Viscous Dissipation Effect on a Steady Generalised Couette Flow of Heat-Generating/Absorbing Fluid in a Vertical Channel

Abstract: This article investigates the viscous dissipation effect on steady generalised Couette flow of heat-generating/absorbing fluid in a vertical channel. Equations of energy and momentum are obtained and solved using the homotopy perturbation method. The influences of the dimensionless flow parameter have been plotted graphically and discussed for varying values of the controlling parameters. During the course of computation, it is found that fluid temperature and velocity increase with an increase in viscous dissipation and also seen that growing mixed convection parameter $\text{Gre}$ leads to a corresponding rise in temperature and velocity. It is further discovered that heat absorption leads to increase in the heat transfer on the heated wall. Finally, it is concluded that heat generation contributes to increase the mixed convection, hence, it requires decrease in mixed convection parameter to bring about a reverse flow near the stationary plate.

Keywords: Couette Flow; Heat Generation/Absorption; Homotopy Perturbation; Mixed Convection; Viscous Dissipation.

1 Introduction

The study of viscous dissipation effect on a steady generalised Couette flow of heat-generating/absorbing fluid in a vertical channel has received a great interest due to its applications in engineering field, geophysics, and environmental problems. Mohamed [1] investigated mixed convection flow of a micropolar fluid from an unsteady stretching surface with viscous dissipation. He discovered that the gradient of wall temperature is increased for $(Ec < 0)$ but is decreased for $(Ec > 0)$, and also the dimensionless temperature increases when the fluid is being heated $(Ec > 0)$ but decreases when the fluid is being cooled $(Ec < 0)$. Mohamed [2] also studied unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. His study revealed that the effect of the mixed convection parameter on the velocity and temperature in the steady flow is more prominent than the unsteady flow. His work further concluded that the velocity profile as well as the temperature profile increase with an increase in Eckert number. Dulal and Hiranmoy [3] studied the effects of temperature-dependent viscosity and variable thermal conductivity on magnetohydrodynamics non-Darcy mixed convective diffusion of species over a stretching sheet. They reported that the velocity profile increases with increasing mixed convection parameter, while the temperature profile decreases with increase in mixed convection parameter due to the fact that the thermal boundary layer thickness decreases with increase in the mixed convection parameter. Srinivasacharya et al. [4] examined non-Darcy mixed convection flow past a vertical porous plate with Joule heating, Hall, and ion-slip effects. They discovered that an increase in Eckert number slightly enhanced the skin friction coefficient. In nature and physical phenomenon, when a fluid flows, there exists a collision viscosity between the fluid particles, which generates internal mechanical energy called viscous dissipation. This viscous dissipation cannot be neglected in a fluid of high gravitational forces or fluid of high Prandtl number. In lubrication industries, viscous dissipation is inevitable, as the fluid temperature and motion are affected by the fluid particles interaction. Gebhart [5] was the first author to investigate the effect of viscous dissipation in natural convection. Later on, researches were carried out to investigate the influence of the viscous dissipation due to its applications in lubrication industries, gas turbines, petroleum industries, fossil fuel combustion, etc. Mohammed et al. [6] studied viscous dissipation effect on the mixed convection boundary layer flow towards solid sphere. They found that the presence of viscous dissipation gives small increment in skin friction coefficient from the middle until end of sphere. Jha and Ajibade [7] investigated the effect of viscous dissipation on natural convection flow between vertical parallel plates with
time-periodic boundary condition. The outcome of their study showed that the fluid temperature increases as a result of increasing dissipation heating within the channel. Ameer Ahmad and Abdulgaphur [8] investigated viscous dissipation on free convective in a vertical annular cylinder embedded with porous medium. They reported that as the viscous dissipation is increased, the isothermal lines are moving towards the cold surface from the hot surface, which increases the generation of heat in the fluid. Ajibade and Taofida [9] considered viscous dissipation effect on steady natural convection Couette flow of heat-generating fluid in a vertical channel. They concluded that fluid temperature and velocity increase with the increase in viscous dissipation. Fahad et al. [10] analysed combined effect of viscous dissipation and radiation on unsteady free-convective non-Newtonian fluid along a continuously moving vertically stretched surface with no-slip phenomena. They reported that fluid velocity enhances with an increasing viscous dissipation parameter Ec. Gireesha and Manjunatha [11] studied the effects of variable viscosity and thermal conductivity on magnetohydrodynamics flow and heat transfer of a dusty fluid. They reported that as the viscous dissipation increases, fluid temperature increases. Hunegnaw and Kisham [12] studied magnetohydrodynamics effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation, and heat source/sink. They reported that an increase in viscous dissipation enhances the temperature profiles. Kabir et al. [13] studied the effect of viscous dissipation on magnetohydrodynamics natural convection flow along a vertical wavy surface with heat generation. They concluded their work that as the dissipation increases, the temperature profile increases. Shashidar [14] analysed the effect of viscous dissipation on slip boundary layer flow of non-Newtonian fluid over a flat plate with convective thermal boundary condition. He discovered that temperature profile increases with the increase of Eckert number in both pseudo-plastic and Newtonian. Machireddy et al. [15] studied the effect of viscous dissipation and heat source on an unsteady magnetohydrodynamics flow over a stretching sheet. They concluded that an increase in viscous dissipation is to increase the temperature distribution. Raihanul Haque et al. [16] examined the effects of viscous dissipation on natural convection flow over a sphere with temperature-dependent thermal conductivity in presence of heat generation. The outcome of their findings showed that temperature and velocity within the boundary layer increase for increasing values of Eckert number. Singh [17] studied the effect of free convection in unsteady Couette motion between two vertical parallel plates. Jha and Ajibade [18] investigated unsteady free-convective Couette flow of heat-generating/absorbing fluid. They reported that an increase in heat absorption increases the rate of heat transfer on the moving plate and decreases the rate of heat transfer on the stationary plate. In another article, Jha and Ajibade [19] studied time-dependent natural convection Couette flow of heat-generating/absorbing fluid. They concluded that an increase in heat absorption parameter enhances the rate of heat transfer on the accelerated plate and decelerates the rate of heat transfer on the stationary plate. Jha and Ajibade [20] studied unsteady free-convective Couette flow having isothermal and adiabatic boundaries. They discovered that the fluid temperature is the same as that of the isothermal moving plate at the steady state, and thus there is no heat transfer between fluids. Jha et al. [21] studied unsteady/steady free-convective Couette flow of reactive viscous fluid in a vertical channel formed by two vertical porous plates. They indicated that increasing the reactant consumption parameter increases the heat transfer on the porous plate. Recently, Mahanthesh et al. [22] considered Marangoni convection in Casson liquid flow due to an infinite disk with exponential space-dependent heat source and cross-diffusion effects. They concluded that viscous and Joule heating aspects showed an enhancement in fluid temperature. Mahanthesh and Gireesha [23] studied scrutinisation of thermal radiation, viscous dissipation, and Joule heating effects on Marangoni convective two-phase flow of Casson fluid with fluid-particle suspension. They discovered that the radiative heat, viscous dissipation, and Joule heating aspects are constructive for the growth of thicknesses of the thermal boundary layer of both Casson fluid and dust particle phases. In another article, Mahanthesh and Gireesha [24] considered thermal Marangoni convection in two-phase flow of dusty Casson fluid. Gireesha et al. [25] investigated nonlinear gravitational and radiation aspects in nanoliquid with exponential space-dependent heat source and variable viscosity. They reported that the heat transfer rate is higher in case of changeable viscosity than constant viscosity. Mahanthesh et al. [26] studied Marangoni convection radiative flow of dusty nanoliquid with exponential space-dependent heat source. The outcome of their study showed that the temperature is higher due to larger heat source. Mahanthesh et al. [27] considered numerical solutions for magnetohydrodynamics flow of nanofluid over a bidirectional nonlinear stretching surface with prescribed surface heat flux boundary.

The objective of this study is to investigate the viscous dissipation effect on steady generalised Couette flow of heat-generating/absorbing fluid in a vertical channel. The equations governing the flow are coupled and have some nonlinear terms arising from the viscous dissipation.
so that obtaining closed form solution is a daunting task. Such problems can therefore be approached by numerical schemes or some approximate solution methods. One of the efficient methods is the perturbation method. However, solutions obtained by perturbation method are restricted to small perturbation parameters; therefore, to overcome this restriction, an alternative method called homotopy perturbation method (HPM) was proposed.

Homotopy perturbation method was introduced by He [28] to solve linear, nonlinear, and coupled problems in partial or ordinary form. He [29, 30] introduced the new method to solve nonlinear and boundary value problems. He [31] examined HPM with two expanding parameters. He reported that the method is effective, and it can be extended to various nonlinear problems with multiple nonlinear terms and further discovered that the solution procedure can be used as a paradigm for many other applications. He [32, 33] established the HPM for solving nonlinear initial and boundary value problems by combining the standard homotopy in topology and the perturbation technique. By this method, a speedy convergent series solution can be obtained in most of the cases. Normally, a few terms of the series solution can be used for numerical calculations. Hossein et al. [34] investigated the application of HPM for solving Gas Dynamics Equation. They concluded that HPM is a powerful and efficient technique in finding exact and approximate solutions for nonlinear differential equations. The method converges as shown by Jafar and Hossein [35], Syed and Muhammad [36] used HPM for solving partial differential equation. They reported that HPM is very powerful and efficient in finding the analytical solutions for a wide class of boundary value problems. Wu and He [37] considered HPM for nonlinear oscillators with coordinate-dependent mass. Liu et al. [38] considered hybridisation of HPM and Laplace transformation for the partial differential equations. The outcome of their study discovered that the result indicates superiority of the method over the conventional method due to its flexibility in choosing its initial approximation. Abou-Zeid [39] studied HPM for MHD non-Newtonian nanofluid flow through a porous medium in eccentric annuli with peristalsis. He discovered that the temperature increases with the increase of Brinkman number. Adamu [40] investigated parametrised HPM. He reported that the new technique is proved to be powerful and efficient. He [41] studied new interpretation of HPM. Due to the nonlinearity and coupling of the governing equations in the present situation, the HPM shall be engaged to obtain the solutions of the momentum and energy equations.

2 Mathematical Analysis

The present problem considers a steady mixed convection flow of viscous incompressible fluid in a vertical channel formed by two infinite parallel plates. The \( x^* \) axis is taken vertically parallel to one of the plates of the channel and normal to the \( y^* \) axis. The plate \((y = 0)\) moves impulsively with constant velocity \( U \), while the other plate is at rest. The fluid flow is set up due to the applied pressure gradient as well as density change caused by the asymmetric heating of the channel boundary plates and under the action of gravitational force. Hence, the present situation describes a mixed convection Couette flow in a vertical channel. The flow configuration and coordinates system are shown in Figure 1.

By the use of Boussinesq’s approximation, the governing dimensional equations of the continuity, momentum, and energy are as follows:

\[
\frac{du^*}{dx^*} = 0, \quad (1)
\]

\[
\alpha \frac{d^2 T^*}{dy^*^2} - \frac{Q_0}{\rho Cp}(T^* - T_0) + \frac{\mu^*}{\rho Cp} \left( \frac{du^*}{dy^*} \right)^2 = 0, \quad (2)
\]

\[
\sqrt{\frac{d^2 u^*}{dy^*^2} + g\beta(T^* - T_0)} - \frac{1}{\rho} \frac{dp^*}{dx^*} = 0. \quad (3)
\]

where \( y^* \) and \( x^* \) are the dimensional distances along and perpendicular to the plate. \( u^* \) and \( T^* \) are the dimensional velocity and temperature. \( Q_0, \mu, k, \rho, Cp, \beta, \) and \( g \) are the dimensional heat generation/absorption coefficient,

![Figure 1: Schematic diagram of the problem.](image-url)
The dimensionless form for the physical system considered is the advection/diffusion parameter. The boundary conditions in (13) are thermal conductivity, heat generation/absorption, and viscous dissipation effect of the fluid. Also, the first, second, and third terms of (3) are the viscosity, thermal buoyancy, and pressure effect of the fluid. We assume that the appropriate boundary conditions of the model are as follows:

\[ u^* = U, \quad T^* = T_w \quad \text{at} \quad y^* = 0, \]
\[ u^* = 0, \quad T^* = T_0 \quad \text{at} \quad y^* = h. \] (4)

Here \( U, T_w, \) and \( T_0 \) are the velocity of the moving plate and temperature of the heated and cold plate, respectively. Below are the dimensionless quantities used:

\[ y^* = \frac{y}{h}, \quad u^* = \frac{u}{U}, \quad T^* = \frac{T - T_0}{T_w - T_0}, \quad S = \frac{Q_0 h^2}{k}, \]
\[ Ec = \frac{U^2}{C_p(T_w - T_0)}, \quad Pr = \frac{\mu C_p}{k}, \quad Gr = \frac{g \beta h^3(T_w - T_0)}{\nu^2}, \quad Re = \frac{U h}{\nu}, \]
\[ x^* = x^* U h^2, \quad P = \frac{P^*}{\rho U^2}. \] (5)

Substituting (5) into (1) to (3), the continuity, momentum, and energy equations can be written in dimensionless form as

\[ \frac{du}{dx} = 0, \] (6)
\[ \frac{d^2 T}{dy^2} + Ec Pr \left( \frac{du}{dy} \right)^2 - ST = 0, \] (7)
\[ \frac{d^2 u}{dy^2} + Gre T \frac{dP}{dx} = 0, \] (8)

where \( Gre \) is the mixed convection parameter, \( Pr \) is the Prandtl number, \( Ec \) is Eckert number, and \( S \) is heat generation/absorption parameter. The boundary conditions in the dimensionless form for the physical system considered are as follows:

\[ u = 1, \quad T = 1 \quad \text{at} \quad y = 0, \]
\[ u = 0, \quad T = 0 \quad \text{at} \quad y = 1. \] (9)

### 2.1 Homotopy Perturbation Method

In order to illustrate the basic ideas of the HPM, we consider the following nonlinear differential equation:

\[ A(u) - f(r) = 0, \quad r \in \Omega, \] (10)

with the boundary conditions

\[ B \left( u, \frac{du}{dn} \right) = 0, \quad r \in \Gamma, \] (11)

where \( A \) is a general differential operator, \( B \) is a boundary operator, \( f(r) \) is known analytical function, and \( \Gamma \) is the boundary of the domain \( \Omega \), respectively. Generally speaking, the operator \( A \) can be divided into two parts, which are \( L \) and \( N \), where \( L \) is linear part and \( N \) is nonlinear part. Therefore, (10) can be written as

\[ L(u) + N(u) - f(r) = 0, \quad r \in \Omega, \] (12)

by the homotopy techniques, we construct a homotopy as follows:

\[ \nu(r, p) : \Omega \times [0, 1] \rightarrow R, \text{ which satisfies:} \]
\[ H(v, p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0. \] (13)

In (13), \( p \in [0, 1] \) is an embedding parameter, while \( u_0 \) is an initial approximation of (10), which satisfies the boundary conditions. Clearly from (13), we have

\[ H(v, 0) = L(v) - L(u_0) = 0, \] (14)
\[ H(v, 1) = A(v) - f(r) = 0. \] (15)

We can assume that the solution of (13) can be written as a power series in \( p \):

\[ v = v_0 + pv_1 + p^2v_2 + ..., \] (16)

setting \( p = 1 \) gives the approximate solution of (10) as

\[ u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + .... \] (17)

Applying the homotopy perturbation technique to solve the governing equations in the present problem, we construct a convex homotopy on (7) and (8) to get

\[ H(T, p) = (1-p) \left( \frac{d^2 T}{dy^2} \right) \]
\[ + p \left( \frac{d^2 T}{dy^2} + Ec Pr \left( \frac{du}{dy} \right)^2 - ST \right) = 0, \] (18)
Substituting (22) into (20) and (21) and simplifying, we have the following:

$$\frac{d^2 T_0}{dy^2} + p \left( EcPr \left( \frac{du_0}{dy} \right)^2 - ST \right) = 0,$$

$$\frac{d^2 u_0}{dy^2} + p \left( GreT - \frac{dp}{dx} \right) = 0. \tag{21}$$

Assume the solutions of (7) and (8) to be written as

$$T = T_0 + pT_1 + p^2 T_2 + \ldots,$$

$$u = u_0 + pu_1 + p^2 u_2 + \ldots. \tag{22}$$

Substituting (22) into (20) and (21) and simplifying, we have the following:

$$\frac{d^2 T_0}{dy^2} + p \frac{d^2 T_1}{dy^2} + p^2 \frac{d^2 T_2}{dy^2} + p^3 \frac{d^2 T_3}{dy^2} + \ldots$$

$$= p \left( EcPr \left( \frac{du_0}{dy} \right)^2 \right)$$

$$- p^2 \left( 2 EcPr \left( \frac{du_0}{dy} \cdot \frac{du_1}{dy} \right) \right)$$

$$- p^3 \left( 2 EcPr \left( \frac{du_0}{dy} \cdot \frac{du_2}{dy} - \left( \frac{du_1}{dy} \right)^2 \right) \right) - \ldots$$

$$+ pST_0 + p^2 ST_1 + p^3 ST_2 + \ldots, \tag{23}$$

$$\frac{d^2 u_0}{dy^2} + p \frac{d^2 u_1}{dy^2} + p^2 \frac{d^2 u_2}{dy^2} + p^3 \frac{d^2 u_3}{dy^2} + \ldots$$

$$= -pGreT_0 - p^2 GreT_1 - p^3 GreT_2 - \ldots. \tag{24}$$

By comparing the coefficient of $p^0$, $p^1$, $p^2$, and $p^3$ of (23) and (24), we have

$$p^0 : \frac{d^2 T_0}{dy^2} = 0, \tag{25}$$

$$p^0 : \frac{d^2 u_0}{dy^2} = 0, \tag{26}$$

$$p^1 : \frac{d^2 T_1}{dy^2} = -EcPr \left( \frac{du_0}{dy} \right)^2 + ST_0, \tag{27}$$

$$p^1 : \frac{d^2 u_1}{dy^2} = \frac{dp}{dx} - GreT_0, \tag{28}$$

$$p^2 : \frac{d^2 T_2}{dy^2} = -2EcPr \frac{du_0}{dy} \cdot \frac{du_1}{dy} + ST_1, \tag{29}$$

$$p^2 : \frac{d^2 u_2}{dy^2} = -GreT_1, \tag{30}$$

$$p^3 : \frac{d^2 T_3}{dy^2} = -2EcPr \frac{du_0}{dy} \cdot \frac{du_2}{dy} - EcPr \left( \frac{du_1}{dy} \right)^2 + ST_2, \tag{31}$$

$$p^3 : \frac{d^2 u_3}{dy^2} = -GreT_2. \tag{32}$$

The boundary conditions (9) are transformed also as

$$T_0(0) = 1, \quad T_1(0) = T_2(0) = T_3(0) = \ldots = 0,$$

$$u_0(0) = 1, \quad u_1(0) = u_2(0) = u_3(0) = \ldots = 0,$$

$$u_0(1) = u_1(1) = u_2(1) = \ldots = 0. \tag{33}$$

As the zeroth order of the homotopy gives a linear ordinary differential equation, it is easily solvable without making recourse to initial guess. Therefore, solving (25) and (26) and applying the boundary conditions $T_0(0) = 1$ and $T_0(1) = 0$, we obtain (34) and (35) as

$$T_0 = A_1 y + A_2, \tag{34}$$

$$u_0 = B_1 y + B_2. \tag{35}$$

Solving (27) and (28) and applying the boundary conditions $T_1(0) = 0$ and $T_1(1) = 0$, we obtain (36) and (37) as

$$T_1 = S \left( \frac{y^2}{2} - \frac{y^3}{6} \right) - \frac{EcPr y^2}{2} + A_3 y + A_4, \tag{36}$$

$$u_1 = \frac{y^2}{2} \frac{dP}{dx} - Gre \left( \frac{y^2}{2} - \frac{y^3}{6} \right) + B_3 y + B_4. \tag{37}$$

Solving (29) and (30) and applying the boundary conditions $T_2(0) = 0$ and $T_2(1) = 0$, we obtain (38) and (39) as

$$T_2 = S \left( \frac{y^5}{24} - \frac{y^{10}}{120} \right) - \frac{EcPrSy^5}{24} + \frac{EcPrSy^3}{12} - \frac{S^2 y^3}{18}$$

$$+ \frac{EcPr^2 \frac{dP}{dx}}{3} - 2EcPrGre \left( \frac{y^3}{6} - \frac{y^4}{24} \right)$$

$$+ EcPrGrey^2 - \frac{EcPr^2 \frac{dP}{dx}}{2} + A_5 y + A_6, \tag{38}$$

$$u_2 = -GreS \left( \frac{y^5}{24} - \frac{y^{10}}{120} \right) + \frac{EcPrGrey^3}{24}$$

$$- \frac{EcPrGrey^3}{12} + \frac{GreSy^3}{18} + B_5 y + B_6. \tag{39}$$
Solving (31) and (32) and applying the boundary conditions \( T_2(0) = 0 \) and \( T_2(1) = 0, u_2(0) = 0 \) and \( u_2(1) = 0 \), we obtain (40) and (41) as

\[
T_3 = S^3 \left( \frac{y^6}{720} - \frac{y^7}{5040} \right) - \frac{EcPrS^2y^6}{720} + \frac{EcPrS^2y^5}{240}
+ \frac{EcPrS^5y^6}{60} - 4EcPrGreS \left( \frac{y^5}{120} - \frac{y^6}{720} \right)
+ \left( \frac{2EcPrGreS}{3} - EcPrS\frac{dP}{dx} \right) \frac{y^4}{24}
+ \left( \frac{S^3}{45} - \frac{EcPrGreS}{24} - EcPrGreS + EcPrS\frac{dP}{dx} \right) \frac{y^3}{6}
- \frac{S^3y^5}{360} - 2EcPrGreS \left( \frac{y^5}{120} - \frac{y^6}{720} \right)
- \left( \frac{Ec^2Pr^2Gre^2}{2} - \frac{EcPrGreS^2}{3} \right) \frac{y^4}{12}
+ \frac{Ec^2Pr^2Grey^2}{24} - \frac{EcPrGreS^2}{45} + \frac{EcPrer^4}{12} \left( \frac{dP}{dx} \right)^2
+ \frac{Ec^2Pr^2Grey^5}{60} + 2EcPrGre \left( \frac{y^4}{12} - \frac{y^5}{120} \right)
- \frac{EcPrGrey}{9} \frac{dP}{dx} - \frac{EcPrGrey^2}{6} \frac{dP}{dx}
- \frac{EcPrGre}{12} \left( \frac{y^5}{12} - \frac{y^6}{120} \right)
- \frac{EcPrGre}{y^4} \left( \frac{y^6}{24} + \frac{y^5}{120} \right)
- \frac{EcPrS^2}{8} \left( \frac{dP}{dx} \right)^2 + \frac{EcPrGreS^2}{18} + A_7y + A_8,
\]

and also

\[
u_3 = -GreS^2 \left[ \frac{y^6}{720} - \frac{y^7}{5040} \right] + \frac{EcPrGreSy^6}{720}
- \frac{EcPrGreS^5}{240} + \frac{GreS^2y^5}{360} - \frac{EcPrGrey^3y^4}{60}
+ 2EcPrGre \left[ \frac{y^5}{120} - \frac{y^6}{720} \right] - \frac{EcPrGrey^3y^4}{36}
+ \frac{EcPrGrey}{24} \frac{dP}{dx} - \frac{GreS^2y^3}{270} - \frac{EcPrGrey^3}{36} \frac{dP}{dx}
+ \frac{EcPrGreyS^3}{144} + \frac{EcPrGreS^2y^3}{72} + B_7y + B_8.
\]

Equations (34) to (41) give the approximate solutions for temperature and velocity as

\[
T = T_0 + T_1 + T_2 + T_3 + \ldots,
\]

where

\[
A_1 = B_1 = -1,
A_2 = B_2 = 1,
A_3 = \frac{EcPr}{2} - \frac{S}{3},
B_3 = \frac{Gre}{3} - \frac{dP}{2dx},
A_4 = B_4 = A_6 = B_6 = A_8 = B_8 = 0,
A_5 = \frac{S^2}{45} - \frac{EcPrS}{24} + \frac{EcPr dP}{6 dx} - \frac{EcPrGre}{12},
B_5 = \frac{EcPrGre}{24} - \frac{GreS}{45},
A_7 = \frac{EcPrS^2}{240} - \frac{S^3}{945} - \frac{EcPrS}{360} \frac{dP}{dx} + \frac{EcPrGreS}{120}
- \frac{Ec^2Pr^2Gre^2}{60} + \frac{EcPr}{24} \left( \frac{dP}{dx} \right)^2 - \frac{17EcPrGre}{360} \frac{dP}{dx}
+ \frac{EcPrGre^2}{72},
B_7 = \frac{2GreS^2}{945} - \frac{EcPrGreS}{240} + \frac{EcPrGre dP}{360} \frac{dx}{dx}.
\]

To obtain the Nusselt number and skin friction at the surfaces of the channel boundaries, the expressions for temperature and velocity are differentiated with respect to \( y \), that is

\[
Nu = \frac{dT}{dy} \bigg|_{y=0,y=1}
\]

\[
Nu_0 = -1 + \frac{EcPr}{2} - \frac{S}{3} + \frac{S^2}{45} \frac{EcPrS}{24}
+ \frac{EcPr dP}{6 dx} - \frac{EcPrGre}{12},
\]

\[
Nu_1 = -1 + \frac{S}{6} - \frac{EcPr}{2} - \frac{7S^2}{360} - \frac{EcPrS}{24}
+ \frac{EcPr dP}{6 dx} - \frac{EcPrGre}{12}.
\]

Also, the skin friction at the boundary surfaces is \( \tau = \frac{du}{dy} \bigg|_{y=0,y=1} \), so that
3 Results and Discussion

The present work investigates viscous dissipation effect on steady generalised Couette flow of heat-generating/absorbing fluid in a vertical channel. The temperature and velocity fields are presented graphically in Figures 2–9 for various values of heat generation/absorption parameter $S$, mixed convection parameter $Gre$, viscous dissipation $Ec$, Prandtl number $Pr$, the constant mass flux $q$, and also the critical values of $Gre$, which signal the onset of reverse flow near the stationary plate, the skin friction $\tau$, rate of heat transfer $Nu$, pressure gradient $\frac{dP}{dx}$, and mean temperature $\theta_m$, which are presented in Tables 1 to 4. For the purpose of this discussion, the value of $S$ is chosen from $-2.0$ to $2.0$ to account for heat generation ($S < 0$) with a corresponding heat absorption

\[
\frac{\tau_0}{EcPrGreS} = \frac{1}{240} \frac{dP}{dx} + \frac{q}{24} - \frac{Gre}{45} + \frac{2Gre^2}{945}
\]

(46)

\[
\tau_1 = -\frac{1}{6} - \frac{Gre}{2} + \frac{1}{2} \frac{dP}{dx} + \frac{7GreS}{360} - \frac{EcPrGre}{24}
\]

(47)

\[
Q = \frac{1}{2} - \frac{1}{12} \frac{dP}{dx} + \frac{Gre}{24} + \frac{GreS}{240} + \frac{EcPrGre}{120} + \frac{17Gre^2}{40320}
\]

(48)

and mean temperature $\theta_m$, we have

\[
\theta_m = \frac{\int_0^1 u(y) dy}{\int_0^1 dy}
\]

(49)

2.2 Pressure Gradient

In order to estimate the pressure gradient in the present problem, we assumed that the flow has a constant mass flux, so that $q$ is obtained such that the mass flux within the channel is maintained at a constant level. That is

\[
\int_0^1 u(y) dy = q,
\]

(50)

where $q$ is the mass flux constant. Then, the pressure gradient can be expressed as

\[
\frac{dP}{dx} = -6 - \frac{Gre}{2} + \frac{GreS}{20} + \frac{EcPrGre}{10} + \frac{17Gre^2}{3360}
\]

\[
-\frac{17EcPrGreS}{1680} + \frac{EcPrGre}{280}.
\]

(51)

The limiting value of $Gre$ for a flow reversal can be found by setting

\[
\frac{du}{dy} \bigg|_{y=1} = 0.
\]

(52)

Applying this condition to (43), we obtain

\[
Gre_c = \frac{17\sqrt{Pr}}{A_0},
\]

(53)

where

\[
A_0 = \frac{EcPr}{90} - \frac{EcPrS}{2000} + \frac{29Ec^2Pr^2}{100800} + \frac{17EcPrS^2}{1209600} - \frac{17Ec^2Pr^2S}{604800}.
\]

\[
\text{Figure 2: Temperature profile for different values of } Gre (Pr = 0.71, Ec = 0.6, S = 2.0, q = 1.0).
\]

\[
\text{Figure 3: Velocity profile for different values of } Gre (Pr = 0.71, Ec = 0.6, S = 2.0, q = 1.0).
\]
Figure 4: Temperature profile for different values of S (Pr = 0.71, Ec = 0.6, Gre = 10.0, q = 1.0).

Figure 5: Velocity profile for different values of S (Pr = 0.71, Ec = 0.6, Gre = 10.0, q = 1.0).

Figure 6: Temperature profile for different values of Ec (Pr = 0.71, S = 2.0, Gre = 10.0, q = 1.0).

Figure 7: Velocity profile for different values of Ec (Pr = 0.71, S = 2.0, Gre = 10.0, q = 1.0).

(S > 0). Similarly, the value of Gre is selected arbitrarily between 1.0 and 20.0. The parameters of interest are arbitrarily chosen between 0.1 ≤ Ec ≤ 2.0, q = 1.0; the value of Pr is considered to be 0.71 for air, which is inversely proportional to the thermal diffusivity of the working fluid.

Figures 2 and 3 show the temperature and velocity distributions within the channel for varying values of the mixed convection parameter. It can be seen that growing Gre leads to a corresponding rise in temperature and velocity. This is physically true as higher value of Gre implies an improved buoyancy force, which is higher when compared to the viscous force. Moreover, fluid temperature increases with growing Gre. This could be attributed to increase in buoyancy force, which causes an increase in fluid velocity and consequently enhanced viscous dissipation that grows the temperature within the channel.

Figures 4 and 5 present the effect of heat generation/absorption S on temperature and velocity profiles. It is clearly seen from the Figure 4 that fluid temperature increases as the heat generation (S < 0) increases, while it decreases with the increase in heat absorption (S > 0). Similarly, fluid velocity increases near the heated plate as a result of increase of heat generation parameter, while the reverse trend is observed on the cold plate. The observed thermodynamic characteristics are traceable to the physical fact that growing heat generation allows the fluid to amplify the applied heat through the boundary as well as the viscous dissipation heating causing the temperature to grow within the channel. The influence of temperature increase is the strengthening of the convection current,
which has the capacity to increase the fluid velocity. However, the pressure gradient application is one that maintains a constant flow rate so that velocity is conditioned to decrease near the cold plate.

Figures 6 and 7 display the response of temperature and velocity profiles for variation in the viscous dissipation effects. It is observed that fluid temperature and velocity increase with an increase in Eckert number $Ec$. More viscous dissipative heat causes rise in the temperature as well as velocity, respectively. Consequently, the thermal boundary layer thickness is much pronounced in the presence of viscous dissipation.

Figures 8 and 9 exhibit the effect of Prandtl number on the fluid temperature and velocity. The temperature profile increases with the increase in Prandtl number $Pr$. On the other hand, the velocity profile increases with increasing Prandtl number $Pr$ on the heated plate, while it decreases towards the cold plate with growing Prandtl number. This is physically true because an increase in $Pr$ decreases the thermal diffusivity of the working fluid; therefore, heat penetration from the cold plate decreases, which leads to a thinning of the thermal boundary layer. However, the different response of velocity to variations in Prandtl number $Pr$ is attributable to the pressure gradient effect, which is dependent on constant mass flux.

Table 1 presents the critical values of $Gre$, which signals the onset of reverse flow near the stationary plate. It is observed that an increase of viscous dissipation and heat generation decrease the critical value of $Gre$ for which reverse flow sets in near the plate ($y = 1$). On the other hand, the table further shows that mixed convection parameter $Gre$ increases due to growing heat absorption. That is, it requires a reduced $Gre$ to bring about a reverse flow near the plate whenever the buoyancy is enhanced. In addition, an increase in the mixed convection parameter increases the reverse flow region and the critical value of the mixed convection parameter leading to the flow reversal.

Table 2 reveals the skin friction on both plates. It is observed that an increase in viscous dissipation, heat generation, as well as mixed convection parameter have tendency to increase the skin friction on both plates. In addition, heat absorption contributes a decrease in the skin friction, and this due to velocity drop caused by increasing heat absorption. The table further shows that the skin friction is higher when the working fluid is air as compared to that of mercury. This is physically expected as fluid velocity increases with growing Prandtl number causing an increase in the skin friction on both plates.

The rate of heat transfer on both walls is simulated and presented in Table 3. From Table 3, it is evident to show that heat absorption leads to increase in the heat transfer on the heated wall, and this due to temperature decrease caused by growing heat absorption, which consequently leads to increase in the rate of heat.
transfer on the heated wall. This is physically true because fluid temperature increases with heat generation, and this decreases the temperature gradient on the heated wall, and heat transfer to the cold wall increases. Moreover, Prandtl number \( Pr \) increases rate of heat transfer decreases on the heated wall, while it increases on the cold wall. This is due to the temperature increases with the increase in \( Pr \) leading to a decrease in the temperature gradient on the heated wall, and the opposite trend is also discovered on the cold wall.

The numerical values of pressure gradient \( \frac{dP}{dx} \) and mean temperature \( \theta_m \) are indicated in Table 4. A general view of this table indicates that the pressure gradient decreases when the heat generation increases, and the reverse trend is observed in mean temperature. Physically this is true as growing heat generation enhances the buoyancy and increases the natural convection with little input from the forced convection. In addition, mean temperature increases with increase in heat absorption. Increases in heat absorption decrease the velocity and mass flux. The table further shows that the pressure gradient decreases with an increase in Prandtl number \( Pr \). An increase in Prandtl number requires a decrease in pressure gradient. This is clearly acquired, consequently for a large value of mixed convection parameter, and viscous dissipation, pressure gradient, and mean temperature increased.

<table>
<thead>
<tr>
<th>( Pr = 0.044 )</th>
<th>( Ec = 0.2, \ Gre = 5.0 )</th>
<th>( Ec = 0.4, \ Gre = 5.0 )</th>
<th>( Ec = 0.4, \ Gre = 8.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( \tau_0 )</td>
<td>( \tau_1 )</td>
<td>( \tau_0 )</td>
</tr>
<tr>
<td>( Ec = 0.2, \ Gre = 5.0 )</td>
<td>0.54044</td>
<td>1.70560</td>
<td>0.54252</td>
</tr>
<tr>
<td>( Ec = 0.4, \ Gre = 5.0 )</td>
<td>0.36756</td>
<td>1.55180</td>
<td>0.37356</td>
</tr>
<tr>
<td>( Ec = 0.4, \ Gre = 8.0 )</td>
<td>0.33178</td>
<td>1.51078</td>
<td>0.31956</td>
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</table>

<table>
<thead>
<tr>
<th>( Pr = 0.71 )</th>
<th>( Ec = 0.2, \ Gre = 5.0 )</th>
<th>( Ec = 0.4, \ Gre = 5.0 )</th>
<th>( Ec = 0.4, \ Gre = 8.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( Nu_0 )</td>
<td>( Nu_1 )</td>
<td>( Nu_0 )</td>
</tr>
<tr>
<td>( Ec = 0.2, \ Gre = 5.0 )</td>
<td>0.64261</td>
<td>1.19308</td>
<td>0.64078</td>
</tr>
<tr>
<td>( Ec = 0.4, \ Gre = 5.0 )</td>
<td>0.82613</td>
<td>1.09534</td>
<td>0.82448</td>
</tr>
<tr>
<td>( Ec = 0.4, \ Gre = 8.0 )</td>
<td>1.15983</td>
<td>0.92904</td>
<td>1.15854</td>
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</table>

<table>
<thead>
<tr>
<th>( Pr = 0.044 )</th>
<th>( Ec = 0.2, \ Gre = 5.0 )</th>
<th>( Ec = 0.4, \ Gre = 5.0 )</th>
<th>( Ec = 0.4, \ Gre = 8.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( \frac{dP}{dx} )</td>
<td>( \theta_m )</td>
<td>( \frac{dP}{dx} )</td>
</tr>
<tr>
<td>( Ec = 0.2, \ Gre = 5.0 )</td>
<td>( Ec = 0.4, \ Gre = 5.0 )</td>
<td>( Ec = 0.4, \ Gre = 8.0 )</td>
<td></td>
</tr>
<tr>
<td>( Pr = 0.71 )</td>
<td>( Ec = 0.2, \ Gre = 5.0 )</td>
<td>( Ec = 0.4, \ Gre = 5.0 )</td>
<td>( Ec = 0.4, \ Gre = 8.0 )</td>
</tr>
<tr>
<td>( S )</td>
<td>( \frac{dP}{dx} )</td>
<td>( \theta_m )</td>
<td>( \frac{dP}{dx} )</td>
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<tr>
<td>( Ec = 0.2, \ Gre = 5.0 )</td>
<td>( Ec = 0.4, \ Gre = 5.0 )</td>
<td>( Ec = 0.4, \ Gre = 8.0 )</td>
<td></td>
</tr>
</tbody>
</table>


Table 5: Numerical comparison between the work of Jha and Ajibade [18] and the present work.

<table>
<thead>
<tr>
<th>$S$</th>
<th>Jha and Ajibade [18]</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gre = 2.0, $y = 0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>Velocity</td>
<td>Temperature</td>
</tr>
<tr>
<td>-1</td>
<td>0.56975</td>
<td>0.63949</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.53296</td>
<td>0.63186</td>
</tr>
<tr>
<td>0.5</td>
<td>0.47030</td>
<td>0.61880</td>
</tr>
<tr>
<td>1</td>
<td>0.44341</td>
<td>0.61318</td>
</tr>
</tbody>
</table>

3.1 Validation of Results

For the validity of our work, we have compared our results with the existing results of Jha and Ajibade [18]; in the absence of viscous dissipation, pressure gradient, and mixed convection parameter, our results appear to be in good agreement with the existing results (Table 5).

4 Conclusion

The present article has considered the viscous dissipation effect on steady generalised Couette flow of heat-generating/absorbing fluid in a vertical channel. The influence of the governing parameters on the temperature and velocity is presented graphically and discussed for varying values of the controlling parameters. The numerical values of skin friction, rate of heat transfer, pressure gradient, and mean temperature are presented in the tables. The work concluded that fluid temperature and velocity increase with an increase in Eckert number and also found that growing mixed convection parameter leads to a corresponding rise in temperature and velocity. It is further discovered that fluid temperature increases as the heat generation increases, while it decreases with the increase in heat absorption. Furthermore, fluid velocity increases near the heated plate as a result of increase of heat generation, and the reverse trend is observed on the cold plate. Finally, it is concluded that heat generation contributes to increase in the mixed convection; hence, it requires a decrease in mixed convection parameter to bring about a reverse flow near the stationary plate.

Nomenclature

- $T^*$: dimensional fluid temperature [K]
- $T_w$: channel wall temperature [K]
- $T_0$: temperature of the ambience [K]
- $T$: dimensionless fluid temperature
- $u^*$: dimensional velocity [ms^{-1}]
- $u$: dimensionless velocity
- $U$: dimensional velocity of the moving plate [ms^{-1}]
- $y^*$: coordinate perpendicular to the plate [m]
- $y$: dimensionless coordinate perpendicular to the plate
- $g$: acceleration due to gravity [ms^{-2}]
- $h$: width of the channel [m]
- $Q_0$: heat generation/absorption coefficient [kgm^{-3}s^{-1}K^{-1}]
- $S$: dimensionless heat generation/absorption parameter
- $Pr$: Prandtl number
- $Gr$: Grashof number
- $Ec$: Eckert number
- $Re$: Reynolds number
- $\theta_m$: mean temperature
- $\beta$: coefficient of thermal expansion [K^{-1}]
- $\mu$: coefficient of viscosity [kgm^{-1}s^{-1}]
- $\nu$: kinematic viscosity [m^2s^{-1}]
- $\rho$: density of the fluid [kgm^{-3}]
- $\sigma$: density of the fluid [kgm^{-3}]
- $\alpha$: thermal diffusivity [m^2s^{-1}]
- $\eta$: embedding parameter

References