Study on Pollution Cost Control Model under Asymmetric Information Based on Principal Agent

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Abstract  Pollution cost control is key to solve pollution problem. The paper takes pollution control contract between management authority and pollutant discharge enterprise as research object, considers pollution control quality level, pollution control quality inspection and pollution control cost model, and establishes pollution control cost model of management authority and pollutant discharge enterprise, including rational constraints of pollutant discharge enterprise. And it analyzes principal-agent relationship between the two under condition of asymmetric information, and un-observability of pollution control level is shown as hiding information of sewage enterprises. In essence, it is problem of adverse selection in principal-agent. Pollution control cost of management is objective function. The first order condition of pollution control cost of sewage enterprise is transformed into state space equation, and optimal control of problem is solved by using maximum principle. In particular, management authority, as principal, uses pollution control provisions to reward, punish and encourage pollutant discharge enterprises as agents.

Keywords  principal agent; asymmetric information; pollution cost control

1 Introduction

Emission cost control is key to solve problem of pollution[1]. Under premise of implementation of emission permit system, emission enterprises can purchase emission rights provided by management authority. In this way, management authority can invest in pollution control projects of emission enterprises, and formulate pollution level within allowable range of emission enterprises[2]. At the same time, emission enterprises can control pollution within allowable pollution level. In order to get corresponding policy preference given by management authority, opposite is also true. Of course, pollutant discharge enterprises may not purchase pollutant
discharge right, however, because they will inevitably discharge pollutants and have no effective pollution control means, they will exceed the pollution standard and bear risk of being forced to close down. The loss of being forced to close down is bound to be greater than the cost of purchasing pollution right provided by management authority. In fact, several literatures have asymmetric study on SCM. Esmaeili, et al. gave models of supply chain management with an asymmetric information structure. Yao, et al. studied vertical cost information sharing in a supply chain with value-adding retailers. Xiao, et al. created price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers. Gaudet, et al. discussed real investment decision under adjustment costs and asymmetric information. Yuan, et al. creatively showed material options optimum investment problem models. Besides, Tsay, et al. told us whole picture of modelling supply chain contracts review. Yeom, et al. analysed role of transfer price for coordination and control within a firm. Wang, et al. tried to make SCM implementation decisions under asymmetric information. In addition, some people turn their research focus on environmental protection. Domeier used intergovernmental approaches to achieve hazards and sustainability. Xing, et al. explained multi-pollution collaborative control technology in iron and steel industry. Zhu, et al. studied emission characteristics of multiple pollutants from iron- steel sinter flue gas. Feng, et al. insisted on research on current ambient and elevated ozone effects on poplar. Pu studied technology of multi-pollution optimization models. Song, et al. analyzed energy consumption atmospheric pollutant emissions and carbon emissions. Hu, et al. gave agglomeration characteristics of industrial pollution and their influencing factors on the scale of cities in China. Innes, et al. figured out collaborative rationality for public policy. Kaufman gave out risk perception and communication in public and environmental decisions. Sam, et al. did an empirical study on voluntary pollution reduction programs, environmental management, and environmental performance. Yudkowsky critically discussed cognitive biases potentially affecting judgements of global risks.

In one word, less literatures focus on study of applying theories of principal-agent, asymmetric information on pollution cost and/or establish relative control models which is an urgent problem to be solved. The paper just chooses the very view point to deploy research. Basically, as a rational enterprise with profit as goal and sustainable operation as premise, it generally needs to purchase emission rights. Therefore, incentive mechanism of pollution control can be designed according to principal-agent theory. In addition, according to theory of asymmetric information, under condition of emission cost control, private information of emission amount owned by agent of emission enterprises is uncontrollable exogenous information, which is in line with concept of hidden information. Under condition of asymmetric information, both sides can realize information asymmetry, which will be reflected in their pollution control behaviour. In information hiding environment, the most important problem is that in process of contract arrangement, contract selection of sewage enterprises may be contrary to expectation of management authorities, resulting in adverse selection. In the case of hidden behaviour, pollutant discharge enterprises may also make some behaviours that are harmful to interests of management authorities (social benefits) from their own interests, forming moral hazard.

In this paper, pollution control cost of pollution control contract between management
authority and pollutant discharge enterprise is taken as research object. Considering pollution control quality level, pollution control quality inspection and pollution control cost model, pollution control cost model of management authority and pollutant discharge enterprise is established, including rational constraints of pollutant discharge enterprise. The paper analyses principal-agent relationship between the two under condition of asymmetric information, and inconceivability of pollution control level is shown as hiding information of sewage enterprises. In essence, it is a problem of adverse selection in principal-agent. The pollution control cost of management is objective function. The first order condition of pollution control cost of sewage enterprise is transformed into state space equation, and optimal control of problem is solved by using maximum principle. In particular, management authority, as principal, uses pollution control provisions to reward, punish and encourage pollutant discharge enterprises as agents.

2 Cost Model of Incentive Mechanism for Pollution Control

Now consider problem of pollution control cost between management and enterprises. This problem includes three parts: Pollution control level, pollution control level test and pollution control cost model. Relative propositions are as follows.

**Proposition 1** Pollution control level is a set of inherent characteristics and degree to which it meets requirements of environmental protection.

**Proposition 2** The only standard of pollution control is to take environmental protection requirements as center and reach level of environmental pollution control.

**Proposition 3** The level of environmental pollution control can be described by an independent continuous random variable, and choice of pollution control level is considered in study of pollution control cost.

**Proposition 4** The inspection of pollution control level is inspection of pollution control level of sewage enterprises by management authority in problem of pollution cost control. According to inspection results, management authorities decide whether to accept or reject pollution control projects of pollutant discharge enterprises.

**Proposition 5** In sampling inspection, management authority randomly selects a sample from pollution control projects of pollutant discharge enterprises. If number of unqualified products in sample is more than number of qualified judgement, management will refuse batch of pollution control items.

Based on above, probability of acceptance in sampling inspection can be described as

$$
\Gamma(m) = \sum_{d=0}^{c} C_{n}^{d} \phi^{d} (1 - \phi)^{n-d},
$$

(2.1)

where $\Gamma(m)$ is acceptance probability in sampling inspection, $m$ is pollution control level; $\phi = \Phi x(T, m, \sigma) = P(x \leq T, m, \sigma)$ is cumulative distribution function of random variables, probability distribution function $\varphi x(T, m, \sigma)$ is function value of variable distribution function when $x = t$, $x$ is a random variable, $t$ is a specific value, $T$ is a low limit, and $\sigma$ is standard deviation of distribution. In sampling inspection, management authority will select $n$ project samples from pollution control results $L$. $d$ is number of project samples that do not meet pollution control standards, and $c$ is qualified judgement number of sampling inspection. $C_{n}^{d}$
is a combination of \( d \) unqualified samples taken from project sample \( n \). \( C_n^d \Phi^d (1 - \Phi)^{n-d} \) is a binomial distribution probability describing \( d \) unqualified project samples in \( n \) projects. The cost of pollution control is composed of two models: Management authority and enterprise.

Management cost model, i.e.,

\[
C_b = P + \frac{rnD}{L}.
\]  

(2.2)

Among them, \( C_b \) is management’s pollution control cost, and the first \( P \) is management’s pollution control supervision provision for sewage enterprises. The provision is a cost that must be paid in operation of economic management, but amount or time has not been determined. Here, provision is incentive of pollution control by management authorities. The meaning of pollution control supervision of provision indicates that if pollution control cost of project provided by pollutant discharge enterprise is satisfactory, pollutant discharge enterprise fee \( P \) will be rewarded; if pollution control cost of product provided is not satisfactory, pollutant discharge enterprise fee \( P \) will be punished; if pollution control cost just reaches satisfactory level, \( P = 0 \). Therefore, pollution control cost provision \( P \) is a real number. The second item is inspection cost of management authority for batch of pollution control items. \( R \) is unit cost of inspected items, \( n \) is number of samples of inspected items, \( D \) is total number of items to be tested, and \( L \) is batch of items. The cost model of pollution control of sewage enterprises, namely

\[
C_s = -P + \frac{hw(Q - L)}{2} + \frac{kD}{TQ} + \frac{s(1 - \Gamma)D}{\Gamma}.
\]  

(2.3)

Among them, \( C_s \) is cost of pollution control. The first \(-P\) is provision acquisition of pollutant discharge enterprise corresponding to pollution control supervision of management authority. The second is holding cost of pollution control project, where \( h \) is cost coefficient of pollution control project, \( W \) is manufacturing cost of project, and \( Q \) is production batch of project. It is assumed that pollution control project lot \( L \) will be directly delivered to management authority, and only \((Q - L)\) products will enter state of waiting for inspection, resulting in holding cost. The third is to set project cost, where \( k \) is to set project cost coefficient and \( D \) is total project demand. The fourth is internal loss cost of pollution control level. Where \( s \) is internal loss cost parameter of pollution control level test. The number of loss control items in pollution control level inspection is also considered in pollution control cost management,

\[
Q \geq L.
\]  

(2.4)

The economic management significance of the above inequality is that production lot size of pollution control project should be greater than or equal to demand lot size.

3 Pollution Level Control Under Asymmetric Information

In pollution control problems (2.2), (2.3) and (2.4) in study of pollution control cost control, if management authority cannot observe pollution control level \( m \) of project provided by sewage enterprise. For management authorities, pollution control level information of products of enterprises is hidden. This is a problem of adverse selection. Pollution control is also an asymmetric information problem. In problem of pollution control, pollution control cost of management authority and pollutant discharge enterprise is described by Equations (2.2) and
respectively, and rational constraints of production lot and demand lot of pollutant control project of pollutant discharge enterprise are described by Equation (2.4). For convenience of problem handling, this paper introduces quadratic constraint of batch size of pollution control enterprises to replace constraint condition (2.4), that is,

\[ C_{IR} = \frac{1}{2}a(Q - L)^2, \]  

where \( a \) is coefficient of quadratic constraint, \( a > 0 \). It can also be considered that \( C_{IR} \) takes rational constraint (3.1) of management authorities on cooperative relationship of sewage enterprises as a cost of batch supply. Thus, objective function of management authority for pollution control under condition of asymmetric information can be established, which is minimum requirement of management authority for pollution control cost under condition of mathematical expectation, that is,

\[ \min_P C_B = E[C_b + C_{IR}] = \int_{m_1}^{m_2} (C_b + C_{IR}) f(m) dm, \]  

where \( f(m) \) is probability density function of pollution control level \( m \), and pollution control level \( m \) is randomly distributed in interval \([m_1, m_2]\). The objective function describes that management should select control provision \( P \) under expected conditions to minimize cost of pollution control \( C_b \) and cost of rational constraint \( C_{IR} \). Under condition of asymmetric information, pollution control level of supply project of pollutant discharge enterprise as an agent is hidden from management. However, according to principle of disclosure in principal-agent theory, management can design incentive contracts to reveal factors close to real level of pollution control. Therefore, in process of minimizing cost of pollution control, the first derivative treatment will be applied to production lot \( Q \) of project which cannot be observed by management authorities, and of course, reserve variable \( P \) will also be included. For model of sewage enterprises (2.3), there is

\[ \arg \min_{\hat{m}} C_s = C_s(P(\hat{m}), Q(\hat{m})). \]  

The conditions of the model (3.3) of pollutant discharge enterprise can be transformed into the first-order conditions, i.e.,

\[ \tilde{P} = \frac{hw}{2}u - \frac{kD}{T(m)Q^2(m)} u, \]  

\[ \tilde{Q} = u. \]  

It is noted that Equation (3.3) can be derived from the first-order condition of Equation (3.3) about hidden information, i.e. sewage control level \( m \), in which control variable \( u \) is introduced to form Equation (3.5), which means marginal effect of project production lot size on sewage control level \( m \). Obviously, pollution cost control problem (2.2), (2.3), (2.4) under asymmetric information has become an optimal control problem (3.2), (3.4), (3.5). In this optimal control problem, all variables including pollution control level have become \( m \), because pollution control level has been close to real value after revealing principle and incentive strategy. Therefore, pollution control level of following problems are described by real value \( m \). Among them,
Equation (3.2) is objective function, Equations (3.4) and (3.5) are state space equations, \( P(m) \) and \( Q(m) \) are state variables, and \( u(m) \) is control variable. According to maximum principle, Hamiltonian function in question, i.e.,

\[
H = - \left[ P + \frac{rnD}{L} + \frac{1}{2} u(Q - L)^2 \right] f(m) + \lambda_P \left( \frac{hw}{2} u - \frac{kD}{T} Q^2 u \right) + \lambda_Q u. \tag{3.6}
\]

The governing equation of the problem is

\[
\frac{\partial H}{\partial u} = h w \lambda_P - \frac{kD}{TQ^2} \lambda_P + \lambda_Q = 0. \tag{3.7}
\]

The co state equation of problem is

\[
\frac{\partial H}{\partial P} = - f(m) = -\tilde{\lambda}_P, \tag{3.8}
\]

\[
\frac{\partial H}{\partial Q} = - a(Q - L)f(m) + \frac{2kDw}{TQ^3} \lambda_P = -\tilde{\lambda}_Q. \tag{3.9}
\]

The solutions of Equations (3.8) and (3.9) are

\[
\lambda_P(m) = F(m), \tag{3.10}
\]

\[
\lambda_Q(m) = \left( -\frac{hw}{2} + \frac{kD}{TQ^2} \right) F(m). \tag{3.11}
\]

In Equations (3.10) and (3.11), \( F(m) \) is distribution function of pollution control level \( m \). The sorting formulas (3.9), (3.10) and (3.11) have cubic equation about production lot \( Q \) of project, i.e.,

\[
2aI^2 fQ^3 + (h\tilde{w}F + hwf - 2aLf)f^2 Q^2 - 2kDf f + 2kD\tilde{f} F = 0. \tag{3.12}
\]

The sum in Equation (3.12) is expressed as follows:

\[
\tilde{w} = \frac{dw}{dm}, \tag{3.13}
\]

\[
\tilde{\Gamma} = \frac{d\Gamma}{dm} = \sum_{d=0}^c C_n^d \Phi^{d-1}(1 - \Phi)^{n-d-1}(1 - 2\Phi) \frac{\partial \Phi}{\partial m}. \tag{3.14}
\]

To solve cubic equation about production lot size of project, we can get solution by eliminating imaginary root, it can be solved as

\[
Q = \left(-\frac{B^3}{27A^3} - \frac{D}{2A} + \frac{1}{A} \sqrt{\frac{B^3 D}{27A^2} + \frac{D^2}{4}}\right)^* + \left(-\frac{B^3}{27A^3} - \frac{D}{2A} + \frac{1}{A} \sqrt{\frac{B^3 D}{27A^2} + \frac{D^2}{4}}\right)^* - \frac{B}{3A}. \tag{3.15}
\]

Among it, \( A = 2aI^2 f, B = (h\tilde{w}F + hwf - 2aLf)I^2, D = 2kD\tilde{f} F - 2kD\tilde{f} f \).

Considering state space Equations (3.4) and (3.5) of pollution cost control problem, production lot \( Q \) and reserve \( P \) under asymmetric information are solutions (3.15) and (3.4), which are solutions of adverse selection problem. In order to consider expression of \( \Phi \) in Equation (2.1) in Equation (3.14), there are

\[
\Phi = \Phi_X(T, m, \sigma) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx, \tag{3.16}
\]
\[
\frac{\partial \Phi}{\partial m} = \frac{\partial}{\partial m} \int_{-\infty}^{T} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-m)^2}{2\sigma^2}} \, dx = -\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(T-m)^2}{2\sigma^2}}. \tag{3.17}
\]

According to Equations (3.14) and (3.17), therefore

\[
\tilde{\Gamma} = \sum_{d=0}^{c} C_d \phi^{d-1}(1 - \phi)^{d-1}(1 - 2\phi) \left( -\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(T-m)^2}{2\sigma^2}} \right). \tag{3.18}
\]

4 Simulation Experiment on Incentive Mechanism of Emission Cost Control

Suppose that in process of pollution control, an environmental treatment company sets data for cubic equation of production lot \(Q\) of project. The quadratic conditional parameters \(a = 1, C = 0, H = 0.2, k = 20000, d = 20000, t = 100, \sigma = 10, l = 200, n = 5, r = 0.5\) for rational constraint of management authority. Binomial distribution probability distribution function

\[
\Gamma(m) = \sum_{d=0}^{0} C_d^d \phi^d (1 - \phi)^{d-1} = (1 - \phi)^5, \tag{4.1}
\]

\[
\tilde{\Gamma} = \phi^{-1}(1 - \phi)^4(1 - 2\phi) \left( -\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(100-m)^2}{2\sigma^2}} \right). \tag{4.2}
\]

Here, project cost function is assumed to be \(w(m) = 100 + \exp(0.075(m - T))\), which is shown by Equation (3.13), there is

\[
\tilde{w} = 0.075 \exp(0.075(m - 100)). \tag{4.3}
\]

The variable \(P\) of management provision has \(s = 7\), and other parameters are set as formula (3.12). Suppose that hidden information pollution control level \(m\) in Equation (3.4) obeys the uniform distribution \([100, 150]\), and discretize it: \(m = m_0 + n\Delta m, m_0 = 100, \Delta m = 1, \) with

\[
P(m_0 + (n + 1)\Delta m) = P(m_0 + n\Delta m) + \frac{hw}{2} \Delta Q - \frac{kQ^2}{T} \Delta Q. \tag{4.4}
\]

Let \(P(m_0) = 0\), then simulate production batch \(Q\) of project under different information conditions, allowance \(P\) of management authority to sewage enterprise, and cost \(C_b, C_s\). The results are shown as follows:

Under condition of asymmetric information, increase of average pollution control level leads to increase of pollution control cost, which leads to corresponding reduction of project batch.

Corresponding incentive (provision) strategy that management needs to pay to pollutant discharge enterprises according to different project pollution control levels required by management under condition of asymmetric information. It can be seen that with increase of average level of pollution control required, provisions that management needs to pay to pollutant discharge enterprises also increase.

Under condition of asymmetric information, with increase of average pollution control level, pollution control cost of sewage enterprises decreases, pollution control cost of management authorities increases, and total cost of sewage enterprises and management authorities decreases.

Therefore, using this incentive mechanism to control pollution cost and enhance pollution control level of project is conducive to reducing whole pollution control cost and improving management level of pollution control cost as a whole.
5 Conclusion

Emission cost control is an important theoretical and applied problem in management of emission cost control. In this paper, a model of information hiding in emission cost control is established, and optimal control theory is applied to analyze and solve this problem. According to optimal control theory, management can test pollution control items of enterprises one by one. According to pollution control standards and efforts of enterprises, management can use pollution control provisions to motivate enterprises. In order to effectively encourage and control negative emission cost control activities of pollutant discharge enterprises, enterprises that exceed expected effect of pollution control should be compensated and rewarded to a certain extent; enterprises that fail to achieve expected effect should be severely punished and cost of pollution control should not be compensated.

Of course, the above pollution cost control model is only one of feasible models based on the principal-agent theory under condition of asymmetric information. The follow-up research work should start with reward and punishment, supervision, incentive and other mechanisms of pollution control. In addition, should possible outsourcing mode be also one of ideas of pollution control, which is possible research orientation in the future.

Acknowledgements The authors gratefully acknowledge the Editor and two anonymous referees for their insightful comments and helpful suggestions that led to a marked improvement of the article.

References


