Two-Sided Matching Decision-Making with Uncertain Information Under Multiple States

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Abstract A novel decision-making method based on evidential reasoning is proposed for solving the two-sided matching problem with uncertain information under multiple states in this paper. Firstly, the discernment frame of evidence is constructed for two-sided matching. Secondly, the preference ordinal values given by two-sided decision-makers are transformed into rank belief degrees. On this basis, and with two-sided satisfaction as the goal, two-sided rank belief degrees are taken as pieces of evidence, and satisfaction degrees of two-sided matching are obtained through evidence fusion. Then, a decision-making model based on satisfaction degrees is constructed in order to obtain the matching solution. Finally, an illustrative example demonstrates the application of the proposed approach.

Keywords matching decision-making; uncertain information; multiple states; evidence fusion

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1 Introduction

Two-sided matching markets are typically characterized as consisting of two kinds of heterogeneous agents, each of whom have preferences regarding potential matches with agents of the other kind; that is, both sides make decisions and rank the opposite side using a matching procedure. For example, hospitals are careful about the doctors they hire, while doctors prefer to work at certain hospitals. Two-sided matching was first introduced by Gale and Shapley^[1] and later extended and refined by other contributors. Therefore, it is also known as the two-sided matching theory, or matching theory, for short. The matching theory can achieve an efficient outcome that advances the common interests or reconciles the conflicting interests of agents wishing to be matched. As result, it has been applied to a wide range of areas including schooling choices^[2, 3], employer-employee job matching^[4, 5], mergers and acquisitions^[6–8], inter-company matching^[9], choices regarding bank lenders^[10], assignment of CEOs to firms^[11, 12], buyer–supplier relationships^[13, 14], firm-university relationships^[15], etc.

While the matching theory has a long and rich history, much of the existing literature assumes complete information; most studies assume that all agents fully know their own preferences. In fact, incomplete information is arguably commonplace in most environments. Thus, solving matching problems under uncertainty needs a novel approach. The matching theory under uncertainty was first developed by Roth^[16] and later extended and improved by other scholars^[17–20]. Notably, recent years have recorded increasing interest in this area. Rastegari, et al.^[21] proposed a novel model of two-sided matching in which agents are endowed with known partially ordered preferences and unknown true preferences drawn from known distributions consistent with the partial order. Chade, Lewis and Smith^[22] devised an equilibrium model of college admissions that analyzes the impact of two previously unexplored frictions in the application process: students find it costly to apply to colleges, while it is uncertain how colleges may evaluate their applications. Liu, et al.^[23] formulated the notion of stable outcomes in matching problems with one-sided asymmetric information and provided sufficient conditions for efficient incomplete information stable matching. In addition, two-sided matching with uncertainty was also studied by Chen, Fan and Li^[25], Yue^[26–28], Liang and Jiang^[29, 30], Zhang and Hu^[24], and Chen, Wang and Shi^[31, 32].

Existing matching models (or matching decision-making methods) have made significant headway in solving matching problems with different uncertain assessment information. Unfortunately, almost all these research results fail to take uncertainty under different states into account. In reality, agents often face multiple future states when they rank the opposite side in a matching procedure. Each of them may have different preferences over potential matches with agents on the other side under different states. For example, under state θ_1 , agent A ranks agent B of the opposite side in the first place. Under state θ_2 , A ranks B in the third place. Under state θ_3 , A does not rank B at all. Suppose that the probabilities of states $\theta_1, \theta_2, \theta_3$ occurring are 0.5, 0.3, and 0.2, respectively. Thus, A 's preference values toward B are distributive belief degrees $\{(1, 0.5); (3, 0.3); (H, 0.2)\}$, with H indicating the unknown preference. As different types of data have different requirements for decision-making methods, it is difficult to directly apply existing methods to solve two-sided matching problems with uncertain information under multiple states. This paper thus explores a matching method based on the evidential reasoning

(ER) approach^[33, 34], which is based on the decision theory and the Dempster-Shafer (D-S) theory of evidence^[35, 36], and it can be used to model various uncertainties in the matching process.

This paper is organized as follows. Section 2 develops an evidence fusion method for matching decision-making problems with uncertain information under multiple states. Section 3 provides a practical example to illustrate the proposed methodology. Finally, Section 4 concludes the paper.

2 Evidence Fusion Method

Our basic reasoning in this paper takes two-sided matching satisfaction as the main goal and uncertain preference information of matching parties A and B under various states as target evidence. We then construct an evidence fusion model and take the two-sided fusion results obtained therein as the two-sided matching satisfaction degrees. Finally, we use a 0–1 assignment model to obtain a two-sided matching solution.

2.1 Constructing Unified Assessment Grades on Both Sides

Let the matching agents on one side be $A_1, A_2, \dots, A_P$ (party A) and those on the other side be $B_1, B_2, \dots, B_Q$ (party B). In order to use the ER approach to conduct decision analysis for the matching problem, it is necessary to define a unified evidence frame of discernment for the two-sided matching agents. Suppose all the agents on both sides are assessed using N crisp assessment grades H_n ($n = 1, 2, \dots, N$), which are mutually exclusive and collectively exhaustive. The N assessment grades form the frame of discernment $H = \{H_1, H_2, \dots, H_N\}$ in the ER approach.

Let Y_n ($n = 1, 2, \dots, N$) be the standard values of assessment grades H_n . For party A , let the standard values of the assessment grades be defined as follows:

$$Y_1 = 1, \quad Y_2 = 1 + \frac{P-1}{N-1}, \quad Y_3 = 1 + 2\frac{P-1}{N-1}, \quad \dots, \quad Y_N = 1 + (N-1)\frac{P-1}{N-1}. \quad (1)$$

For party B , let the standard values of assessment grades be defined as follows:

$$Y_1 = 1, \quad Y_2 = 1 + \frac{Q-1}{N-1}, \quad Y_3 = 1 + 2\frac{Q-1}{N-1}, \quad \dots, \quad Y_N = 1 + (N-1)\frac{Q-1}{N-1}. \quad (2)$$

In Equations (1) and (2), the standard values of assessment grades Y_n ($n = 1, 2, \dots, N$) are equal to the relative positions on both sides. For instance, suppose $P = 9$ and $Q = 5$, and we use five crisp assessment grades. Then, the calculated results of the standard values of party A are 1, 3, 5, 7, and 9 according to Equation (1), while those of party B are 1, 2, 3, 4, and 5 according to Equation (2). Clearly, for matching party A , the relative position with the highest value will be ranked first; the lowest value, ninth; and the middle value, fifth. For matching party B , the relative position with the highest value will be ranked first; the lowest value, fifth; and the middle value, third. It follows that a person ranked in the fifth place among a group of nine people and a person ranked in the third place among a group of five people will occupy the same relative position; thus, we ascertain that using relative positions to determine the standard values of two-sided evaluation of rankings is reasonable.

In order to model quantitative agents using the defined assessment grades, grade utilities need to be known. The utilities for each assessment grade can be elicited from decision-makers

in different ways, but in this paper, we suppose that the utilities denoted by $u(H_n)$ ($n = 1, 2, \dots, N$) are set to

$$u(H_1) = 1, \quad u(H_2) = (N-2)/(N-1), \quad u(H_3) = (N-3)/(N-1), \quad \dots, \quad u(H_N) = 0.$$

2.2 Transforming Preference Ordinals into Belief Degrees of Evidence

To use the ER approach to conduct decision analysis for matching problems, preference ordinals on both sides need to be transformed into belief degrees. Depending on three different types of data relationships between the preference ordinals and the standard values of the assessment grades, any preference ordinal can be transformed into a belief degree of the assessment grades.

In the first type of data relationship, preference ordinal y_i happens to be equivalent to a certain standard value of assessment grade Y_n ($n = 1, 2, \dots, N$). Then, value y_i can be equivalently expressed as the following belief degree:

$$y_i \Leftrightarrow \{(H_n, \beta_n)\}, \quad (3)$$

where the symbol “ $\Leftrightarrow$ ” represents “is equivalent to”, and $\beta_n = 1$. This means that preference ordinal y_i is equivalent to Y_n with probability 100%. In Equation (3), β_n represents the belief degree of H_n .

In the second type of data relationship, preference ordinal y_i lies between two adjacent assessment grades H_n, H_{n+1} ($n = 1, 2, \dots, N-1$), that is, $Y_n < y_i < Y_{n+1}$. For

$$y_i = \frac{Y_{n+1} - y_i}{Y_{n+1} - Y_n} \cdot Y_n + \frac{y_i - Y_n}{Y_{n+1} - Y_n} \cdot Y_{n+1},$$

value y_i can be equivalently expressed as the following belief degree:

$$y_i \Leftrightarrow \{(H_n, \beta_n); (H_{n+1}, \beta_{n+1})\}, \quad (4)$$

where

$$\beta_n = \frac{Y_{n+1} - y_i}{Y_{n+1} - Y_n}, \quad \beta_{n+1} = \frac{y_i - Y_n}{Y_{n+1} - Y_n}.$$

This means that preference ordinal y_i is either equivalent to Y_n with probability β_n or equivalent to Y_{n+1} with probability β_{n+1} . Note that $\beta_n + \beta_{n+1} = 1$.

In the third type of data relationship, preference ordinal y_i is totally unknown. Thus, value y_i can be equivalently expressed as the following belief degree:

$$y_i \Leftrightarrow \{(H, \beta_H)\}, \quad (5)$$

where $H = \{H_1, H_2, \dots, H_N\}$ and $\beta_H = 1$. This means that preference ordinal y_i must lie at a certain assessment grade, but its specific location is unknown.

According to Equations (3)~(5), all the preference ordinal values of two-sided matching agents under various states can be transformed into belief degrees.

2.3 Determining the Degree of Satisfaction Between Two-Sided Matching Agents

Taking the evaluation values of two-sided matching agents under various states as evidence of the degree of satisfaction, and then using ER^[28] for evidence fusion, we can determine the satisfaction degree of the two-sided matching. The process is briefly described in the following steps.

The first step is to transform the original belief degrees into basic probability masses by combining the relative weights using the following equations:

$$\begin{cases} m_i(H_n) = w_i \beta_{n,i}, & i = 1, 2, \dots, L; n = 1, 2, \dots, N, \\ \tilde{m}_i(H) = w_i \left(1 - \sum_{n=1}^N \beta_{n,i} \right), & i = 1, 2, \dots, L, \\ \bar{m}_i(H) = 1 - w_i, & i = 1, 2, \dots, L, \\ m_i(H) = \bar{m}_i(H) + \tilde{m}_i(H), & i = 1, 2, \dots, L, \end{cases} \quad (6)$$

with $\sum_{i=1}^L w_i = 1$. Note that $m_i(H)$, which is not assigned to any assessment grade, contains two parts, $\bar{m}_i(H)$ and $\tilde{m}_i(H)$, where $\bar{m}_i(H)$ is caused by the relative importance of the i^{th} evidence, and $\tilde{m}_i(H)$, by the incompleteness of the evaluation.

The second step is to aggregate all pieces of evidence by combining the basic probability masses generated above. All pieces of evidence are combined using the following equations:

$$\begin{cases} m_1 \oplus m_2 \cdots \oplus m_L(H_n) = \frac{\sum_{A_1 \cap A_2 \cdots \cap A_L = H_n, A_1, A_2, \dots, A_L \subseteq H} m_1(A_1) m_2(A_2) \cdots m_L(A_L)}{\sum_{A_1 \cap A_2 \cdots \cap A_L \neq \emptyset, A_1, A_2, \dots, A_L \subseteq H} m_1(A_1) m_2(A_2) \cdots m_L(A_L)}, \\ \bar{m}_1 \oplus \bar{m}_2 \cdots \oplus \bar{m}_L(H) = \frac{\bar{m}_1(H) \bar{m}_2(H) \cdots \bar{m}_L(H)}{\sum_{A_1 \cap A_2 \cdots \cap A_L \neq \emptyset, A_1, A_2, \dots, A_L \subseteq H} m_1(A_1) m_2(A_2) \cdots m_L(A_L)}. \end{cases} \quad (7)$$

The third step is to normalize the aggregated basic probability masses into overall belief degrees using the following equations:

$$\begin{cases} \beta_n = \frac{m_1 \oplus m_2 \cdots \oplus m_L(H_n)}{1 - (\bar{m}_1 \oplus \bar{m}_2 \cdots \oplus \bar{m}_L(H))}, & H_n \subset H, \\ \beta_H = \frac{\tilde{m}_1 \oplus \tilde{m}_2 \cdots \oplus \tilde{m}_L(H)}{1 - (\bar{m}_1 \oplus \bar{m}_2 \cdots \oplus \bar{m}_L(H))}, \end{cases} \quad (8)$$

where β_n and β_H are the belief degrees of the aggregated assessment. Note that the belief degree β_H may be assigned to any assessment grade.

The final step is to determine the degree of satisfaction using the following equations:

$$f(\beta_1, \beta_2, \dots, \beta_N, \beta_H) = \sum_{n=1}^N [\beta_n \cdot u(H_n)] + \beta_H \cdot u(H_H). \quad (9)$$

When β_H is assigned to the most preferred assessment grade H_1 , the degree of satisfaction f achieves its maximum, which is defined by Equation (10). However, when it is assigned to the least preferred assessment grade H_1 , the degree of satisfaction f achieves its minimum, which is defined by Equation (11). So, f is, in fact, an interval if $\beta_H \neq 1$. Without loss of generality, we can use the midpoint of the interval, which is expressed as Equation (12), to determine f .

$$f_{\max} = \sum_{n=1}^N [\beta_n \cdot u(H_n)] + \beta_H \cdot u(H_1). \quad (10)$$

$$f_{\min} = \sum_{n=1}^N [\beta_n \cdot u(H_n)] + \beta_H \cdot u(H_N). \quad (11)$$

$$f_{ave} = (f_{\max} + f_{\min})/2. \quad (12)$$

2.4 Constructing the Two-Sided Matching Model

Using the above-mentioned steps to find the average degree of satisfaction f_{ij} of agent $A_i (i = 1, 2, \dots, P)$ in party A and $B_j (j = 1, 2, \dots, Q)$ in party B , we can construct a two-sided matching model as seen below:

$$\max Z = \sum_{i=1}^P \sum_{j=1}^Q f_{ij} x_{ij} \quad (13)$$

$$\text{s.t. } \sum_{j=1}^Q x_{ij} = 1, \quad i = 1, 2, \dots, P, \quad (14)$$

$$\sum_{i=1}^P x_{ij} \leq 1, \quad j = 1, 2, \dots, Q, \quad (15)$$

$$x_{ij} = 0, 1, \quad i = 1, 2, \dots, P; \quad j = 1, 2, \dots, Q. \quad (16)$$

Equation (13) is a target function, enabling the greatest possible degree of satisfaction between the two-sided agents. Equation (14) is a constraining condition, indicating that each agent in party A can only match with one other agent in party B . Equation (15) is a constraining condition, indicating that each agent in party B matches with no more than one agent in party A . Equation (16) is a constraining condition, indicating that $x_{ij} = 1$ when there is successful match between A_i and B_j , and $x_{ij} = 0$ when there is a match failure. This model is a standard 0–1 assignment model and can be solved using software such as Lingo or Cplex.

3 Illustration

Solving matching problems to select a business partner is a task of critical importance in supply chain management, where suppliers and retailers must find suitable business partners in order to form effective strategic alliances. Nine existing supply companies ($A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$) provide an assortment of different products. They receive cooperation offers from five retail companies (B_1, B_2, B_3, B_4, B_5). According to market surveys, product sales are related to the current supply and sales of other products. There are three types of natural states {ample supply of other products, average supply of other products, and insufficient supply of other products}, represented by $\{\theta_1, \theta_2, \theta_3\}$. Under different natural states, retail companies will consider supply companies based on product quality, wholesale price, service, and other similar criteria. Using this evaluation, they put forth a preference order, as seen in Table 1. At the same time, the uncertainty in capital turnover causes retail companies to also have two different types of natural states {prompt payment, delayed payment}, represented by $\{\theta_1, \theta_2\}$. Under different natural states, suppliers also consider retail companies based on corporate reputation, cooperation experience, and other factors, which also results in a preference order, as seen in Table 2. The data in the tables contain probability values for natural state occurrence and preference ordinal values; for example, in Table 1, A_1 's preference value for B_1 $\{(1, 0.5); (-, 0.5)\}$ indicates that the probability of the first natural state occurring is 0.5, and in this scenario, A_1 will rank B_1 in first place. The probability of the second scenario occurring is also 0.5, wherein A_1 will not rank B_1 among its preferences (indicated by “-”).

Table 1 Suppliers' uncertain preference ordinal values for retailers under different states

	B_1	B_2	B_3	A_4	A_5
A_1	(1, 0.5); (-, 0.5)	(-, 0.4); (-, 0.6)	(2, 0.6); (3, 0.4)	(4, 0.7); (5, 0.3)	(1, 0.2); (3, 0.8)
A_2	(2, 0.5); (-, 0.5)	(2, 0.4); (-, 0.6)	(1, 0.6); (3, 0.4)	(1, 0.7); (3, 0.3)	(2, 0.2); (-, 0.8)
A_3	(3, 0.5); (4, 0.5)	(1, 0.4); (2, 0.6)	(-, 0.6); (5, 0.4)	(2, 0.7); (-, 0.3)	(3, 0.2); (4, 0.8)
A_4	(-, 0.5); (5, 0.5)	(3, 0.4); (4, 0.6)	(3, 0.6); (4, 0.4)	(3, 0.7); (4, 0.3)	(-, 0.2); (5, 0.8)
A_5	(2, 0.5); (3, 0.5)	(4, 0.4); (5, 0.6)	(-, 0.6); (5, 0.4)	(-, 0.7); (5, 0.3)	(2, 0.2); (3, 0.8)
A_6	(1, 0.5); (-, 0.5)	(3, 0.4); (4, 0.6)	(2, 0.6); (-, 0.4)	(1, 0.7); (-, 0.3)	(1, 0.2); (-, 0.8)
A_7	(2, 0.5); (-, 0.5)	(2, 0.4); (-, 0.6)	(1, 0.6); (-, 0.4)	(2, 0.7); (-, 0.3)	(2, 0.2); (-, 0.8)
A_8	(2, 0.5); (3, 0.5)	(1, 0.4); (2, 0.6)	(4, 0.6); (5, 0.4)	(2, 0.7); (3, 0.3)	(2, 0.2); (3, 0.8)
A_9	(4, 0.5); (5, 0.5)	(2, 0.4); (3, 0.6)	(2, 0.6); (3, 0.4)	(2, 0.7); (3, 0.3)	(4, 0.2); (5, 0.8)

Table 2 Retailers' uncertain preference ordinal values for suppliers under different states

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
B_1	(2, 0.3);	(1, 0.3);	(-, 0.2);	(-, 0.4);	(2, 0.5);	(3, 0.3);	(-, 0.2);	(-, 0.3);	(5, 0.2);
	(3, 0.4);	(2, 0.3);	(7, 0.3);	(-, 0.3);	(3, 0.4);	(3, 0.4);	(6, 0.3);	(4, 0.4);	(7, 0.4);
	(-, 0.3)	(3, 0.4)	(8, 0.5)	(8, 0.3)	(-, 0.1)	(-, 0.3)	(8, 0.5)	(7, 0.3)	(9, 0.4)
B_2	(3, 0.3);	(2, 0.3);	(-, 0.2);	(4, 0.4);	(5, 0.5);	(2, 0.3);	(2, 0.2);	(7, 0.3);	(2, 0.2);
	(4, 0.4);	(3, 0.3);	(6, 0.3);	(5, 0.3);	(6, 0.4);	(3, 0.4);	(3, 0.3);	(8, 0.4);	(3, 0.4);
	(5, 0.3)	(5, 0.4)	(7, 0.5)	(6, 0.3)	(9, 0.1)	(-, 0.3)	(-, 0.5)	(9, 0.3)	(-, 0.4)
B_3	(2, 0.3);	(-, 0.3);	(-, 0.2);	(2, 0.4);	(3, 0.5);	(5, 0.3);	(-, 0.2);	(-, 0.3);	(-, 0.2);
	(3, 0.4);	(7, 0.3);	(-, 0.3);	(3, 0.3);	(3, 0.4);	(7, 0.4);	(4, 0.3);	(4, 0.4);	(6, 0.4);
	(-, 0.3)	(8, 0.4)	(8, 0.5)	(4, 0.3)	(-, 0.1)	(9, 0.3)	(7, 0.5)	(7, 0.3)	(8, 0.4)
B_4	(5, 0.3);	(-, 0.3);	(-, 0.2);	(3, 0.4);	(1, 0.5);	(2, 0.3);	(-, 0.2);	(-, 0.3);	(1, 0.2);
	(7, 0.4);	(4, 0.3);	(6, 0.3);	(3, 0.3);	(2, 0.4);	(3, 0.4);	(-, 0.3);	(7, 0.4);	(2, 0.4);
	(9, 0.3)	(7, 0.4)	(8, 0.5)	(-, 0.3)	(3, 0.1)	(-, 0.3)	(8, 0.5)	(8, 0.3)	(3, 0.4)
B_5	(1, 0.3);	(-, 0.3);	(-, 0.2);	(5, 0.4);	(-, 0.5);	(2, 0.3);	(3, 0.2);	(-, 0.3);	(1, 0.2);
	(2, 0.4);	(7, 0.3);	(6, 0.3);	(7, 0.3);	(-, 0.4);	(3, 0.4);	(3, 0.3);	(4, 0.4);	(2, 0.4);
	(3, 0.3)	(8, 0.4)	(8, 0.5)	(9, 0.3)	(8, 0.1)	(-, 0.3)	(-, 0.5)	(7, 0.3)	(3, 0.4)

The method proposed in this paper is used to find a fully satisfactory solution for both parties. The process is briefly described in the following steps:

1) Assuming that a five-level ranking system $\{H_1, H_2, H_3, H_4, H_5\}$ is used, based on Equation (1), we know that suppliers' standard ranking values are $\{Y_1, Y_2, Y_3, Y_4, Y_5\} = \{1, 3, 5, 7, 9\}$, and the retailers' standard ranking values are $\{Y_1, Y_2, Y_3, Y_4, Y_5\} = \{1, 2, 3, 4, 5\}$.

2) To determine the data relationship between preference order information and standard ranking values, in accordance with Equations (3)~(5), we transform the suppliers' and retailers' preference ordinal values under different uncertain states into rank belief degree information.

3) From Equations (6)~(12), we obtain two-sided average satisfaction degrees, as shown in Table 3.

4) From Equations (13)~(14), by employing satisfaction degrees, we find the following two-sided matching solution: A_1 and B_5 , A_2 and B_2 , A_4 and B_3 , A_5 and B_1 , A_6 and B_4 ; that is, $A_1 \leftrightarrow B_5, A_2 \leftrightarrow B_2, A_4 \leftrightarrow B_3, A_5 \leftrightarrow B_1, A_6 \leftrightarrow B_4$.

Table 3 Retailer and supplier average satisfaction degrees

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
B_1	0.769	0.756	0.311	0.384	0.755	0.744	0.433	0.581	0.200
B_2	0.574	0.683	0.514	0.429	0.358	0.658	0.653	0.369	0.615
B_3	0.737	0.569	0.326	0.654	0.666	0.465	0.626	0.371	0.480
B_4	0.244	0.702	0.488	0.602	0.786	0.826	0.508	0.500	0.741
B_5	0.800	0.390	0.270	0.156	0.514	0.684	0.613	0.523	0.675

4 Conclusion

Research on two-sided matching decision-making with uncertainty under various states is a beneficial addition to the existing body of matching decision-making theory, as it has considerable application value and research significance. Using the foundation of ER, this paper proposed tools for solving two-sided matching problems with uncertain preference ordinal values under multiple states. This reasoning is simple and logical, and the method was verified as feasible through an illustration. The proposed method is suitable for solving two-sided matching problems with uncertainty under different states in fields such as economic management and human resources. However, more questions remain, such as how we may obtain the occurrence probabilities of states and so forth.

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