Optimal Two-Part Tariff Licensing in a Differentiated Mixed Duopoly

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Abstract This paper considers the two-part tariff licensing by an innovating firm to its potential competitor in a differentiated mixed duopoly, in which one firm sets a quantity and the other firm charges a price. Based on the development cost incurred by the rival, we derive the optimal behavior of the firms under full information case and partial information case respectively. Information difference on the equilibrium strategies is also investigated.

Keywords two-part tariff; full information; partial information; mixed duopoly

1 Introduction

Technology licensing is one of the main forms of technology transactions in the technology market and takes place in almost all industries. The method to determine the price item in the technology transfer contract is the core of the technology licensing. The common modes of patent licensing include the followings forms: (i) A royalty on per-unit of output produced with the patented technology; (ii) A fixed-fee that is independent of the quantity produced with the patented technology; and (iii) a two-part tariff, i.e., a fixed-fee plus royalty. By using the data from United States, early work by Rostoker^[1] finds that 46% of licensing contracts use two-part tariff, a fixed fee alone in 13%, and 39% use royalties alone.

Theoretical research of patent licensing can be divided into two types of class: One, where the patentee is outside the market of operation and not a competitor in the product; the other where the patentee is inside the market and competes with licensee. In the case when the patentee is an insider and competition in the output market is Cournot, Wang^[2] compared licensing by means of fixed fees and royalty licensing in a homogeneous good duopoly. Then Wang^[3] extended the homogeneous model to a differentiated goods duopoly and found that licensing by means of royalty may be superior to licensing by means of a fixed fee from the viewpoint of the patent-holding form. While Muto^[4] and Wang and Yang^[5] considered the licensing contract in the Bertrand structure with differentiated products. Recently, some works

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address the question of licensing contracts in a Stackelberg competition when the innovator is also an insider in the industry. Ferreira and Bode^[6] investigated a Stackelberg model where the leader firm engages in an R&D process that gives endogenous cost-reducing innovation and study the licensing of the cost-reduction by a two-part tariff method. Nguyen, et al.^[7] developed a duopoly model of vertical product differentiation and outcomes between Bertrand and Cournot competition are compared, among many others.

Most of the existing literature on technology licensing describes studies of pure duopoly setup. Recently, the economic implications of the mixed duopoly market have been an issue with respect to the change in competition for market structure. The mixed duopoly is an intermediate model between a pure Cournot duopoly and a pure Bertrand duopoly, in which one firm adjusts its quantity and the other firm its price. The mixed duopoly model was first considered by Bylka and Komar^[8] and then extended in various directions. Kreps and Scheinkman^[9] argue that whether firms compete in output or price is ultimately an empirical question. Tremblay, et al.^[10] revealed that mixed duopoly behavior is observed in the small cars market, where Saturn and Scion dealers set prices and Honda and Subaru dealers set quantities. The interested readers are referred to Choi^[11] and the references therein for related research under mixed duopoly market structure.

In the present paper, we will analyze the licensing by means of a two-part tariff in a differentiated good mixed duopoly, where an incumbent innovator competes with a potential rival who may self-develop the technology. Following Kitagawa, et al.^[12], we investigate the market equilibrium based on two types of scenarios for the technology development cost. The purpose of the present paper is two fold. The first is to study two-part tariff licensing by an incumbent innovator to its competitor in a differentiated product duopoly with mixed competition. The other examines how the market information affects the equilibrium.

The remainder of the paper is organized as follows. Section 2 introduces the model. In Section 3, we analyze the optimal licensing mechanism with full information in which the behavioral strategy of the two firms is common knowledge. Section 4 examines the optimal contracts with partial information in which the behavioral strategy of the rival firm is unknown. Information difference on the equilibria is also discussed. Concluding remarks are given in Section 5.

2 The Basic Model

There are two firms in a market, denoted by firm 1 and firm 2. One of the firms, which without loss of generality we will assume is firm 1, owns a technology for a new product and can license the technology to firm 2, who may alternatively self-develop the technology for an imperfectly substitutable product.

The sequence of events is summarized as follows. In the first period, firm 1 decides whether to offer the licensing to firm 2. If firm 1 offers licensing, firm 2 may accept or reject it in period 2. In the later case, firm 2 may stay out of the market or enter the market by self-developing the technology, which may incur a cost $J > 0$. If firm 1 does not offer licensing, firm 2 has two options in period 2: stay out of the market or enter the market by investing the technology. If firm 2 enters the market, both firms engage in the mixed duopoly competition in period 3.

More precisely, firm 1 behaves as a Cournot-type firm by competing in output, firm 2 behaves as a Bertrand-type firm competing in price. This kind of mixed duopoly model is called Cournot-Bertrand (CB) competition in literature. We remark that there exists another heterogeneous competition in which different firms take different strategies: firm 1 is pricing and firm 2 is quantity adjusting (the so-called Bertrand-Cournot competition). Since the analysis is similar, we restrict our study only to the optimal two-part tariff licensing in *CB* competition.

The basic structure of our research is a differentiated duopoly, which is a simplified version of Sing and Vives^[13]. The inverse demand is characterized by

$$p_i = a - q_i - \theta q_j, \quad i \neq j, \quad i, j = 1, 2, \quad (1)$$

where p_i is firm i 's market price and q_i denotes the output of firm $i = 1, 2$. The parameter $\theta \in (0, 1)$ represents the degree of substitution among the competing products. The two goods become closer substitute as θ approaches to one.

As mentioned before, in the present paper we are mainly focus on the two-part tariff licensing contracts. Firm 1 licenses the use of the innovation in exchange for a fixed fee $F \geq 0$ and a royalty rate of $r \geq 0$, where F is independent of the level of production. Hereafter, we denote such a two-part tariff contract as (r, F) . If firm 2 accepts the offer, it should pay a licensing fee of $r q_2 + F$ to firm 1. For analytical convenience, the following assumptions are adopted throughout the rest of the paper.

Assumption 1 The unit cost of the production for the two firms is equal and normalized to zero.

Assumption 2 If firm 1 is indifferent about offering and not offering the licensing, it chooses not to offer the contract.

Assumption 3 In the licensing process, when firm 2 is indifferent between accepting and rejecting the offer, it chooses to accept the offer.

Assumption 4 If firm 2 is indifferent between entering and staying out the market, it chooses to enter the market.

3 The Optimal Licensing Contract in Full Information Case

In this section we assume that the behavioral strategy of the firms is common knowledge. Under this case, firm 1 knows that its rival is price adjusting and firm 2 knows that firm 1 is quantity adjusting.

Consider the subgame that firm 1 chooses not to offer the licensing. If firm 2 enters the market by self-developing the technology, the two firms engage in *CB* competition. From (1), we can write the demand system in the two strategic variables, q_1 and p_2 :

$$p_1 = a(1 - \theta) - (1 - \theta^2)q_1 + \theta p_2, \quad q_2 = a - p_2 - \theta q_1, \quad (2)$$

then the profit function for firm i can be expressed in terms of q_1 and p_2 as:

$$\pi_1 = [a(1 - \theta) - (1 - \theta^2)q_1 + \theta p_2]q_1, \quad \pi_2 = [a - p_2 - \theta q_1]p_2 - J. \quad (3)$$

Solving the first-order conditions for profit maximization of π_i yields the following linear best reply functions:

$$q_1 = \frac{(1 - \theta)a + \theta p_2}{2(1 - \theta^2)}, \quad p_2 = \frac{a - \theta q_1}{2}, \quad (4)$$

Equations (2) and (4) imply that the outputs at the equilibrium are

$$q_1^* = \frac{(2-\theta)a}{4-3\theta^2}, \quad q_2^* = \frac{(2-\theta-\theta^2)a}{4-3\theta^2}. \quad (5)$$

Substituting the outputs and prices into (3) yields the CB equilibrium profits:

$$\pi_1^* = \frac{(1-\theta^2)(2-\theta)^2 a^2}{(4-3\theta^2)^2}, \quad \pi_2^* = \frac{(2-\theta-\theta^2)^2 a^2}{(4-3\theta^2)^2} - J. \quad (6)$$

Introduce the notation

$$J_1 = \frac{(2-\theta-\theta^2)^2 a^2}{(4-3\theta^2)^2},$$

then (6) implies that firm 2 enters the market if and only if $J \leq J_1$. In the case of $J > J_1$, firm 2 will stay out of the market and duopoly competition turns into monopoly. Thus, we get the payoff $\pi_1 = \frac{a^2}{4}$ for firm 1 after some simple algebras.

Now we investigate the subgame that firm 1 chooses to offer the contract (r, F) , we have the following result.

Lemma 1 *In CB competition with full information, suppose that firm 1 offers the contract (r, F) and firm 2 accepts it. If the royalty rate $r \leq \hat{r} \doteq \frac{(2-\theta-\theta^2)a}{2(1-\theta^2)}$, the profits for the two firms are determined by*

$$\pi_1^*(r, F) = \frac{-(8-11\theta^2+4\theta^4)r^2 + (8-12\theta^2+\theta^3+4\theta^4)ar + (1-\theta^2)(2-\theta)^2 a^2}{(4-3\theta^2)^2} + F, \quad (7)$$

$$\pi_2^*(r, F) = \frac{[(2-\theta-\theta^2)a - 2(1-\theta^2)r]^2}{(4-3\theta^2)^2} - F. \quad (8)$$

If the royalty rate $r > \hat{r}$, the only contract accepted by firm 2 is $(r, 0)$. In this case, the payoffs of the two firms are given by

$$\pi_1^*(r, 0) = \frac{a^2}{4}, \quad \pi_2^*(r, 0) = 0. \quad (9)$$

Proof Since firm 2 accepts the contract (r, F) , the payoffs of the two firms are modified to the following forms:

$$\pi_1 = p_1 q_1 + r q_2 + F, \quad \pi_2 = (p_2 - r) q_2 - F, \quad (10)$$

substitution (2) into (10), the first-order conditions ultimately yield the equilibrium outputs:

$$q_1^*(r) = \frac{(2-\theta)a - \theta r}{4-3\theta^2}, \quad q_2^*(r) = \frac{(2-\theta-\theta^2)a - 2(1-\theta^2)r}{4-3\theta^2}. \quad (11)$$

By noting that the inequality $\frac{(2-\theta-\theta^2)a}{2(1-\theta^2)} < \frac{(2-\theta)a}{\theta}$ holds for any $\theta \in (0, 1)$, the nonnegativity condition for CB outputs is given as $r \leq \hat{r}$. By (11), we obtain the CB equilibrium profits $\pi_i^*(r, F)$ after some calculations.

When the royalty rate $r > \hat{r}$, the only contract accepted by firm 2 is $(r, 0)$. In this case, it is obvious that the profit functions in (9) hold.

Now we are ready to give the optimal licensing contracts. We first consider the case $J > J_1$. Under this scenario, firm 2 will stay out of the market if it rejects the offer. We have the following result:

Theorem 1 *In CB competition with full information, suppose that the self-developing cost of firm 2 satisfies $J > J_1$. Then the optimal two-part tariff (r^*, F^*) is given by*

$$r^* = \frac{\theta a}{2}, \quad F^* = \frac{(1 - \theta)^2(2 - \theta^2)a^2}{(4 - 3\theta^2)^2}, \quad (12)$$

the equilibrium payoffs are

$$\pi_1^*(r^*, F^*) = \frac{(8 - 8\theta + \theta^2)a^2}{4(4 - 3\theta^2)}, \quad \pi_2^*(r^*, F^*) = 0. \quad (13)$$

Proof Since $J > J_1$, firm 2 will stay out of the market without accepting the licensing. By Lemma 1, if the royalty rate $r \leq \hat{r}$, firm 2 accepts the offer if and only if $\pi_2^*(r, F) \geq 0$, or equivalently,

$$F \leq F(r) \doteq \frac{[(2 - \theta - \theta^2)a - 2(1 - \theta^2)r]^2}{(4 - 3\theta^2)^2}. \quad (14)$$

Using (7) and (14), we conclude that the optimal contract (r, F) should satisfy $F = F(r)$. Then substitution the expression $F(r)$ in (7) we obtain

$$\pi_1(r, F(r)) = \frac{-r^2 + \theta ar + 2(1 - \theta)a^2}{4 - 3\theta^2}. \quad (15)$$

By (15), we know that the quadratic function is maximized at $r^* = \frac{\theta a}{2}$. For $\theta \in (0, 1)$, it is straight to verify that $r^* < \hat{r}$ holds. Further, substitution $r = r^*$ into (15) leads to the first equality in (13), where $F^* \doteq F(r^*)$ is calculated as (13).

For $r > \hat{r}$, if firm 2 accepts the licensing, by Lemma 1, the payoff of firm 1 becomes $\frac{a^2}{4}$. On the other hand, if firm 2 reject the offer, firm 1 becomes a monopolist and the profit also equals to $\frac{a^2}{4}$. However, note that the inequality $\frac{(8 - 8\theta + \theta^2)a^2}{4(4 - 3\theta^2)} > \frac{a^2}{4}$ holds for any $\theta \in (0, 1)$, we conclude that (r^*, F^*) is the optimal licensing contract.

Next we turn to the scenario that $J \leq J_1$. To enter to market, firm 2 has two options between self-developing the technology and accepts the licensing. Define

$$J_2 = \frac{\theta(1 - \theta^2)(4 - 3\theta - 2\theta^2 + \theta^3)a^2}{(4 - 3\theta^2)^2},$$

then straight calculations reveal that $J_2 < J_1$ holds for any $\theta \in (0, 1)$.

Theorem 2 *Consider CB competition with full information, we have*

(i) *If the self-developing cost of firm 2 satisfies $J_2 \leq J \leq J_1$, the optimal two-part tariff (r^*, F_1^*) is given by $r^* = \frac{\theta a}{2}$ and $F_1^* = J - J_2$, the equilibrium payoffs are*

$$\pi_1^*(r^*, F_1^*) = \frac{[(4 - 3\theta^2)\theta^2 + 4(1 - \theta^2)(2 - \theta^2)]a^2}{4(4 - 3\theta^2)} + J, \quad (16)$$

$$\pi_2^*(r^*, F_1^*) = \frac{(2 - \theta - \theta^2)^2 a^2}{(4 - 3\theta^2)^2} - J. \quad (17)$$

(ii) If the developing cost of firm 2 satisfies $J < J_2$, the optimal two-part tariff contract degenerates to a pure royalty contract $(\tilde{r}, 0)$ with

$$\tilde{r} \doteq \frac{a(2 - \theta - \theta^2) - \sqrt{\Delta(\theta, J)}}{2(1 - \theta^2)}, \quad (18)$$

where $\Delta(\theta, J) = (2 - \theta - \theta^2)a^2 - (4 - 3\theta^2)^2 J$, the equilibrium payoffs are

$$\pi_1^*(\tilde{r}, 0) = \frac{-(1 - \theta)d(\theta)a^2 + (2 - \theta^2)(4 - 3\theta^2)a\sqrt{\Delta(\theta, J)}}{(1 - \theta)(4 - 3\theta^2)^2(1 + \theta^2)} + \frac{(8 - 11\theta^2 + 4\theta^4)J}{4(1 - \theta^2)^2}, \quad (19)$$

in which $d(\theta) = 12 + 4\theta - 13\theta^2 - 4\theta^3 + 2\theta^4 + \theta^5 + \theta^6$, and firm 2's profit $\pi_2^*(\tilde{r}, 0) = \pi_2^*(r^*, F_1^*)$.

Proof Consider the two-part tariff contract (r, F) with $r \leq \hat{r}$. By Eqs. (6) and (8), firm 2 accepts the offer if and only if

$$\frac{[(2 - \theta - \theta^2)a - 2(1 - \theta^2)r]^2}{(4 - 3\theta^2)^2} - F \geq \frac{(2 - \theta - \theta^2)^2 a^2}{(4 - 3\theta^2)^2} - J,$$

or equivalently

$$F \leq F_1(r) \doteq \frac{4(1 - \theta^2)r[(1 - \theta^2)r - (2 - \theta - \theta^2)a]}{(4 - 3\theta^2)^2} + J. \quad (20)$$

From (7) and (20), to maximize $\pi_1^*(r, F)$, it must be the case that $F = F_1(r)$ with $F_1(r) \geq 0$. It is easy to verify that $F_1(r) \geq 0$ is equivalent to $r \leq \tilde{r}$, where $\tilde{r}$ is defined by (18). Plugging the expression $F_1(r)$ in (7), we obtain

$$\pi_1^*(r, F_1(r)) = \frac{-(4 - 3\theta^2)r^2 + \theta(4 - 3\theta^2)ar + (1 - \theta^2)(2 - \theta)^2 a^2}{(4 - 3\theta^2)^2} + J. \quad (21)$$

Since $\tilde{r} \leq \hat{r}$ holds for $\theta \in (0, 1)$, firm 1 now solves the problem $\max_{0 \leq r \leq \tilde{r}} \pi_1^*(r, F_1(r))$. This implies that (21) is maximized at $r^* = \frac{\theta a}{2}$ if $r^* \leq \tilde{r}$, or at the corner solution $\tilde{r}$ if $r^* > \tilde{r}$.

By some simple algebras, we know that $r^* \leq \tilde{r}$ is equivalent to $J \geq J_2$. Substitution $r = r^* = \frac{\theta a}{2}$ into $F_1(r)$ and (23), we get $F_1^* = F_1(r^*) = J - J_2$ and (16) respectively.

In the case of $J > J_2$, since $F_1(\tilde{r}) = 0$, the two-part tariff contract degenerates to a pure royalty contract $(\tilde{r}, 0)$. Careful calculations lead to (19). Firm 2's equilibrium profit function in (17) follows apparently.

Lastly, we consider the special case $r > \hat{r}$. Under this scenario, if $J < J_1$, firm 2 will reject the licensing and enter the market by self-developing. The *CB* equilibrium profits are given by (6). However, it is obvious that the contract $(0, J)$ with $J > 0$ satisfies (9) and $\pi_1^*(0, J) > \pi_1^*$. On the other hand, if $J = J_1$, the only contract firm 2 can accept is $(r, 0)$ and firm 1 gains $\frac{a^2}{4}$. Again, direct calculations yield $\pi_1^*(0, J_1) > \frac{a^2}{4}$. Therefore, in order to maximize the profit of firm 1, it is sufficient to consider the case $r \leq \hat{r}$. This completes the proof of Theorem 3.

4 Partially Informed Case and Information Difference

In this section we consider the licensing contract in *CB* competition with partial information, in which none of the firms have enough information to observe its rival's strategic behavior and hence each of them determines its best choice, assuming that its rival takes the same strategy. More precisely, firm 1 believes that it is in a Cournot competition and firm 2 thinks that it is

in a Bertrand competition. To distinguish the situation with full information, we use “ $\star$ ” to indicate the equilibrium solution in the partial information case.

Following the procedure processed in Section 3, we first study the subgame that firm 1 chooses not to offer the licensing and firm 2 enters the market by self-developing the technology. From (1), we can write q_2 in terms of p_1 and p_2 as

$$q_2 = \frac{a(1-\theta) + \theta p_1 - p_2}{1-\theta^2}. \quad (22)$$

As explained above, firm 1's profit-maximizing choice of output is q_1 , subject to the believed strategic variable q_2 of firm 2:

$$\max_{q_1} \pi_1 = (a - q_1 - \theta q_2)q_1,$$

firm 2's profit-maximizing choice of price is then p_2 :

$$\max_{p_2} \pi_2 = p_2 q_2 - J,$$

subject to (22). Jointly solving the first-order conditions yields:

$$q_1 = \frac{a - \theta q_2}{2}, \quad p_2 = \frac{(1-\theta)a + \theta p_1}{2}. \quad (23)$$

Due to the partial information, expressions in (23) depend on the variable that the rival firm is supposed to pick. To express both best response functions in terms of the true strategic variables q_1 and p_2 , substitution (2) into (23) gives the equilibrium outputs in partially informed case:

$$q_1^* = \frac{(2-\theta-\theta^2)a}{4-3\theta^2}, \quad q_2^* = \frac{(2-\theta)a}{4-3\theta^2}, \quad (24)$$

and the equilibrium payoffs are:

$$\pi_1^* = \frac{(2-\theta-\theta^2)^2 a^2}{(4-3\theta^2)^2}, \quad \pi_2^* = \frac{(1-\theta^2)(2-\theta)^2 a^2}{(4-3\theta^2)^2} - J. \quad (25)$$

Introduce a new symbol as

$$J_3 = \frac{(1-\theta^2)(2-\theta)^2 a^2}{(4-3\theta^2)^2},$$

then (25) implies that firm 2 enters the market in status quo if and only if $J \leq J_3$. In the case of $J > J_3$, firm 1 monopolizes the market and gains $\frac{a^2}{4}$.

Now we investigate the subgame that firm 1 chooses to offer the contract (r, F) , we have the following lemma, which is the counterpart with full information.

Lemma 2 *In CB competition with partial information, suppose that firm 1 offers the contract (r, F) and firm 2 accepts it. If the royalty rate $r \leq \bar{r} \doteq \frac{(2-\theta)a}{2}$, the profits for the two firms are determined by*

$$\pi_1^*(r, F) = \frac{-(8-7\theta^2)r^2 + (8-8\theta^2+\theta^3)ar + (2-\theta-\theta^2)^2 a^2}{(4-3\theta^2)^2} + F,$$

$$\pi_2^*(r, F) = \frac{(1 - \theta^2)[(2 - \theta)a - 2r]^2}{(4 - 3\theta^2)^2} - F.$$

If the royalty rate $r > \tilde{r}$, the only contract accepted by firm 2 is $(r, 0)$. In this case, the payoffs of the two firms are given by

$$\pi_1^*(r, 0) = \frac{a^2}{4}, \quad \pi_2^*(r, 0) = 0.$$

Proof The proof is similar to Lemma 1 except for distinguishing the strategic variables selected by the firms. Imitating the process discussed above for the partial information case, we ultimately obtain the equilibrium outputs in the present scenario:

$$q_1^*(r) = \frac{(2 - \theta - \theta^2)a + \theta r}{4 - 3\theta^2}, \quad q_2^*(r) = \frac{(2 - \theta)a - 2r}{4 - 3\theta^2}. \quad (26)$$

By (26), the nonnegativity condition for CB outputs in partial information is given as $r \leq \tilde{r}$. The rest of results in Lemma 4 involve only simple calculations.

Along much the same lines proceeded in Section 3, in what follows we give results based on two types of scenarios for the developing cost: $J > J_3$ and $J \leq J_3$. Since the discussions are similar to the full informed case, we only list the conclusions and omit the concrete proofs.

Theorem 3 *In CB competition with partial information, suppose that the self-developing cost of firm 2 satisfies $J > J_3$. Then the optimal two-part tariff (r^*, F^*) is given by*

$$r^* = \frac{\theta a}{2}, \quad F^* = \frac{4(1 - \theta^2)(2 - \theta)^2 a^2}{4 - 3\theta^2},$$

the equilibrium payoffs are

$$\pi_1^*(r^*, F^*) = \frac{(8 - 8\theta + \theta^2)a^2}{4(4 - 3\theta^2)}, \quad \pi_2^*(r^*, F^*) = 0.$$

Next we turn to the scenario that $J \leq J_3$. Before proceeding further, we define

$$J_4 = \frac{\theta(1 - \theta^2)(4 - 3\theta)a^2}{(4 - 3\theta^2)^2},$$

one can easily check that $J_4 < J_3$ for any $\theta \in (0, 1)$.

Theorem 4 *Consider CB competition with partial information, we have*

(i) *If the self-developing cost of firm 2 satisfies $J_4 \leq J \leq J_3$, the optimal two-part tariff (r^*, F_1^*) is given by $r^* = \frac{\theta a}{2}$ and $F_1^* = J - J_4$, the equilibrium payoffs are*

$$\begin{aligned} \pi_1^*(r^*, F_1^*) &= \frac{[\theta^2(4 - 3\theta^2) + 4(2 - \theta - \theta^2)]a^2}{4(4 - 3\theta^2)} + J, \\ \pi_2^*(r^*, F_1^*) &= \frac{(1 - \theta^2)(2 - \theta)^2 a^2}{(4 - 3\theta^2)^2} - J. \end{aligned} \quad (27)$$

(ii) *If the developing cost of firm 2 satisfies $J < J_4$, the optimal two-part tariff contract degenerates to a pure royalty contract $(\tilde{r}, 0)$ with*

$$\tilde{r} \doteq \frac{a(2 - \theta)(1 - \theta^2) - \sqrt{\nabla(\theta, J)}}{2(1 - \theta^2)},$$

where $\nabla(\theta, J) = (1 - \theta^2)[(1 - \theta^2)(2 - \theta)^2 a^2 - (4 - 3\theta^2)^2 J]$, and the equilibrium payoffs are

$$\pi_1^*(\tilde{r}, 0) = \frac{-(1 - \theta^2)k(\theta)a^2 + (4 - 3\theta^2)a\sqrt{\nabla(\theta, J)}}{(1 + \theta)(4 - 3\theta^2)^2} + \frac{(8 - 7\theta^2)J}{4(1 - \theta^2)},$$

in which $k(\theta) = 4 - 4\theta - 3\theta + 4\theta^3$, and $\pi_2^*(\tilde{r}, 0) = \pi_2^*(r^*, F_1^*)$, given by (27).

It is interesting to note that the royalties are identical to each other ($r^* = r^*$) in the optimal two-part tariff contracts under different informed scenarios. However, the fixed fees are varied with the cost parameter and information condition. Therefore, the equilibria must be affected by the uncertainty about the rival firm's behavior. To this end, we compare the equilibrium strategies under different information occasions to show the effects caused by information asymmetry. Observing the results obtained under full information case and those under partial information, we know that the comparison depends on the relationships among $J_i, i = 1, 2, 3, 4$. From the definitions, the inequalities $J_1 < \max\{J_2, J_4\} < J_3$ is always true for $\theta \in (0, 1)$. One can also verify numerically that $J_2 < J_4$ for $\theta > 0.6889$ and the inequality is reversed otherwise. Applying the conclusions derived in Theorems above, we summarize our results as follows.

Theorem 5 *If the self-developing cost of firm 2 satisfies $J > J_3$, information difference has no effect on the equilibrium payoffs for both firms. In parameter region $J \leq J_3$, firm 1 (firm 2) is better off (worse off) under fully informed case than under the partially informed case.*

5 Concluding Remarks

This paper extends Kitagawa, et al.'s^[12] differentiated Cournot duopoly model to a mixed duopoly market, in which an incumbent innovator offers the licensing by a two-part tariff to a potential competitor, who may alternatively develop a compatible technology. Most patent licensing papers in literature deal with Cournot or Bertrand competition in oligopolistic market. To the best of our knowledge, very little has been done with respect to differentiated mixed duopoly and the effects caused by the behavioral uncertainty on the equilibrium. However, empirical evidence of mixed duopoly is abundant (see, for example, [10]). The main aim of this paper is to shed lights on these problems. We derive optimal two-part tariff licensing contracts in mixed duopoly under full information case and incomplete information case, respectively. We also discuss the information difference on the equilibrium strategies. Note that we get different conclusions depending on the development cost incurred by the licensee.

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