A New Class of Production Function Model and Its Application

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Abstract Under some circumstances, the studies on economic growth theory can be translated into the researches on production function which will be beneficial for the government to analyze the pattern of economic growth and then make reasonable policies. The commonly used production functions include C-D production function, CES production function, VES production function with different elasticity of substitution. This paper will put forward to a new class of production function which elasticity of substitution σ is a non-linear function of K/L . With this new model, a calculation formula for accurately measure the influence rates of various factors to economic growth will be derived, which is significant for in-depth studies on functions and scientific measurement. The empirical analysis on the influence rates of China's economic growth factors and its good results will be presented in the end of this paper.

Keywords production function; elasticity of substitution; economic growth; influence rate of factors; empirical analysis

1 Introduction

Since the mathematician Cobb and the economist Douglas from the America proposed the concept of production function, and later the production function had been widely used in practices. The production function represents the relationship between input factors and the output. Through production function model, it is allowable that the influence rate of capital, labor, scientific and technological progress and other factors to economic development to be analyzed. The production functions have always been an active field in economy studies and applications^[1–8], most researchers used C-D (Cobb-Dauglas) production function to analyze the economic issues, but it has limitations, the most notably is the elasticity of substitution σ is constant at 1, while in CES (constant elasticity of substitution) production function, the elasticity of substitution σ is not constant at 1, but this constant variable will keep invariable for one specific production function, such as the production function for certain industry. In fact, the elasticity of substitution will vary with different sample points, for example, it may

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change according to the ratio of capital to labor K/L , and it is based on such thought that variable elasticity of substitution (VES) is proposed. For VES, there are many theoretical and methodical achievements from researches^[9–11], the most well-known is the model proposed by Revankar in 1971 and the model proposed by Sato and Hoffman in 1968, the former assumes the elasticity of substitution σ is a linear function of K/L , while the latter assumes σ is a function of the time t . As a matter of fact, in some cases, the elasticity of substitution σ perhaps is a non-linear function of K/L , therefore, this paper present a circumstance where σ is a quadratic function of K/L and we call this new function as MVES function model. Finally, based on this new model, this paper derives a calculation formula to accurately measure the influence rate of economic growth factors^[12–14], and finally analyzes the influence rate of China's economic growth factors from empirical perspectives.

2 General Form of Common Production Function Models

The production function is the mathematical expression of relationship between changes in input factors, such as the capital K , the labor L , the technology A and the output Y during production process, that is, $Y = F(K, L)$. In case that $Y = F(\lambda K, \lambda L) = \lambda^m F(K, L)$, namely, the production function is m homogeneous expression, it means the output Y will grow at λ^m times if input of the capital K and the labor L increase at m times simultaneously. $m = 1$ denotes constant returns to scale, $0 < m < 1$ denotes decreasing returns to scale, and $m > 1$ denotes increasing returns to scale.

Under the circumstance that two production factors can substitute each other, it can produce the same products with different factor combinations, so, the elasticity of substitution between two factors would be the ratio of the increase percentage rate of proportion two factors account for to the increase percentage rate of marginal rate of substitution of these two factors, that is

$$\sigma = \frac{d(\frac{K}{L})}{(\frac{K}{L})} \bigg/ \frac{d(\frac{\partial Y}{\partial L} / \frac{\partial Y}{\partial K})}{(\frac{\partial Y}{\partial L} / \frac{\partial Y}{\partial K})}.$$

It assumes the production function $Y = F(K, L)$ is m homogeneous, that is,

$$Y = F(K, L) = L^m F\left(\frac{K}{L}, 1\right).$$

Set $k = \frac{K}{L}$, $y = f(k)$, so the above expression is transformed into

$$Y = L^m y.$$

Set the elasticity of substitution of two factors $\sigma = g(k)$, then

$$g = \frac{d(\frac{K}{L})}{(\frac{K}{L})} \bigg/ \frac{d(\frac{\partial Y}{\partial L} / \frac{\partial Y}{\partial K})}{(\frac{\partial Y}{\partial L} / \frac{\partial Y}{\partial K})}.$$

Given that $Y = L^m y$, we have

$$\begin{aligned} \frac{\partial Y}{\partial L} &= mL^{m-1}y + L^{m-1}y' \cdot \left(-\frac{K}{L^2}\right) = L^{m-1}(my - ky'), \\ \frac{\partial Y}{\partial K} &= L^m y' \cdot \frac{1}{L} = L^{m-1}y'. \end{aligned}$$

So

$$g = \frac{dk}{k} \cdot \frac{\left(\frac{my-ky'}{y'}\right)}{d\left(\frac{my-ky'}{y'}\right)} = \frac{myy' - ky'}{(m-1)k(y')^2 - mky'y''},$$

that is,

$$myy' - k(1 + (m-1)g)(y')^2 + mky'y'' = 0.$$

The above expression so is transformed into

$$\frac{1}{kg} - \frac{1 + (m-1)g}{mg} \cdot \frac{y'}{y} + \frac{y''}{y'} = 0.$$

Set $z = \frac{y'}{y}$, then $y' = zy$, take its derivative, we have $y'' = z'y + zy'$. As a result,

$$\frac{1}{kg} - \frac{1 + (m-1)g}{mg}z + \frac{z'}{z} + z = 0,$$

that is

$$z' = -\frac{1}{kg}z + \frac{1}{m}\left(\frac{1}{g} - 1\right)z^2.$$

Set $w = -\frac{1}{z}$, we have

$$\frac{dw}{dk} = \frac{1}{kg}w + \frac{1}{m}\left(\frac{1}{g} - 1\right).$$

The general solution of the above equation is

$$w = -\frac{1}{m}k - c \exp\left(\int \frac{dk}{kg}\right).$$

Then

$$\frac{y'}{y} = z = -\frac{1}{w} = \frac{1}{\frac{k}{m} + c \exp\left(\int \frac{dk}{kg}\right)}.$$

By the integration, we have

$$y = A \exp \int \frac{dk}{\frac{k}{m} + c \exp\left(\int \frac{dk}{kg}\right)}.$$

We have the general expression of production function in this way,

$$Y = L^m y = AL^m \exp \int \frac{dk}{\frac{k}{m} + c \exp\left(\int \frac{dk}{kg}\right)},$$

where $k = \frac{K}{L}$.

3 A New Class of Production Function Model and Parameters Estimation

3.1 Expression of the New Class of Production Function

If $g(k) = 1$,

$$Y = L^m y = AL^m \exp \int \frac{dk}{\frac{k}{m} + c \exp\left(\int \frac{dk}{k}\right)}$$

$$\begin{aligned}
&= AL^m \exp \int \frac{dk}{\left(\frac{1}{m} + c\right)k} \\
&= AL^m \exp \left(\frac{m}{1 + cm} \right) \ln k \\
&= AL^m k^{\frac{m}{1+cm}} \\
&= AL^m \left(\frac{K}{L} \right)^{\frac{m}{1+cm}} \\
&= AK^{\frac{m}{1+cm}} L^{m(1-\frac{1}{1+cm})}.
\end{aligned}$$

The above expressions are C-D production function.

$$\text{If } g(k) = \frac{1}{1+b},$$

$$\begin{aligned}
Y = L^m y &= AL^m \exp \int \frac{dk}{\frac{k}{m} + c \exp \left(\int \frac{(1+b)dk}{k} \right)} \\
&= AL^m \exp \int \frac{dk}{\frac{k}{m} + ck^{1+b}} \\
&= AL^m \exp \int \frac{mdk}{k(1 + cmk^b)} \\
&= AL^m \frac{k^m}{(1 + cmk^b)^{\frac{m}{b}}} \\
&= AL^m \left(\frac{K}{L} \right)^m \left(1 + cm \left(\frac{K}{L} \right)^b \right)^{-\frac{m}{b}} \\
&= A(K^{-b} + cmL^{-b})^{-\frac{m}{b}}.
\end{aligned}$$

The above expressions are CES production function.

$$\text{If } g(k) = 1 + bk,$$

$$\begin{aligned}
Y = L^m y &= AL^m \exp \int \frac{dk}{\frac{k}{m} + c \exp \left(\int \frac{dk}{k(1+bk)} \right)} \\
&= AL^m \frac{\left(k + cmk^{\frac{1}{1+b}} \right)^{m+\frac{m}{b}}}{k^{\frac{m}{b}}} \\
&= AL^m \frac{\left(\frac{K}{L} + cm \left(\frac{K}{L} \right)^{\frac{1}{1+b}} \right)^{m+\frac{m}{b}}}{\left(\frac{K}{L} \right)^{\frac{m}{b}}} \\
&= AK^{\frac{m}{1+c}} \left(L + \frac{1}{1+c} K \right)^{\frac{cm}{1+c}}.
\end{aligned}$$

The above expressions are VES production function.

It can be seen that if $g(k)$ changes, the type of production function will change also. This paper will only discuss the case of $g(k) = 1 + bk^2 (b \neq 0)$, and due to the complexity of calculation, we use MATLAB2014a and have

$$y = A \exp \int \frac{dk}{\frac{k}{m} + c \exp \left(\int \frac{dk}{k(1+bk^2)} \right)}$$

$$= A \exp \left(\frac{\log(k - \#5)\#7}{\#8} - \frac{3m \log(k)}{\#10} + \frac{\log(k + \#5)\#7}{\#8} + \frac{\log \left(\frac{\#1 + \#6 + 1.5}{k\#9 - \#3} \right) \#4}{\#2} \right) \\ \cdot \exp \left(\frac{\log \left(\frac{\#1 + \#6 + 1.5}{k\#9 + \#3} \right) \#4}{\#2} - \frac{\sqrt{6}cm^2 \log \left(\frac{\sqrt{\frac{9bk^2}{4} + 2.25} + 1.5}{k\#9} \right)}{\#10} \right).$$

In this equation, $k = \frac{K}{L}$, $\#1 = \sqrt{1.5c^2m^2(bk^2 + 1)}$, $\#2 = cm\#8$, $\#3 = \frac{\sqrt{3b\#10}\#9}{3b}$, $\#4 = 3cm^2 + cm^2\#10$, $\#5 = \frac{\sqrt{3b\#10}}{3b}$, $\#6 = \frac{\sqrt{3b\#10}k}{2}$, $\#7 = 3m + m\#10$, $\#8 = 4c^2m^2\#10$, $\#9 = \sqrt{1.5b}$, $\#10 = 2c^2m^2 - 3$.

We have the expression of production function, $Y = L^m y$, and the above model is called as MVES production function.

3.2 Estimation of Parameters of the New Class of Production Function Model

We express MVES production function as

$$Y = f(K, L, A, b, c, m) + \varepsilon,$$

where f is the non-linear function of the independent variables K and L , and four parameters, A, b, c, m . Use the nonlinear least square method^[15–17], that is error sum of squares and least, to estimate the parameters A, b, c, m . If we get N number of observations of Y and K, L , let

$$\sum_{i=1}^N \varepsilon_i^2 = \sum_{t=1}^N [Y_t - f(K, L, A, b, c, m)]^2$$

have the least. However, it is not easy to implement the calculation of non-linear estimations, hence software is often used, and this paper uses MATLAB2014a to estimate the parameters.

4 The Method to Measure the Influence Rates of Economic Growth

Express MVES production function as

$$Y = AF(K, L, A, b, c, m),$$

where (A, K, L) are inputs, Y is the output, (A, b, c, m) are parameters to be estimated.

Take its differential, we have

$$dY = FdA + A \frac{\partial F}{\partial K} dK + A \frac{\partial F}{\partial L} dL.$$

Set the elasticity coefficient of the capital $\alpha = \frac{\partial Y}{\partial K} \cdot \frac{K}{Y} = \frac{\partial F}{\partial K} \cdot \frac{K}{F}$, and the elasticity coefficient of the labor $\beta = \frac{\partial Y}{\partial L} \cdot \frac{L}{Y} = \frac{\partial F}{\partial L} \cdot \frac{L}{F}$, then

$$dY = \frac{Y}{A} dA + \alpha \frac{Y}{K} dK + \beta \frac{Y}{L} dL.$$

Provided that the economic vector (A, K, L, Y) changes into $(A^{(t+1)}, K^{(t+1)}, L^{(t+1)}, Y^{(t+1)})$ of the year $t + 1$ from $(A^{(t)}, K^{(t)}, L^{(t)}, Y^{(t)})$ of the year t , then

$$\Delta Y^{(t)} = \int_{L^{(t)}} dY = \int_{L^{(t)}} \frac{Y}{A} dA + \int_{L^{(t)}} \alpha \frac{Y}{K} dK + \int_{L^{(t)}} \beta \frac{Y}{L} dL.$$

Record as

$$\Delta Y_A^{(t)} = \int_{L^{(t)}} \frac{Y}{A} dA, \quad \Delta Y_K^{(t)} = \int_{L^{(t)}} \alpha \frac{Y}{K} dK, \quad \Delta Y_L^{(t)} = \int_{L^{(t)}} \beta \frac{Y}{L} dL, \quad t = 1, 2, \dots, N,$$

then

$$\Delta Y^{(t)} = \Delta Y_A^{(t)} + \Delta Y_K^{(t)} + \Delta Y_L^{(t)}.$$

Given

$$\frac{\partial Y}{\partial A} = \frac{Y}{A}, \quad \frac{\partial Y}{\partial K} = \alpha \frac{Y}{K}, \quad \frac{\partial Y}{\partial L} = \beta \frac{Y}{L}.$$

So

$$\Delta Y_A^{(t)} = \int_{L^{(t)}} \frac{\partial Y}{\partial A} dA, \quad \Delta Y_K^{(t)} = \int_{L^{(t)}} \frac{\partial Y}{\partial K} dK, \quad \Delta Y_L^{(t)} = \int_{L^{(t)}} \frac{\partial Y}{\partial L} dL, \quad t = 1, 2, \dots, N.$$

It is obvious that $\Delta Y_A^{(t)}$ is the absolute influence rate value of technological progress to economic growth at t period; $\Delta Y_K^{(t)}$ is the absolute influence rate value of capital to economic growth at t period; $\Delta Y_L^{(t)}$ is the absolute influence rate value of labor to economic growth at t period.

Then, the influence rate of capital growth to economic growth at t period is $KCI^{(t)} = \frac{\Delta Y_K^{(t)}}{Y^{(t)}}$, the influence rate of labor growth to economic growth at t period is $LCI^{(t)} = \frac{\Delta Y_L^{(t)}}{Y^{(t)}}$.

Use the method of residues to calculate the influence rate of technological progress, from

$$\frac{\Delta Y^{(t)}}{Y^{(t)}} = \frac{\Delta Y_A^{(t)}}{Y^{(t)}} + \frac{\Delta Y_K^{(t)}}{Y^{(t)}} + \frac{\Delta Y_L^{(t)}}{Y^{(t)}}.$$

We have the influence rate of technological progress to economic growth at t period is

$$ACI^{(t)} = \frac{\Delta Y_A^{(t)}}{Y^{(t)}} = \frac{\Delta Y^{(t)}}{Y^{(t)}} - \frac{\Delta Y_K^{(t)}}{Y^{(t)}} - \frac{\Delta Y_L^{(t)}}{Y^{(t)}},$$

where

$$\begin{aligned} \Delta Y_K^{(t)} &= \int_{L^{(t)}} \frac{\partial Y}{\partial K} dK = \int_{L^{(t)}} L^m y' \frac{1}{L} dK = \int_{L^{(t)}} L^{m-1} y' dK, \\ \Delta Y_L^{(t)} &= \int_{L^{(t)}} \frac{\partial Y}{\partial L} dL = \int_{L^{(t)}} \left[mL^{m-1} y + L^m y' \left(-\frac{K}{L^2} \right) \right] dL = \int_{L^{(t)}} (mL^{m-1} y - L^{m-2} K y') dL, \end{aligned}$$

where

$$\begin{aligned} y' &= -A \exp \left(\frac{\log(\#11)\#8}{\#1} - \frac{3m \log(k)}{\#17} + \frac{\log(\#12)\#8}{\#1} + \frac{\log(\frac{\#5}{k\#15-\#10})\#7}{\#1\#2} \right) \\ &\cdot \exp \left(\frac{\log(\frac{\#5}{k\#15})\#7}{\#1\#2} - \frac{\sqrt{6}cm^2 \log(\frac{\#3+1.5}{k\#15})}{\#17} \right) \\ &\cdot \left(\frac{3m}{k\#17} - \frac{\#8}{\#1\#11} + \frac{(k\#15 + \#10) \left(\frac{0.5\#16-\#9}{k\#15+\#10} + \frac{\#15\#5}{(k\#15+\#10)^2} \right) \#7}{\#1\#2\#5} \right. \\ &\quad \left. - \frac{(k\#15 - \#10) \left(\frac{0.5\#16}{k\#15-\#10} - \frac{\#15\#6}{(k\#15-\#10)^2} \right) \#7}{\#1\#2\#6} \right) \end{aligned}$$

$$+ \frac{\sqrt{6}ckm^2 \left(\frac{9b}{4\#15\#3} - \frac{\#3+1.5}{k^2\#15} \right) \#15}{\#17(\#3+1.5)} \Bigg).$$

In above equation, $k = \frac{K}{L}$, $\#1 = 4c^2m^2 - 6$, $\#2 = cm$, $\#3 = \sqrt{\frac{9bk^2}{4} + \frac{9}{4}}$, $\#4 = -\sqrt{6}cm^2 + 3m$, $\#5 = \#14 - \#13 + 1.5$, $\#6 = \#14 + \#13 + 1.5$, $\#7 = 3cm^2 + cm^2\#17$, $\#8 = 3m + m\#17$, $\#9 = \frac{1.5bc^2km^2}{\#14}$, $\#10 = \frac{\sqrt{3b\#17\#15}}{3b}$, $\#11 = k - \frac{\#16}{3b}$, $\#12 = k + \frac{\#16}{3b}$, $\#13 = \frac{\sqrt{3b\#17k}}{2}$, $\#14 = cm\sqrt{1.5bk^2 + 1.5}$, $\#15 = \sqrt{1.5b}$, $\#16 = \sqrt{3b\#17}$, $\#17 = 2c^2m^2 - 3$.

The curve $L^{(t)}$ has many forms, this paper selects 3rd order polynomial,

$$\begin{cases} A = A_i + \frac{(A_{i+1} - A_i)}{3}(t + t^2 + t^3), \\ K = K_i + \frac{(K_{i+1} - K_i)}{3}(t + t^2 + t^3), \\ L = L_i + \frac{(L_{i+1} - L_i)}{3}(t + t^2 + t^3), \\ Y = Y_i + \frac{(Y_{i+1} - Y_i)}{3}(t + t^2 + t^3), \end{cases} \quad 0 \leq t \leq 1.$$

We can get the calculation of $\Delta Y_K^{(t)}$ and $\Delta Y_L^{(t)}$ by numerical integration.

5 Empirical Analysis on Influence Rates of China's Economic Growth Factors

In order to intensively study China's economic growth and explore its growth pattern, as well as measure the influence rate of input factors to economic growth, we take the gross domestic product (Y , in a hundred million Yuan) as the comprehensive representative indicator of economic growth, and take the number of employees (L , in ten thousand), fixed asset input (K , in a hundred million Yuan) as the influence factors to the economy, and the information is shown in Table 1.

Here, it establishes the MVES production function model, based on non-linear least square method, and take advantages of MATLAB 2014a, we have

$$(A, b, c, m) = (26.9580, 939.9444, -0.3132, 0.7725).$$

The coefficient of determination is

$$R^2 = 1 - \frac{\sum (Y_t - \hat{Y}_t)^2}{\sum (Y_t - \bar{Y})^2} = 0.9885.$$

It can be seen that the model has high fitting accuracy.

We have $\Delta Y_A^{(t)} = \Delta Y^{(t)} - \Delta Y_K^{(t)} - \Delta Y_L^{(t)}$ by taking numerical integration of $\Delta Y_K^{(t)}$, $\Delta Y_L^{(t)}$, and further have the influence rate of capital growth to economic growth, $KCI^{(t)}$ at each year; the influence rate of labor growth to economic growth, $LCI^{(t)}$ at each year; the influence rate of technological progress to economic growth, $ACI^{(t)}$ at each year; as shown in Table 1.

The annual average influence rate of capital growth to economic growth during 1994–2013 is

$$ECI = \sqrt[19]{\prod_{t=2}^{20} (1 + ECI^{(t)})} - 1 = 10.06\%.$$

Table 1 The data of China's economic growth and the results

Year	Y	K	L	$KCI^{(t)}$	$LCI^{(t)}$	$ACI^{(t)}$
1994	48197.9	17042.1	67455.0	-	-	-
1995	60793.7	20019.3	68065.0	14.3251	0.4793	11.3293
1996	71176.6	22913.5	68950.0	11.8548	0.6891	4.5349
1997	78973.0	24941.1	69820.0	7.2561	0.6687	3.0288
1998	84402.3	28406.2	70637.0	4.3924	0.6202	1.8623
1999	89677.1	29854.7	71394.0	4.1814	0.5680	1.5002
2000	99214.6	32917.7	72085.0	8.4129	0.5130	1.7095
2001	109655.2	37213.5	72797.0	7.7011	0.5235	2.2987
2002	120332.7	43499.9	73280.0	6.8521	0.3516	2.5336
2003	135822.8	55566.6	73736.0	10.7465	0.3298	1.7964
2004	159878.3	70477.4	74264.0	13.0040	0.3795	4.3274
2005	184937.4	88773.6	74647.0	12.2875	0.2733	3.1130
2006	216314.4	109998.2	74978.0	11.6051	0.2350	5.1262
2007	265810.3	137323.9	75321.0	20.3704	0.2425	2.2686
2008	314045.4	172828.4	75564.0	15.2008	0.1710	2.7747
2009	340902.8	224598.8	75828.0	6.5630	0.1852	1.8040
2010	401512.8	251683.8	76105.0	9.8886	0.1936	7.6970
2011	473104.0	311485.1	76420.0	15.4836	0.2194	2.1274
2012	519470.1	374694.7	76704.0	6.6402	0.1970	2.9632
2013	568845.0	447074.0	76977.0	5.8398	0.1886	3.4764

The annual average influence rate of labor growth to economic growth during 1994–2013 is

$$LCI = \sqrt[19]{\prod_{t=2}^{20} (1 + LCI^{(t)})} - 1 = 0.37\%.$$

The annual average influence rate of technological progress to economic growth during 1994–2013 is

$$ACI = \sqrt[19]{\prod_{t=2}^{20} \left(1 + \frac{\Delta Y^{(t)}}{Y^{(t)}}\right)} - \sqrt[19]{\prod_{t=2}^{20} (1 + ECI^{(t)})} - \sqrt[19]{\prod_{t=2}^{20} (1 + LCI^{(t)})} = 3.44\%.$$

Namely, from 1994 to 2013, the average annual growth rate of China's economy was 13.97%, in which, 10.06% was contributed by the capital growth, and 0.37% was contributed by the labor growth, and 3.44% was contributed by technological progress. It is observed that China's economic growth is mainly dependent on capital input, and then on technological progress, and the influence of labor is less, therefore, the results are consistent with the law of China's economic growth.

6 Conclusion

This paper derived the general form of common production function model by use of differential equation, and under the circumstance that the elasticity of substitution σ is a quadratic function of K/L , a new class of production function model (MVES) was provided. And then the new model was used to derive the calculation equation that is able to precisely measure the influence rate of economic growth factors, which is very significant for the general application of MVES production function and scientific measurement and calculation. In the end of this paper, the influence ratio of various factors to China's economic growth was analyzed from empirical perspective, and good results were attained. This paper will provide valuable advices for reasonable arrangement of limited resources, and acceleration of economic growth, as well as powerful theoretical foundation for the government leaders to make decisions.

References

- [1] Avvakumov S N, Kiselev Yu N, Orlov M V, et al. Profit maximization problem for Cobb-Douglas and CES production functions. *Computational Mathematics and Modeling*, 2010, 21(3): 336–378.
- [2] Choi E J, Moon H R, Ridder G. Estimation of an education production function under random assignment with selection. *American Economic Review*, 2014, 104(5): 206–211.
- [3] Epple D, Gordon B, Sieg H. A new approach to estimating the production function for housing. *American Economic Review*, 2010, 100(3): 905–924.
- [4] Fayissa B, Gutema P. Estimating a health production function for Sub-Saharan Africa (SSA). *Applied Economics*, 2005, 37(2): 155–164.
- [5] Kuosmanen T, Pemsil D, Wesseler J. Specification and estimation of production functions involving damage control inputs: A two-stage, semiparametric approach. *American Journal of Agricultural Economics*, 2006, 88(2): 499–511.
- [6] Levinsohn J, Petrin A. Estimating production functions using inputs to control for unobservables. *The Review of Economic Studies*, 2003, 70(243): 317–341.
- [7] Mundlak Y. Production function estimation: Reviving the primal. *Econometrica*, 1996, 64(4): 31–38.
- [8] Zhengfei G, Lansink A O, van Ittersum M, et al. Integrating agronomic principles into production function specification: A dichotomy of growth inputs and facilitating inputs. *American Journal of Agricultural Economics*, 2006, 88(1): 203–214.
- [9] Cheng M L. China forecasting model of economic growth based on production function. *Statistics and Decision*, 2010, 20: 34–36.
- [10] Li Z N, Pang W Q. *Econometrics*. Beijing: Higher Education Press, 2010: 156–167.
- [11] Sun J S, Ma S Q. *Econometrics*. Beijing: Tsinghua University Press, 2004: 203–212.
- [12] Cheng M L. A new measure model of factor contribution ratio for economic growth. *Mathematics in Economics*, 2004, 21(3): 235–239.
- [13] Cheng M L. Correction and empirical analysis of the CES production function model. *Journal of Engineering Mathematics*, 2013, 30(4): 535–544.
- [14] Cheng M L, Han Y. The measure model and analysis of contribution ratio of economic growth factor on Suzhou foreign capital manufacturing. *Application of Statistics and Management*, 2009, 28(3): 381–385.
- [15] Liu H L. A trust region algorithm for nonlinear least squares problems. *Mathematics in Economics*, 2007, 24(2): 213–216.
- [16] Wang L Q, Leblanc A. Second-order nonlinear least squares estimation. *Annals of the Institute of Statistical Mathematics*, 2008, 60(4): 883–900.
- [17] Argyros I K, Magreñán Á A. Local convergence analysis of proximal Gauss-Newton method for penalized nonlinear least squares problems. *Applied Mathematics and Computation*, 2014, 241: 401–408.