. In all, the designed structure in this article will provide a simple and practical platform to carry out research in PT-symmetric optics.

Appendix: Determination of Transmittance and Reflectance from the Scatter Matrix

In Figure 10, \( a_i \) and \( b_i \) are the amplitudes of forward and backward waves in an arbitrary layer \( l \), respectively. For the two neighbouring layers \( l' \) and \( l \), the four wave amplitudes \( a_l, b_l, a_{l'}, \) and \( b_{l'} \) are related by a scatter matrix \( S(l', l) \):

\[
\begin{bmatrix}
  a_l \\
  b_l
\end{bmatrix} = S(l', l) \begin{bmatrix}
  a_{l'} \\
  b_{l'}
\end{bmatrix}.
\]  

(A.1)

By analogy, the four wave amplitudes in layers \( l' \) and \( l + 1 \) can be written as

\[
\begin{bmatrix}
  a_{l+1} \\
  b_{l+1}
\end{bmatrix} = S(l', l + 1) \begin{bmatrix}
  a_{l'} \\
  b_{l+1}
\end{bmatrix}.
\]  

(A.2)

4 Conclusions

In conclusion, we have presented a theoretical and numerical study of two designed coupled resonators. Given the peak value occurs at \( \tau = 0.008533 \), which is not at the EP. The result is different from that of the TM waves. To find the reason, we plot the two eigenvalues of the scatter matrix, together with the maximum reflection peak, in Figure 9. It is clear that the maximum reflection peak also occurs at the poles, but the poles have left the EP. The transmission and reflectance peaks are at the same position. Therefore, without plasmonic resonance, the maximum reflectance or transmittance occurs not at the EP but at the pole. Also, to achieve the maximum reflectance or transmittance, TE waves needs a smaller \( \tau \), i.e. a smaller pumping energy. Such pole-induced reflection or transmission peaks are also found to be direction-independent. For brevity, we do not show the results.

Figure 7: Spectra of reflectance and transmittance for TE waves with \( \tau = 0.0011 \).

Figure 8: Spectra of reflectance for TE waves with different values of \( \tau \).

Figure 9: Two eigenvalues of the scatter matrix together with the maximum reflection peak for TE waves with \( \tau = 0.008533 \).

Figure 10: Special structure and incidence conditions, the system can excite unique merging and splitting of the EP and pole.
We label a layer with an integer \( n \). For a structure with \( N \) layers, the initial scatter matrix is \( S(0, 0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). Based on (A.4)–(A.7), we can deduce the total scatter matrix \( S_0(0, N) \), and the transmittance \( T \) and reflectance \( R \) are

\[
T = S_{11}(0, N)^2, \quad (A.8)
\]
\[
R = S_{22}(0, N)^2. \quad (A.9)
\]

### References