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Tackseung Jun, *Kyung Hee University*

Jeong-Yoo Kim, *Kyung Hee University*

Hee-Su Kim, *Korea Information Society Development
Institute*

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Endogenous Internet Structure and Bargaining Power in Interconnection Agreements

Tackseung Jun, Jeong-Yoo Kim, and Hee-Su Kim

Abstract

We consider a model of Internet interconnection. We characterize the Myerson value as the bargaining outcome in interconnection agreements in given networks and provide its interpretation in terms of "betweenness centrality." Then, in an extended network formation model, we characterize the structures of the efficient network and the stable network and confirm the disparity between them. We also identify the condition for the disparity to disappear under bill-and-keep agreements. The possibility of multi-homing and secondary peering is briefly discussed.

KEYWORDS: endogenous network formation, Myerson value, centrality, Internet interconnection

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1 Introduction

The value of the Internet comes from its universal connectivity, the potential for any user to communicate with any other anywhere and to have access to any information anywhere. Such any-to-any connectivity can be achieved as a consequence of Internet interconnection agreements among independent Internet service providers (ISPs).¹ Currently, two types of interconnection arrangements coexist; peering agreements and transit agreements. In a peering agreement,² which is usually made by ISPs of similar size, ISPs do not pay interconnection charges to each other (a bill-and-keep arrangement), while in a transit (or customer) agreement, the customer ISP purchasing transits pays the ISP selling transits for the transit services.

In this paper, we are mainly interested in how Internet interconnection charges should be determined. Our main argument is that the network structure must be a crucial determinant of interconnection charges. It would be far from efficient to connect among all ISP routers. Rather, it would be better that ISPs maintain just indirect links with some other ISPs, as long as other ISPs are willing to provide connectivity to them. In fact, it is better (both from the private and the social point of view) that an ISP sells its connectivity to others rather than denies providing connectivity. Hence, an ISP connected with more ISPs will usually be in a stronger bargaining position in interconnection charge agreements. Then, even if the market shares of two ISPs are similar, they may not end up with a bill-and-keep arrangement. This suggests that the network structure should be seriously taken into consideration in determining interconnection charges.

¹How any-to-any connectivity of the Internet emerged can be summarized as follows. In the early Internet, the NSFNET was the single backbone to achieve any-to any connectivity, interconnecting the research networks of the government, universities and private firms. The NSF encouraged the local/regional networks to seek commercial non-academic customers to expand their facilities, thus leading to the emergence of competitive, long-haul networks such as PSI and UUNET. With a view to phasing out its funding for connections to the Internet, the NSF made a new interconnection architecture whereby Network Access Points (“NAPs”) were created to serve as interconnection nodes between commercial networks and a Routing Arbiter would manage the ever-growing routing tables. In this way, the NSF successfully transferred the management of the backbone infrastructure to industry. What began as reciprocal, traffic-sharing arrangements between networks that interconnected at the centralized NAPs has developed into a number of innovative traffic sharing and pricing arrangements and the emergence of decentralized, private interconnections between networks. See Frischmann (2001) for more details.

²There is no agreed-upon definition of the peering agreement. See Cukier (1998) for various definitions.

The determination of interconnection charges will depend on the specificity of the bargaining procedure and the bargaining solution concept. For example, Besen *et al.* (2001) use the Nash (1953) bargaining solution in bilateral bargaining between any pair among n ISPs for determining their interconnection fees. We show, however, that bilateral bargaining in a network is highly likely to fail to reach an agreement especially when the number of ISPs is large, because the possibility of communication with a large number of other ISPs makes the disagreement payoff of individual ISPs so high as to leave no Pareto superior feasible outcome than the disagreement outcome. Thus, instead of relying on the (extended) Nash solution in bilateral bargaining, we will resort to the Myerson value (1977) as the solution concept of our multilateral bargaining game.

Jackson (2005) raises some criticisms against the Myerson value and proposes some alternative solution concepts. His criticisms basically rely on the possibility of network reformation. Although our game allows the possibility of network change as well, his criticisms are not applicable to our game, because a network is formed in the first stage and given the network bargaining ensues in the second stage in our game.³ The reason that we model the situation as two stages is two-fold. First, the link-forming decision especially between two geographically remote ISPs (e.g., an ISP in U.S.A. and an ISP in a Far Eastern country) is rather long-term relative to the bargaining decision. This implies that it is not always easy for an ISP failing in an interconnection agreement to find an alternative path, due to high connection costs, collusion between ISPs or the possibility of causing a congestion to the alternative route etc. Second, ISPs may renegotiate. Unless renegotiations are prohibited, ISPs do not have to be concerned about the possibility of future network changes in advance when they determine interconnection charges, because they can renegotiate whenever a change in the network structure occurs. Thus, as far as one admits the possibility of renegotiation, it seems reasonable to assume that bargaining among ISPs occurs only after the network formation process is completed.

³In footnote 8 of Jackson (2005), he argues that even if the network is fixed, the Myerson value is not attractive from the normative perspective, since it does not guarantee symmetric allocations to *ex ante* symmetric players. This argument is, however, based on the exclusion of *ex post* renegotiation possibility. If *ex post* renegotiation is possible, this argument is weakened, since *ex ante* players are not necessarily *ex post* symmetric once a certain network is formed.

As the extended Nash solution does, the Myerson value in our game has an interesting interpretation in terms of “(betweenness) centrality” widely used in social network analysis. If Internet users’ utility function is linear, the Myerson value of an ISP can be interpreted as the average of its centrality plus its own connectivity in every possible coalition.

The concept of centrality was initiated by Moreno (1934) who is the founder of sociometry, the precursor to social network analysis. Following him, various formal or informal concepts of importance or prominence in a network, e.g., degree centrality, eigenvector centrality, closeness centrality, betweenness centrality etc. have been proposed.⁴ The betweenness centrality of a person in a network is, roughly speaking, the number of pairs for which he is between them in the shortest path (geodesic) connecting the pair. If he is between the pair, he may charge a fee for their communication or at least control the flow of information exchange between them. In this sense, he is important or central to the pair. This concept was first suggested by Bavelas (1948), Shimbel (1953) and Shaw (1954), and later quantified by Anthonisse (1971).

Having the bargaining game in mind, we also examine how Internet structure will evolve, in other words, what structure of Internet will emerge as a result of the network evolution driven by profit-oriented ISPs. ISPs must decide whether and to whom to connect a link with by taking into account the effect on the subsequent bargaining outcome. We will use the concept of pairwise stability proposed by Jackson and Wolinsky (1996) as our solution concept. This static concept can be understood as a long-run stable state of the dynamic evolution process.⁵ By comparing the stable network with the efficient network, we will demonstrate the possible conflict between the social planner and ISPs. Specifically, we show that if the interconnection charge is determined based on the Myerson value, the stable network cannot be underconnected when the efficiency of a network requires full connectivity.

The structures of the efficient network and the stable network are affected by the possibility of friction in information transmission, in other words, efficiency losses in transmitting information via a longer communication path. The efficient network cannot contain a closed loop for any positive connection cost of a link if there is no friction. However, it turns out not to be the case with friction. For example, in a line network, the communication between two end ISPs takes a very long route, thereby incurring high communication costs

⁴See Wasserman and Faust (1994) for comprehensive reading about the definitions.

⁵Watts (2001) showed the possible disparity between the static stability and the dynamic stability in a deterministic dynamic model. But, if the dynamic process of network formation is stochastic so that any network can be reached with positive probability, the process will not get stuck at an unstable network. See Jackson and Watts (2002) for this point.

if there is friction. In this case, connecting the two end ISPs by a direct link would significantly save the costs. We also show that a star is the most efficient tree in the friction case, while all trees are equally efficient in the no-friction case.

To the best of our knowledge, no paper so far has examined the endogenous network structure to address issue of interconnection charges in telecommunications or Internet. This paper is the first step to the direction of researches. Several papers used the approach of network theory to explain the Internet market. Sgroi (2008) adapted the endogenous network formation approach to analyze the structure of the evolving web. He characterized the conditions for the star network or the complete network to be stable, depending on the connection speed and the complexity cost. D'Ignazio and Giovannetti (2009) showed empirically that the betweenness centrality can explain the Internet interconnection quality and claimed that the regulatory authority should pay attention to the centrality as well as the market share as a measure of market power. Such an empirical finding seems to be consistent with our theoretical result that the betweenness centrality of an ISP determines its bargaining power (monopoly power).

There is vast literature on Internet interconnection. Laffont *et al.* (2003) was the first to analyze the determination of the Internet interconnection charges between two ISPs, based on the consumer benefits and the transmission cost. Weiss and Shin (2004) considered the incentive to make a peering agreement, and showed that using maximum of inbound traffic volume and outbound traffic volume as an alternative criterion to determine the Internet settlement will increase the joint incentive of two ISPs to make a peering arrangement between them. They showed that ISPs have a stronger incentive to enter into a peering arrangement when the traffic volumes are more symmetric. Badasyan and Chakrabarti (2008) considered two ISP's choices between peering and transit contracting and obtained a similar result that the ISP's peer if the traffic flows are fairly symmetric, otherwise they choose transit. Our paper assumes transit agreements in a multilateral bargaining mode, and demonstrates that a similar size or a similar traffic volume of two ISPs is not a sufficient condition for a bill-and-keep agreement between them. Cremer *et al.* (2003) and Economides (2006) examined the possible anti-competitive incentive of a larger Internet Backbone Provider (IBP) in Internet interconnection. Economides criticized the claim by Cremer *et al.* that a larger IBP will degrade interconnection with a targeted rival. He argued that the result of Cremer *et al.* is not robust to variations in their assumptions, especially to the assumption that consumer migration is allowed. However, both of the results hinge on neglecting the feature of paying charges for interconnection.

Outside the realm of Internet, Goyal and Vega-Redondo (2007) consider a similar issue of network formation when nodes in a position to connect a pair of other nodes can earn intermediary rents. There are two major differences between our model and theirs. First, as an allocation rule given network, they use the concept of kernel rather than Myerson value. Second, for stability of a network, they do not use pairwise stability but what they call strictly bilateral equilibrium.⁶ The latter is a stronger concept than the former in the sense that more possible deviations are allowed, for example, for a pair of nodes to form a link and delete their links simultaneously. Consequently, only the star network survives as a nontrivial stable network in their model, while a circle or a complete network can be also stable in our model. In their model, a circle cannot be stable, because a pair of eccentric nodes in a circle have an incentive to increase their centrality by establishing a direct link between them and at the same time severing one of their links. Such a deviation is not allowed in our model. Considering the reality that the Internet structure has many cycles, pairwise stability seems to be better suited to the current model of the Internet.

The organization of the paper goes as follows. We start by introducing some definitions of terminology in the graph theory in Section 2. In Section 3, we set up the basic model of interconnection agreements. Section 4 discusses the concept of the extended Nash solution in a bilateral bargaining in a network and its inefficiency. In Section 5, we introduce the Myerson value as the solution concept for multilateral bargaining among ISPs in a network and characterize the values for homogeneous ISPs in specific networks. In Section 6, we present an extended model of network formation. Characterizations of the efficient network and the stable network are provided in Section 7. We consider the case in which there is friction in information transmission in Section 8. Section 9 contains discussions on possible variations of our model and concluding remarks.

2 Preliminaries

A *network* (or *graph*), denoted by G , consists of a finite set of elements called nodes and a finite set of pairs of nodes called links. We denote the node set of G by $N(G)$ and the link set by $L(G)$. Each node is interpreted as a router of an ISP. Each link, $ij \in L(G)$, can be interpreted as routers of ISP i and ISP j being directly connected. A subgraph of G is defined by $G' = \langle N(G'), L(G') \rangle$

⁶Roughly speaking, it requires no player or no pair to be benefited from a unilateral or bilateral deviation.

such that $N(G') \subset N(G)$ and $L(G') \subset L(G)$. As in most literature, we will use G and $L(G)$ interchangeably unless it causes some confusion. We denote by $G + ij$ the graph obtained from adding the link ij to G and by $G - ij$ the graph obtained from deleting the link ij from G .

We say that, if $ij \in G$, nodes i and j are *adjacent* or i and j are incident with ij . We will use the notation of $A_i(G) = \{j \in N(G) \mid ij \in G\}$.

The degree of a node $i \in N(G)$ is defined by the number of links incident with i and denoted by $d(i)$. A network is called *regular* if each node has the same degree. If, for some integer $k \leq n - 1$, $d(i) = k$ for all $i \in N(G)$, the network, G , is called *regular of degree k* . The regular network of degree $n - 1$ is called the *complete network*.

We say that there exists a path Q_{ij} from i to j in G , or simply, i and j are connected in G , if either $ij \in G$ or there exists a sequence of distinct nodes $\{i, i_1, i_2, \dots, i_m, j\}$ such that $ii_1, i_1i_2, \dots, i_mj \in G$. We will use the notation of $P_i(G) = \{j \in N(G) \mid \text{There exists } Q_{ij}\}$. A network is called *connected* if and only if there is a path from i to j for all i and $j \in N(G)$. In other words, a network G is connected if $P_i(G) = N(G) \setminus \{i\}$ for all $i \in N(G)$. If G is not connected, it is decomposed into several connected components, $\{C_\tau(G)\}$. We call $\{C_\tau(G)\}$ components of G if (i) $G = \bigcup_{\tau=1}^k C_\tau(G)$ and (ii) i and j are connected in G if and only if $i, j \in C_\tau(G)$ for some $\tau = 1, \dots, k$.

A link ij is called “*bridge*” in a connected network G if $G - ij$ is disconnected. A *cycle* is defined by a path such that $i = j$.⁷ A graph which does not contain a cycle is a *forest*. A connected forest is called a *tree*. Every link in a tree is a bridge.

The *distance* between connected nodes i and j in G is defined by the number of links in the shortest path between them and denoted by $t(ij; G)$. We call $\max_{j \in P_i(G)} t(ij; G)$ the *eccentricity* of i and $\arg \max_{j \in P_i(G)} t(ij; G)$ the *eccentric node* from i .

A *coalition* in $N(G)$ is a subset of nodes. We define G *restricted to a coalition S* by $G|_S = \{ij \in G \mid i, j \in S\}$. For example, if $G = \{12, 23, 34\}$ and $S = \{1, 3, 4\}$, we have $G|_S = \{34\}$. Note that $G|_S$ is a subgraph of G so that it is defined on S , not on the whole set of nodes $N(G)$. We say that S is connected in G if $G|_S$ is connected. Node $i \notin S$ is said to be linked with a connected set $S \subset N(G)$ in G or linked with a graph $G|_S$ if $ij \in G$ for some $j \in S$.

⁷By a graph, we mean only a simple graph. So, a loop which is a link from a node to itself is disallowed.

3 Model

We follow the model of Internet interconnection developed by Besen *et al.* (2001) except that we explicitly consider the network structure. There are $n(< \infty)$ ISPs in the Internet market. Let $I = \{1, 2, \dots, n\}$ be the set of ISPs. There are also Internet customers the size of which is normalized to one. ISP i serves a fraction α_i of end users. The connectivity among ISPs is represented by a network G . For simplicity, we assume that Internet users are all homogeneous.⁸ An end user served by ISP i enjoys a utility of $v(\alpha_i + \sum_{k \in P_i} \alpha_k)$ where $v'(\cdot) > 0$ and $v(0) \geq 0$. That is, the utility increases in the number of customers affiliated with ISPs connected to ISP i . For example, a user gets a utility of $v(\alpha_i)$ if ISP i is connected to no other ISP $j (\neq i)$, and gets a utility of $v(1)$ if it is connected to all other ISPs. We will call $\alpha_i + \sum_{k \in P_i} \alpha_k$ the connectivity of ISP i . If $\alpha_k = \alpha$ for all $k \in I$, we may call $1 + \#P_i$ the connectivity of ISP i . We assume that no users multihome, i.e., are served by more than one ISP.⁹

ISPs charge their customers the price equal to their willingness to pay for their connectivity, i.e., $v(\cdot)$. Here, we are implicitly assuming that switching costs of users are huge, so that there is no competition over users among ISPs.¹⁰ Then, once a consumer subscribes to an ISP, he will stay in it, meaning that all α 's are exogenous. As long as ISPs have monopoly power over its users, it can extract consumer surplus by charging a monopoly price. Thus, most of the assumptions in our model could be justified.¹¹ For the time being, we assume that ISPs incur no costs,¹² à la Besen *et al.* (2001).

⁸In reality, there are heavy users and light users. However, as long as ISPs can distinguish between the two from their past usage records, the subsequent analysis remains qualitatively unaffected, although quite complicated.

⁹Multihoming can be considered as a substitute for compatibility in enhancing network benefits. See Doganoglu and Wright (2006). In this model, however, all ISPs provide compatible services as long as they are interconnected and there is no value loss from indirect connection as assumed in the above $v(\cdot)$ function. Thus, no multihoming is a reasonable assumption in this model.

¹⁰Imagine a situation where the consumer contracts are long-term. According to ISP Review (May 28, 2010), a survey of 6000 broadband users in the UK has revealed that more than half have not changed ISP for at least three years.

¹¹If ISPs have no monopoly power, the prices they charge to consumers should depend on $n - 1$ prices and connectivities of other ISPs, and it would be extremely complicated to obtain the Nash equilibrium prices in general asymmetric network structures. The incentive for multihoming of users would be also weak under the assumption of high switching (setup) cost, since their consumer surplus should be zero.

¹²This can be justified by the implicit assumption that fixed transmission costs including purchasing routers etc. are all sunk. Then, pure marginal transmission costs are usually

Interconnection charges can be determined by bargaining among ISPs. Note that we distinguish an agreement in bargaining between two ISPs from a (physical) link between them. In our model, a link simply implies the possibility of interconnection by bargaining. If they fail in an agreement, interconnection between them is not guaranteed,¹³ unless there is a bypass.

4 Bilateral Bargaining and Its Inefficiency

Suppose ISPs negotiate bilaterally to determine interconnection charges. Abstracting from detailed bargaining procedure, we can adapt the Nash (1953) bargaining solution as a possible solution concept.

Consider a bilateral bargaining between ISP i and ISP j such that $ij \in G$. If $n = 2$ so that $d(i) = d(j) = 1$, this is just a standard Nash bargaining problem. Let s be the amount to be divided and $\mathbf{d} = (d_i, d_j)$ be disagreement payoffs of ISP i and ISP j respectively. Then, given $(s, \mathbf{d})$, the Nash bargaining outcome $\mathbf{x}^* \equiv (x_i^*, x_j^*)$ is given by

$$\mathbf{x}^*(s, \mathbf{d}) = \arg \max_{x_i + x_j \leq s, x_i \geq d_i, x_j \geq d_j} (x_i - d_i)(x_j - d_j). \quad (1)$$

In this case, one can define the interconnection revenue that ISP i gets from ISP j as the difference between ISP i 's share in the bargaining and its revenue under a bill-and-keep arrangement, i.e., $a_i = x_i^* - \alpha_i v(1)$. This yields a simple principle governing interconnection arrangements.

Proposition 1 (Besen *et al.* (2001)) *If $d(i) = d(j) = 1$, then the bargaining outcome between ISP i and ISP j is the bill-and-keep arrangement if (i) $\alpha_i = \alpha_j$ or (ii) $v(\cdot)$ is linear.*

Proof. See Besen *et al.* (2001).

Proposition 1 provides sufficient conditions for bill-and-keep arrangements when there are only two ISPs. In particular, it suggests that even if the market shares of the two ISPs are different, the bill-and-keep arrangement can emerge if the utility function is linear in the network size. The intuition is that the smaller ISP loses more per customer due to the bargaining breakdown but it has less customers; hence, the scale effect of the bargaining gain is zero.

small enough to be negligible compared to the revenues.

¹³This distinction corresponds to the reality. Some ISPs often refuse to renew its interconnection contract with another by insisting on a higher interconnection fee. For example, in 1997, UUNET denied peering with some networks, which triggered a subsequent series of transit agreements.

If $d(i) \geq 2$, ISP i must bargain with more than one ISP. Thus, any ISP i with $d(i) \geq 2$ in a network G faces multiple bilateral bargaining problems.¹⁴ In this situation, a breakdown in an agreement between a pair of bargainers does not necessarily imply breakdowns in agreements between all other pairs. This problem of bilateral bargaining with multiple players can be solved in two stages.

The motivation for the two-stage bilateral bargaining is as follows. For a bilateral bargaining between ISPs, it is essential to define the total surplus. However, since an ISP bargains with more than one ISP, the surplus to be divided between them is not well defined. Thus, ISPs first determine the surplus to be divided between each pair of bargaining partners and then make bilateral bargaining.

Let $l = \#L(G)$. In the first stage, ISPs solve the l -person bargaining problem among l pairs for all $ij \in L(G)$ to get the share for their respective bilateral bargaining problem. Let disagreement payoffs of ISP i and ISP j in a bilateral bargaining between i and j be d_{ij} and d_{ji} . Assume that $\sum_{ij \in G} d_{ij}^+ \leq s$ where $d_{ij}^+ = d_{ij} + d_{ji}$. Then, considering a pair as a bargaining identity, l pairs of bargainers determine s_{ij} with $\sum_{ij \in G} s_{ij} = s$ by solving

$$\mathbf{s} = (s_{ij}) = \arg \max_{\sum_{ij \in G} s_{ij} = s, s_{ij} \geq d_{ij}^+} \prod_{ij \in G} (s_{ij} - d_{ij}^+). \quad (2)$$

In the second stage, two ISPs within a pair make a two-person bargaining game. The final bargaining solution is $\mathbf{x}^*(G) = (x_1^*(G), x_2^*(G), \dots, x_n^*(G))$, satisfying

$$(x_{ij}^*(G), x_{ji}^*(G)) = \arg \max_{x_{ij} + x_{ji} \leq s_{ij}, x_{ij} \geq d_{ij}, x_{ji} \geq d_{ji}} (x_{ij} - d_{ij})(x_{ji} - d_{ji}), \quad (3)$$

and $x_i^*(G) = \sum_{j \in A_i(G)} x_{ij}^*(G)$,¹⁵ where

$$d_{ij} = x_i^*(G - ij) \text{ and } s = \sum_{i \in C_\tau(G)} \alpha_i v \left(\sum_{k \in N(C_\tau(G))} \alpha_k \right).$$

We can easily check that this two-stage solution satisfies all of the four Nash axioms.¹⁶

¹⁴In Besen *et al.*, they implicitly assume that ISP i and ISP j already reached agreements with remaining $(n - 2)$ ISPs.

¹⁵This two-stage bargaining outcome is similar to the position value in its spirit in the sense that a value is assigned to a pair of players (or a link). But in this outcome, the value is distributed between the two players through another round of bargaining, whereas it is simply split equally in the latter.

¹⁶In the bargaining game on a network, the symmetry axiom should be modified as stating,

If they reach a bargaining agreement, their direct connectivity will be maintained, but if they fail, the link between them will be disrupted. We assume that if two ISPs enter into the interconnection agreement, they deliver to each other not only the traffic destined to each other's customers, but also the traffic destined to customers of the third ISP connected to them. This is the typical feature of a transit agreement. Under transit agreements, even if ISP i and ISP j fail to reach an agreement in the interconnection arrangement, they may maintain their (indirect) connectivity if they also have an indirect link with each other.

Solving two maximization problems sequentially leads to an extended Nash solution to the problem of n -person bilateral bargaining in a network.¹⁷ Assuming that G is connected,¹⁸ the explicit Nash solution is as follows.

Proposition 2 *The extended Nash bargaining solution in G is $(x_i^*(G) = \sum_{j \in A_i(G)} x_{ij}^*(G))_{i=1, \dots, n}$ where*

$$x_{ij}^*(G) = \frac{1}{2}(s_{ij} + d_{ij} - d_{ji}),$$

$$s_{ij} = \frac{1}{l}(s + \sum_{kk' \in G} (d_{ij}^+ - d_{kk'}^+)).$$

Proof. This is direct from solving the maximization problems (2) and (3).

The Nash bargaining solution is to divide the surpluses from bargaining between bargainers fairly (in terms of their disagreement payoffs). In particular, in the second stage of bargaining in a network, the outcome relies on a difference in ISP i 's bargaining power advantage over ISP j measured by $\delta_{ij} = d_{ij} - d_{ji}$. Thus, the bargaining solution depends on the network structure and the resultant connectivity of the ISPs, $(\sum_{k \in P_i(G-ij)} \alpha_k, \sum_{k \in P_j(G-ij)} \alpha_k)$, determining the disagreement payoffs, as well as on the market shares of ISP i and j .

"Let the bargaining solution in G be $(x_1, x_2, \dots, x_n)$. Suppose that (i) $d_{ij} = d$ for some integer d for all $ij \in G$, (ii) that the feasible set is symmetric among all $i \in I$ and (iii) that G is regular so that any relabeling of nodes does not change the degree of any node. Then, $x_1 = \dots = x_n$."

¹⁷By n -person bilateral bargaining, we mean that n persons maintain the spirit of bilateral bargaining (in the second stage), not depending entirely on the multilateral agreement.

¹⁸If G is disconnected, the same analysis can be applied to each connected component.

In this case, however, the interconnection payment between ISP i and ISP j becomes obscure because each ISP's revenue under a bill-and-keep arrangement cannot be isolated for each individual bargaining partner. But, we can define the total interconnection revenue that ISP i can get from all its adjacent ISPs by $a_i = x_i^* - \alpha_i v(1)$. Then, we can demonstrate that the result of Proposition 1 is not carried over to a general network if $d(i) \geq 2$.

To illustrate, consider a star network G in which there is a center $j \in I$ such that $ij \in G$ for all $i \neq j$, assuming that $\alpha_i = \alpha$ for all $i \in I$. Let $x(k)$ be the payoff of the center j in the bargaining game in a substar with k nodes. We have

$$x(k) = \frac{1}{2} [k\alpha v(k\alpha) + (k-1)(x(k-1) - \alpha v(\alpha))],$$

$$x(2) = \alpha v(2\alpha),$$

$$x(1) = \alpha v(\alpha),$$

for $k = 3, \dots, n$. Then, we can demonstrate that the center's payoff is always greater than the periphery's payoff as long as $n \geq 3$. Intuitively, this is because the disagreement payoff of the center, $x(n-1)$, is higher than that of the periphery, $x(1)$.

Proposition 3 *In the extended Nash bargaining solution, the center of a star receives interconnection charges from all peripheries, if $\alpha_i = \alpha$ for all $i \in I$.*

Proof. See the appendix.

Since the result of Proposition 3 is valid regardless of the form of $v(\cdot)$, one can see that an ISP can get paid the positive interconnection charge even if all ISPs have the same market share and $v(\cdot)$ is linear. This suggests that interconnection charges are determined by the network structure as well as the market shares of the associated ISPs.

The intuition behind Proposition 3 is illuminating for our subsequent discussion. Consider a bargaining between one peripheral ISP i and the center j . Traffic between ISP i and any other ISP $k \in P_j(G')$ must pass ISP j because ISP j is on the unique path between ISP i and ISP k , where $G' = G - ij$. This implies that ISP j is important to ISP i by $1 + \#P_j(G') = n - 1$. Similarly, ISP i is important to ISP j by $1 + \#P_i(G') = 1$ since ISP j communicates with ISP i only for traffic between ISP i and itself. Accordingly, the center has the highest bargaining power, thus receiving interconnection charges from all other ISPs.

The two numbers $\#P_i(G')$ and $\#P_j(G')$ are conceptually akin to “betweenness centrality” widely used in social network analysis. One difference is that the betweenness centrality is measured for one node independently of other nodes (how important role a node plays in a given network), while the numbers are measured for one node, say i , but with respect to another node, say j , (how much important a node i is to another node j in a given network). In other words, our concept is a relative concept, whereas the betweenness concept in sociology is an absolute concept.

We will say that ISP i is *critical* or *essential* in communication between j and k if $i \in Q_{jk}$ for all Q_{jk} . Then, “the centrality of i with respect to j ” in G can be generally defined by $b_{ij}(G) = \sum_{k \in I \setminus \{j\}} \phi_{jk}(i)$, where $\phi_{jk}(i) = 1$ if $i \in Q_{jk}$ for all Q_{jk} and $\phi_{jk}(i) = 0$ if $i \notin Q_{jk}$ for some Q_{jk} ,¹⁹ in words, the number of k such that i is critical in communication between j and k . In the case of a star, the centrality of a peripheral node is one and the centrality of the center is $n - 1$. We will call $b_i(G) = \sum_{j \neq i} b_{ij}(G)$ the centrality of i in G , which corresponds to the absolute centrality concept in sociology. Intuitively, the centrality of i in G tells us the number of pairs of ISP that can communicate only through ISP i in G . Thus, the betweenness centrality can be interpreted as a degree of unavoidability or bottleneck in the context of telecommunications.²⁰

Our concept of absolute centrality is distinguished from that in sociology in two aspects. First, if ISP i is on one of $m(\geq 2)$ shortest paths between ISPs j and k , the pair (j, k) is counted only by $1/m$ in calculating i ’s centrality in sociologists’ definition, whereas the pair is not counted at all in our definition. This difference occurs because sociologists care about centrality in situations in which there is no bargaining possibility. Our concept is based on a (bilateral) bargaining situation where an ISP i is nothing to another ISP j if the latter can reach others even if the communication between i and j is disrupted. In this sense, our concept of centrality is more strict. Second, in our definition, the pair including an end node of a path is also counted in calculation of its centrality, while only the pair including a between node is counted in their definition. This will turn out to be more convenient for our purpose.

The extended Nash solution for the n -person bilateral bargaining in a network, however, has a serious drawback. Generally, the solution does not ensure efficiency, because it is very unlikely that all disagreement points (d_{ij}, d_{ji}) in the second stage bargaining remain in the feasible set, i.e., $d_{ij}^+ \leq s_{ij}$.

¹⁹We are abusing the definition of a path. A path is a sequence of ISPs, but we treat it as if it is a set of ISPs.

²⁰This interpretation for betweenness centrality appears earlier in D’Ignazio and Giovannetti (2006).

In particular, if n is large, it is very likely that $\sum_{ij \in G} d_{ij}^+ > s$, implying that $d_{ij}^+ > s_{ij}$ for some $ij \in G$; hence, no agreement between i and j possible.²¹ For example, in a star with center 1 and $n = 4$, if $v(\alpha)$ is linear, say, $v(\alpha) = \alpha$, we have $d_{14}^+ = \alpha v(\alpha) + x(3) = \alpha v(\alpha) + (13/2)\alpha v(\alpha) > \alpha v(1) = s_{14}$, implying that none of the pairs will reach an agreement in a bilateral bargaining. If $v(\cdot)$ is concave, this possibility becomes more severe. This suggests that the bilateral bargaining between directly linked ISPs is not an efficient way to determine interconnection charges, as long as we agree on the extended Nash solution as a natural solution concept under the circumstance.²²

5 Myerson Value as a Multilateral Bargaining Solution

As an alternative bargaining solution concept, we can use the Myerson value developed by Myerson (1977) in the context of the communication game and adapted by Jackson and Wolinsky (1996) to the network game which is a broader class of game. Although it is a normative concept, it is typically used as a bargaining solution in a multilateral bargaining situation. The use of the Myerson value as a surplus-division rule is to abstract from details of bargaining procedures that may depend on underlying situations and to focus only on final outcomes according to players' bargaining powers under the implicit assumption that the bargaining procedures are fair. Also, the concept of Myerson value is in accord with the transit agreement in the senses that ISPs pay each other and that they allow for the delivery of transit traffics. The interconnection charge of an ISP is then interpreted as its Myerson value (the share in the bargaining) minus the revenues from its customers.

Although all the subsequent analysis can be applicable for any $(\alpha_i)_{i=1}^n$, we will assume that $\alpha_i = \alpha$ for all $i \in I$ just for simplicity. The case of asymmetric ISPs will be briefly discussed in Section 9.

Let us start from definitions of some basic concepts.

Definition 1 *A value function is a function $V : \mathcal{G} \rightarrow \mathbb{R}$, where $\mathcal{G}$ is the set of graphs.*

²¹This does not imply that the two stage bargaining solution we derived violates the efficiency axiom of the Nash Theorem. The efficiency axiom assumes that the disagreement point is in the feasible set.

²²The efficiency of the bilateral bargaining may be achieved under the assumption that $v(\alpha)$ is very convex.

A value function specifies the total value generated by a graph G . For example, in our model, $V(G) = \alpha kv(\alpha k)$ for any graph in which k nodes are connected, and $V(G_i) = \alpha v(\alpha)$ where G_i is a graph consisting only of a node i .

Definition 2 A value function V is component additive if

$$V(G) = \sum_{\tau=1, \dots, k} V(C_\tau(G)).$$

Definition 3 A value function V is monotonic if $V(G') \geq V(G)$ whenever $G \subset G'$.

Component additivity of V means that if a graph is not connected, the total value generated in each component is not spilled over to other components, that is, the total value of each component is independent of the network structure of other components. Monotonicity of V implies that the total value is increased if a node or a link is added to a graph. The value function in our model satisfies both component additivity and monotonicity.

Definition 4 A network game Γ is $\langle I, G, V \rangle$ where I is the set of players, G is a graph consisting of I , and $V : \mathcal{G}(G) \rightarrow \mathbb{R}$ is a value function assigning a value to a subgraph of G .

The network game is distinguished both from the standard cooperative game and from the communication game defined by Myerson (1977). While all kinds of coalition are allowed in the standard cooperative game, a coalition is possible only among connected players both in a communication game and in a network game.²³ In a communication game, however, what matters is who are connected with each other, but not how they are connected. That is, the value function does not depend on how a group of players are connected with each other in a connected component, while it depends in a network game. For example, in a network game with a given number of players, the values generated from a line and from a circle may be different. Since a communication game is a special class of the network game, we will use the term of the network game consistently throughout the paper.

²³Strictly speaking, a coalition is meaningful only among connected players in a network game in the sense that it can increase the value.

The value function in our game can be written as

$$V(G) = \sum_{\tau=1}^k V(C_\tau(G)) = \sum_{\tau=1}^k \alpha m_\tau v(\alpha m_\tau), \quad (4)$$

for $k \geq 1$, where $\#N(C_\tau) = m_\tau$. If $\alpha v(\alpha) \equiv w(\alpha)$ is convex, it can be interpreted as the total consumer surplus or the profits of ISPs when the connectivity is α . Henceforth, we assume that $w''(\alpha) \geq 0$.²⁴ This includes the case that $v''(\alpha) \geq 0$. If $v(\alpha)$ is concave but not very much, this assumption can be satisfied.

Definition 5 *An allocation rule is a function $y : \mathcal{G} \times \mathcal{V} \rightarrow \mathbb{R}^n$ such that $\sum_{i \in I} y_i(G, V) = V(G)$ where $\mathcal{V}$ is the set of value functions $V : \mathcal{G} \rightarrow \mathbb{R}$.*

We call $y(G, V)$ an allocation induced by an allocation rule y , a graph G and a value function V , and call $y_i(G, V)$ node i 's allocation.

The Myerson value is an allocation rule y^M defined by

$$y_i^M(G, V) = \sum_{S \subset I \setminus \{i\}} \Delta_i(S) \frac{\#S!(n - \#S - 1)!}{n!}, \quad (5)$$

where $\Delta_i(S) \equiv U(S \cup \{i\}) - U(S)$ and $U(S) \equiv V(G|_S)$. It is a simple application of the Shapley value to the network game, and thus can be interpreted as the average of player i 's marginal contributions to all possible coalitions in a graph G .

We can interpret a coalition as an identity maximizing the joint profit. Thus, in our game, a coalition among ISPs connected to one another can be interpreted as successful agreements in bargaining among them. Accordingly, a coalition is meaningful only when the ISPs in the coalition are connected. A coalition between ISPs unlinked to each other is possible but meaningless because they have nothing to bargain over.

As the extended Nash bargaining solution, the Myerson value in our game has the centrality interpretation if $v(\cdot)$ is linear. Generally, ISP i 's contribution to S can be written as $\Delta_i(S) = [b_i(G|_S) + (1 + \#P_i(G|_{S \cup \{i\}}))] \alpha^2$ where $b_i(G|_S) = \sum_{j \in S} b_{ij}(G|_S)$. The first term is the value added to ISPs in S and the second term is the value added to ISP i due to network externalities. In other words, an ISP contributes to a coalition by the sum of its centrality and its connectivity in the coalition.

²⁴This assumption implies that the network game with the complete graph and the value function given by (4) is a convex game, although the convexity is not inherited by a game with an arbitrary network, as is well known.

Since the Myerson value is based on the Shapley-style calculations, it satisfies the properties of symmetry, dummy player and additivity. Myerson (1977) also shows that it further satisfies the properties of component balance and equal bargaining power. Among others, we state three properties that will be explicitly used in this paper.

Property 1 (Additivity) *For any two value functions, V_1 and V_2 defined on G , $y^M(G, a_1V_1 + a_2V_2) = a_1y^M(G, V_1) + a_2y^M(G, V_2)$ for any scalar a_1 and a_2 .*

Property 2 (Component Balance) *For any G and for any component additive V ,*

$$\sum_{i \in N(C_\tau(G))} y_i^M(G, V) = V(C_\tau(G)) \text{ where } G = \cup_{\tau=1}^k C_\tau(G).$$

Property 3 (Equal Bargaining Power) *For any G and for any component additive V ,*

$$y_i^M(G, V) - y_i^M(G - ij, V) = y_j^M(G, V) - y_j^M(G - ij, V) \text{ for any } ij \in G.$$

Under the property of equal bargaining power, all the losses in the total value from a deletion of ij may not go to ISP i and ISP j but ISP $k (\neq i, j)$ may bear some of the losses, just as an addition of a link ij can benefit ISP k . The property, however, requires that ISP i and ISP j suffer equally from deleting ij .²⁵

As far as V is fixed as in our game, we will abbreviate $y_i^M(G, V)$ simply as $y_i^M(G)$ by suppressing V . The following proposition gives another useful property of the Myerson value that holds in our game.

Proposition 4 *For an arbitrary G , let $U = \{ij \mid ij \notin G\}$. Then, for any $ij \in U$ and for any $k = i, j$, (i) $y_k^M(G + ij) > y_k^M(G)$, and (ii) $y_k^M(G + ij) - y_k^M(G) \geq y_k^M(G' + ij) - y_k^M(G')$ for any $G' \supset G$.*

Proof. See the appendix.

The first part of this proposition says that adding a new link between a pair of ISPs always increases their Myerson values. However, it does not imply that the Myerson values for all ISPs are increased when a new link is added. It is, in fact, possible that $y_k^M(G + ij) < y_k^M(G)$ for some $k \neq i, j$. For

²⁵See Jackson (2005) for the criticism against properties of component balance and equal bargaining power when a network can be changed.

instance, consider a graph $12 \in G$ with $I = \{1, 2, 3\}$. Then, $y_1^M(G) = \alpha v(2\alpha)$ but $y_1^M(G + 23) = \alpha v(3\alpha) - \frac{\alpha}{3}(v(2\alpha) - v(\alpha))$. Thus, $y_1^M(G + 23) < y_1^M(G)$ if v is concave enough that $\frac{v(3\alpha) - v(2\alpha)}{v(2\alpha) - v(\alpha)} < \frac{1}{3}$. The second part of the proposition implies that an additional link between two ISPs contributes to the increase in their Myerson values less in a richer network structure.

We will characterize Myerson values of our game in specific graphs. It is clear that if the graph is a complete network (G^N) or a circle (G^C), $y_i^M(G) = \alpha v(1)$ for all $i \in I$.

Consider a star G^* in which j is the central ISP. The Myerson value for ISP j is then

$$y_j^M(G^*) = \frac{1}{n} \sum_{k=0}^{n-1} (\alpha(k+1)v(\alpha(k+1)) - \alpha k v(\alpha)). \quad (6)$$

The Myerson value for ISP $i \neq j$ is

$$\begin{aligned} y_i^M(G^*) &= \sum_{k=1}^{n-1} \left(\frac{\frac{(n-2)!}{(k-1)!(n-k-1)!} (\alpha(k+1)v(\alpha(k+1)) - \alpha k v(\alpha k)) + \frac{(n-1)!}{k!(n-1-k)!} - \frac{(n-2)!}{(k-1)!(n-k-1)!} (\alpha(k+1)v(\alpha) - \alpha k v(\alpha)) \right) \frac{k!(n-k-1)!}{n!} \\ &\quad + \frac{1}{n} \alpha v(\alpha). \end{aligned} \quad (7)$$

The first term (the second term, respectively) corresponds to the case that S contains the center (consists only of peripheries respectively). The third term corresponds to the case that $S = \emptyset$.

The Myerson value in a star can be interpreted as follows. ISP j contributes to the coalition S with k nodes by enabling k ISPs to communicate with each other. Since ISP $i \in S$ can communicate with any ISP $l \in S$ only through ISP j , the centrality of ISP j with respect to ISP i is $b_{ji} = k$. If $v(\cdot)$ is linear, ISP j contributes to each member i in a coalition S by b_{ji} .

Next, consider a line G^L in which $ij \in G^L$ if $j = i+1$, $ij \notin G^L$ otherwise, for all $i = 1, \dots, n-1$. It is not straightforward to find the general Myerson value for any n in a line network, but we can demonstrate the monotonicity of the value. To summarize, we have

Proposition 5 (i) In a star with $n \geq 3$, the center receives interconnection charges from all peripheral ISPs. (ii) $y_i^M(G^L)$ is strictly increasing in i up to $i = i_m$ where $i_m = m$ if $n = 2m$ and $i_m = m + 1$ if $n = 2m + 1$.

Proof. See the appendix.

The second part of this proposition implies that an ISP gets higher interconnection revenues as it is closer to the middle of a line, while an ISP pays higher charges for interconnection as it is farther from it. Note that in a line network the centrality of the mid-ISP is highest, while the centrality of end ISPs is lowest.

6 Extended Model of Network Formation

In this section, we extend our analysis by considering the decisions of network formation before the bargaining game $\Gamma = \langle I, G, V \rangle$ is played given the network.²⁶

Forming a link between ISPs costs each of the two $f > 0$. A link can be formed by the mutual consent of the two ISPs. An ISP can sever its link without the consent of the other ISP.

Let the net profit and the gross profit of ISP i obtained from multilateral agreements in a network G be $\pi_i(G)$ and $\tilde{\pi}_i(G)$. Then, we have $\tilde{\pi}_i(G) = y_i^M(G)$ and $\pi_i(G) = y_i^M(G) - d(i)f$.

We introduce two definitions proposed by Jackson and Wolinsky (1996).

Definition 6 *A network G is efficient if $\sum_{i \in I} \pi_i(G) \geq \sum_{i \in I} \pi_i(G')$ for all $G' \in \mathcal{G}$.*

In words, a graph is efficient if it maximizes the sum of each ISP's net profits.

Definition 7 *A network G is (pairwise) stable if (i) $\pi_i(G) \geq \pi_i(G - ij)$ for all $ij \in G$ and (ii) if $\pi_i(G) < \pi_i(G + ij)$, then $\pi_j(G) > \pi_j(G + ij)$ for all $ij \notin G$.*

In words, a network is stable if no ISP has an incentive to sever a direct link unilaterally, and no pair of ISPs have mutual incentives to form a link between them.

While the concept of efficiency seems indisputable, the concept of pairwise stability has been under controversy. Many alternative concepts have been proposed,²⁷ but we stick to this concept of pairwise stability because of its undoubted merit of simplicity.

²⁶The order of the sequence reflects the reality that the network structure affects the interconnection agreement outcome, not the converse.

²⁷To name a few, see Dutta & Mutuswami (1997) and Jackson and van den Nouweland (2005) for strong stability and Bala & Goyal (2000a) for the concept of Nash network. Strong stability requires that the network be stable against all possible coalitions of more than two

7 Efficiency and Stability

The efficiency of a network can be measured by summing the profits of all ISPs, that is, $E(G) = \sum_{i \in I} \pi_i(G)$. A network G can be decomposed into finite connected components, i.e., $G = \bigcup_{1 \leq \tau \leq k} C_\tau$ for some $k(\leq n)$. Let $\#N(C_\tau) = m_\tau$ and $\#L(C_\tau) = l_\tau$. Then, $E(G) = \sum_{1 \leq \tau \leq k} E(C_\tau) = \sum_{1 \leq \tau \leq k} (m_\tau \alpha v(m_\tau \alpha) - 2l_\tau f)$. It is important to note that $E(C_\tau)$ does not depend on specific bargaining outcomes, but only on the connectivity $(m_\tau$ and $l_\tau)$, since the Myerson value is an allocation rule satisfying the property of component balance, i.e., $\sum_{i \in N(C_\tau)} y_i^M(G) = V(C_\tau) = m_\tau \alpha v(m_\tau \alpha)$.

Intuitively, the efficient network cannot contain a cycle for any $f > 0$, because there is no efficiency loss involved with roundabout traffic. Thus, a deletion of a link consisting of a cycle will only save the link cost without incurring any efficiency loss. Thus, we have

Proposition 6 (i) *The efficient connected network must be a tree for any $f > 0$.* (ii) *Any tree is efficient if $0 < f < \alpha(v(2\alpha) - v(\alpha))$.*

A tree is a connected network and accordingly must have exactly $(n-1)$ links. Thus, all trees are equivalent from the efficiency point of view, because $E(T)$ is the same for all $T \in \mathcal{T}$ where $\mathcal{T}$ is the set of trees.

Since efficiency requires no cycle, the efficient network is a forest, a union of disjoint trees. The number of components in a forest can be from one (which corresponds to a tree) to n (which corresponds to the empty network). Under the assumption that $w''(\alpha) \geq 0$, the second result follows. Hereafter, we will assume that $0 < f < \alpha(v(2\alpha) - v(\alpha))$, reflecting the widely accepted perception that full connectivity of the Internet is socially desirable.

If ISPs make decisions to form their links in a decentralized fashion, it is naturally anticipated that a stable network will emerge. Stable networks can be characterized as follows:

- (i) If $f < \frac{2}{n(n-1)} \alpha(v(2\alpha) - v(\alpha)) \equiv \alpha v(1) - y_i^M(G^N - ij)$, the unique stable network is the complete network G^N .
- (ii) A star G^* is stable if $y_i^M(\hat{G}^*) - y_i^M(G^*) < f < y_i^M(G^*) - \alpha v(\alpha)$ where $\hat{G}^* = G^* + ii'$ for any peripheral i and i' .

players. Although this concept seems more appealing, it exhibits the serious nonexistence problem. Bala and Goyal propose the concept of Nash network in a situation where each player can choose whether to connect or sever a link unilaterally without the consent of the other player. Although it corresponds more to the spirit of noncooperative game theory, it is not relevant in our model because it can be hardly imaginable for an ISP to connect a link to another without its consent.

(iii) A line network G^L is stable if $\alpha v(1) - y(n) < f < y(n) - \alpha v(\alpha)$ where $y(n) = y_1^m(G^L)$ and G^L is such that $i(i+1) \in G^L$ for $1 \leq i \leq n-1$.

(iv) A circle G^C is stable if $y_i^M(\hat{G}^C) - \alpha v(1) < f < \alpha v(1) - y(n)$ where $\hat{G}^C = G^C + ii^*$ and i^* is the eccentric node of i .²⁸

Part (i) says that the complete network is stable, although not efficient, if $f < \frac{2}{n(n-1)}\alpha(v(2\alpha) - v(\alpha))$ for $n \geq 3$, implying that the network can be overconnected. This can be intuitively explained as follows. Consider deleting any link ij from the complete network with $n \geq 3$. While it reduces no total value in the network and saves each of ISP i and ISP j the connection cost f , it reduces their bargaining power. So, they will have an incentive to maintain the link if f is very small.

It is worth noticing that if $n \geq 4$, the complete network cannot be strongly stable for any $f < \frac{2}{n(n-1)}\alpha(v(2\alpha) - v(\alpha))$. This is because the deviation by the grand coalition to a circle is always profitable for all members. This implies that the strongly stable network does not exist at all, since the complete network is the unique stable network.

Part (ii) - (iii) suggest that all trees are not equivalent in terms of stability. Part (iv) says that a circle can be stable, although it cannot be efficient for any $f > 0$. The intuition is the same for the case of the complete network. A deletion of a link from a circle does not reduce the total value generated from the graph and only improves efficiency by saving the connection cost. Thus, it is clear that a circle is less efficient than a line. But, ISPs deleting the link will lose their bargaining power. This consideration can lead them into maintaining their link.

Figure 1. Stable Networks without Friction

Figure 1 illustrates stable networks for various values of f when $n = 4$ given $v(\alpha) = \alpha$. This figure shows that the stable network is, in general, not unique.

²⁸Proofs are provided in the appendix.

We argued before that the efficient network must be connected if $f < \alpha(v(2\alpha - v(\alpha)))$. The following proposition and its corollary establishes a relationship between efficiency and stability.

Proposition 7 *If $f < \alpha(v(2\alpha - v(\alpha)))$, a stable network must be (i) connected and (ii) cannot be underconnected.*

Proof. See the appendix.

This proposition says that if the interconnection charge is determined based on the Myerson value, the stable network cannot be underconnected when the efficiency of a network requires full connectivity.

Jackson and Wolinsky (1996) noted the tension between efficiency and stability in their model with and without side payments. To see what really leads to the overconnection result in our model, we will investigate the structure of stable networks under bill-and-keep arrangements, which corresponds with the case of no side payments, and compare it with the previous outcome, which corresponds with the case of side payments.

Proposition 8 *Under bill-and-keep agreements, (i) a stable network cannot contain a cycle for any $f > 0$. (ii) If $f < \alpha(v(1) - v(1 - \alpha))$, any tree is stable and no other network than a tree is stable.*

Proof. See the appendix.

Proposition 8 (i) implies that neither the complete network nor a cycle can be stable for any $f > 0$ if ISPs must make bill-and-keep arrangements for interconnection charges. It is not difficult to see that a network is less connected in bill-and-keep agreements than in interconnection agreements based on the Myerson value. The intuition is quite clear. Under bill-and-keep agreements, an ISP is not rewarded for providing connectivity to adjacent ISPs, which weakens the incentive to form a link.

A stable network can be overconnected (if $v''(\alpha) > 0$) or underconnected (if $v''(\alpha) < 0$) under bill-and-keep agreements, depending on the shape of $v(\alpha)$. Interestingly, if $v(\alpha)$ is linear, the condition for a tree to be efficient exactly coincides with the condition to be stable under bill-and-keep agreements. In other words, there is no tension between efficiency and stability if $v(\alpha)$ is linear and the interconnection agreements are bill-and-keep, as long as efficiency requires full connectivity.²⁹ This suggests that in this range of f , the government can duplicate the efficient network in a decentralized way by imposing the duty of interconnection on *all* ISPs, i.e., mandating interconnection with prohibiting ISPs in a stronger bargaining position from charging a positive price for the interconnection.³⁰

8 Friction

In the simple model given in Section 3, there is no friction, i.e., no loss in the value is incurred from roundabout traffic. In reality, however, if communication takes an indirect route, there can be efficiency loss (friction), for example, a delay may occur, some information may be lost or a higher transmission cost may be incurred. There are various ways to model friction.³¹ In this section, we incorporate friction into our model by assuming that the transmission cost per unit distance is $c > 0$.

The model in Section 3 is based on the implicit assumption that any pair of users communicate the same amount of information.³² Assuming that G is a tree and all market shares are α , the traffic volume of a link $ij \in G$ with centrality b_{ji} and b_{ij} is computed as $V_{ij} = 2b_{ij}b_{ji}\alpha^2$ and the transmission cost associated with this link is cV_{ij} . Note that the traffic volume of both ways is the same. But, if G is not a tree, V_{ij} is not well defined, because a communication path between ISP i and ISP j is not unique. However, we can compute the total transmission cost in a network G as $T(G) = \sum_{i \neq j} 2t(ij; G)\alpha^2 c$. This is under the implicit assumption that traffic between any pair of ISPs takes the

²⁹If $f > \alpha^2$, i.e., full connectivity is suboptimal, only the empty network is stable implying that the network is underconnected.

³⁰In fact, the U.K. began to impose the duty of Internet interconnection on some major ISPs, and Australia adopted the policy of designating Internet interconnection as "declared service," meaning the service whose provider the duty of interconnection is imposed on. See OFTEL (2001) and ACCC (2003).

³¹One alternative way to model friction is to assume that users get the utility of $\delta^t v$ from traffic with transmission length equal to t where $0 < \delta < 1$.

³²This assumption of "balanced calling pattern" has been used by many authors following Rohlfs (1974).

shortest path between them and that the transmission cost for traffic within ISP i is zero.³³ Also, for simplicity, we assume that $v(\alpha) = \alpha$. To avoid the unrealistic case that ISPs lose money, we assume that $c < 1$.³⁴

The network formation game under friction can be analyzed based on the subgame $\hat{\Gamma} = \langle I, G, \hat{V} \rangle$ where $\hat{V}(G) = V(G) - T(G)$, a combination of a revenue-sharing problem and cost-sharing problem.³⁵ We will assert that Proposition 6 no longer holds in $\hat{\Gamma}$. The following proposition provides a condition under which the complete network is efficient.

Proposition 9 (i) *The complete network is efficient if $f < \min\{\alpha^2 c, \alpha^2(1 - c)\}$.* (ii) *The most efficient tree is a star.*

Proof. See the appendix.

If a link between a pair of ISPs is severed, it saves $2f$ but increases the transmission cost by $2\alpha^2 c$, since traffic between them must be roundabout through an indirect route. Thus, the complete network can be efficient if the cost of severing a link, $2\alpha^2 c$, exceeds the benefit, $2f$. As n is larger, i.e., α is smaller, the social loss is smaller when an ISP severs its link; hence, the complete network is less likely to be efficient. The intuition for the second result is that the lengths of all indirect paths are two in a star, while all trees have the same number of indirect paths. This proposition implies that all trees are not equally efficient unlike in the case of no friction.

For stability, new Myerson values need to be computed, since links in a network generate transmission costs as well as users' utility. Let the Myerson value of ISP i in the game $\hat{\Gamma}$ be $y_i^M(G, \hat{V})$. By the additivity property of the Myerson value, we have $y_i^M(G, \hat{V}) = y_i^M(G, V) - y_i^M(G, T)$. In other words, the

³³If some ISPs make a peering agreement, it is not the case that all traffics take the shortest path, because the peer ISPs may deny traffics destined to the third ISP. This increases the volume of the whole traffic and degrade the average quality of communications, although the peers may maintain the high quality of their own. Moreover, in reality, transit agreements include the service level agreement which assures a higher quality than a certain level.

³⁴We could take an alternative model of friction that there is informational loss in roundabout traffics. This alternative formulation would enable us to discuss the quality of communications. It is clear that the higher quality is warranted under transit agreements than under peering agreements, as long as we maintain the shortest path assumption in transit agreements. Under peering agreements, some traffics destined to non-peering partners are supposed to take longer routes, unless the underlying network is a tree. This leads to the degraded quality.

³⁵Henriet and Moulin (1996) also address the problem of cost sharing in a network. But they only consider the situation in which the costs are not related to usage. Moreover, in their model, a consumer's benefit from joining in a network is ignored, while it is central in our model.

new Myerson value can be found by subtracting the Myerson value of the cost generating game $\langle I, G, T \rangle$ from the Myerson value of the pure utility generating game $\langle I, G, V \rangle$. A high centrality of a node in this case has two conflicting effects on its bargaining share. On one hand, it increases the bargaining power, since it means a high contribution to communications between arbitrary pairs of ISPs and the generation of network benefits. On the other hand, however, a high centrality means a high degree of unavoidability, thus a high degree of bottleneck in the presence of friction. Since such a node generates a high delay cost, that is, contributes much to the total cost generation, it decreases the bargaining power. The second effect would not be present without friction.

Letting $y_i^M(G, T) \equiv y_i^C(G)$, we can characterize stable networks as follows.

Proposition 10 (i) *The complete network is stable under friction if $f < \frac{2\alpha(v(2\alpha)-v(\alpha))}{n(n-1)} + \frac{(n^2-n-4)}{n(n-1)}\alpha^2c$ for $n \geq 2$.*

(ii) *The star G^* is stable under friction if $y_i^M(\hat{G}^*) - y_i^M(G^*) + \frac{1}{3}\alpha^2c < f < y_i^M(G^*) - \alpha v(\alpha) - \frac{1}{3}(4n-5)\alpha^2c$.*

(iii) *A line network G_L is stable if $\alpha v(1) - y(n) + \Delta y < f < y(n) - \alpha v(\alpha) - y^C(n)$ and $c \leq \bar{c}$ for some small $\bar{c}(> 0)$ where $\Delta y = y^C(n) - y_i^C(G^C)$ and $y^C(n) = y_1^C(G_L)$.*

(iv) *A circle G^C is stable if $y_i^M(\hat{G}^C) - \alpha v(1) + \Delta z < f < \alpha v(1) - y(n) + \Delta y$ where $\Delta z = y_i^C(G^C) - y_i^C(\hat{G}^C)$.*

Proof. See the appendix.

If $n = 2$, adding a link incurs extra transmission costs, so the two ISPs have less incentive to add the link under friction. Hence, the complete network is less likely to be stable under friction. If $n \geq 3$, however, by adding a link, a pair of ISPs make an extra contribution of reducing the total transmission cost in the whole network. Since this contribution is reflected in their Myerson value, they bear smaller part of the total transmission costs thereby ensuring a higher bargaining payoff by adding a link. This gives an ISP a stronger incentive to add a link under friction. Thus, the complete network is more likely to be stable under friction.

Also, note that the upper bound of f for the stability of the complete network is strictly higher than the upper bound for its efficiency when $v(\alpha) = \alpha$ unless $\alpha = 1/2$.³⁶ This implies that if the complete network is efficient, it is stable but not vice versa. This overconnectivity result is again due to the fact that the effect of increasing the bargaining power by adding a link is not captured in the notion of efficiency.³⁷

If a pair of ISPs add a link to a star, they contribute to reducing the total communication length and consequently communication costs. Since this contribution is reflected in their Myerson value, an ISP has a stronger incentive to add a link to a star under friction. This reduces the likelihood that a star will be stable. On the other hand, if a peripheral ISP deletes its link in a star, it can save the transmission cost as well as the linking cost. Thus, it has a stronger incentive to delete its link under friction. This also makes it more difficult for a star to be sustainable. Overall, the star is less likely to be stable under friction.

Figure 2. Stable Networks with Friction

The condition for a line network to be stable is rather complex. The complexity comes from the fact that end ISPs have the lowest Myerson value in the cost generating game as well as in the pure utility generating game. So, it is generally not clear whether end ISPs have the strongest incentive to add a link. However, if the transmission cost c is sufficiently low, they do have the strongest incentive to do so, as in the pure utility generating game, since the effect of adding a link on the Myerson value in the utility generating game dominates its effect in the cost generating game (See Figure 2 for stable networks under friction when $n = 4$ and $c < 4/29$).

³⁶If $\alpha = 1/2$, they coincide so that the complete network is stable if and only if it is efficient.

³⁷A similar insight can be found in Jackson (2003). His overconnectivity result under the Myerson value allocation holds for some cost structure of $T(G)$, so our result is not a special case of his.

9 Discussion and Conclusions

In this paper, we examined how Internet interconnection charges should be determined in a given network and how network structure can be formed.

Although we presented several models, we believe that the models provided in this paper are still so simplified as to neglect some interesting features of the Internet industry. We think, however, that our work is worthy as a stepping stone to future theoretical developments along this line. Below, we suggest several future research directions.

Asymmetric ISPs In this paper, we considered ISPs which are identical in sizes. Suppose there are n_i ISPs with market share α_i , $i = 1, 2$ where $\alpha_1 > \alpha_2$ and $\sum_{i=1}^2 n_i \alpha_i = 1$ and $n_1 + n_2 = n$. We call ISPs with market share α_1 primary ISPs or core ISPs. ISPs with market share α_2 are called secondary ISPs or non-core ISPs.

If the Internet industry consists of two groups of ISPs, the network structure can be characterized by three features; (i) how ISPs at the primary tier are connected with one another, (ii) how ISPs at the secondary tier are connected to ISPs at the primary tier and (iii) how ISPs at the secondary tier are connected with one another.

Since the analysis for the general case is too complicated, we will assume that $n_1 = n_2 = 2$. Despite its simplicity, this case provides some insights into the emergence of multi-homing and secondary peering agreements, two recent trends in the Internet interconnection. Let the set of α_k -ISPs be I_k where $k = 1, 2$. Specifically, let $I_1 = \{i_1, i_2\}$ and $I_2 = \{j_1, j_2\}$. We define some special forms of network.

Definition 8 An *LS (link – single)* network is a graph in which (i) two ISPs in I_1 form a link between them and (ii) each ISP $j \in I_2$ is linked to an ISP i for some $i \in I_1$.

Definition 9 An *LSL (link – single – link)* network is a graph in which (i) two ISPs in I_1 form a link between them, (ii) each ISP $j \in I_2$ is linked to an ISP i for some $i \in I_1$ and (iii) two ISPs in I_2 form a link between them.

Definition 10 An *LM (link – multi)* network is a graph in which (i) two ISPs in I_1 form a link between them and (ii) both ISPs in I_2 are linked to both ISPs in I_1 .

We maintain the assumption of no friction. Since an efficient network cannot contain a cycle then, no network except LS network is efficient.³⁸ However, we can illustrate that LSL network or LM network can be stable. Let us pick $\alpha_1 = 1/3$ and $\alpha_2 = 1/6$ with $v(\alpha) = \alpha$. If f is small, ISP j_1 and ISP j_2 may increase their bargaining power by adding a link between them to an LS network. Specifically, LSL network of a circular shape is stable if $0.0185 < f < 0.0694$; hence, secondary peering agreements. Or, they may want to increase their bargaining power by forming each additional link with a primary ISP, say ISP i_2 and ISP i_1 respectively. In fact, LM network is stable if $0.0093 < f < 0.0370$; hence, multi-homing.³⁹ Emergence of either secondary peering or multi-homing will depend on the size of the linking cost f , while both are the result of secondary ISPs' endeavor to increase their bargaining power.⁴⁰

Capacity Constraint We assumed in this paper that ISPs deliver transit traffic destined to a third ISP. It is in fact in their interest to allow transit traffic because they are paid for this extra service and the price is reflected in the interconnection charge. That is, ISPs always prefer allowing transit traffic. However, this reasoning relies heavily on the implicit assumption that there is no capacity constraint in a link. If a link capacity is constrained, an ISP will be either reluctant to relay transit traffic or want to be paid more for relaying it. Especially, if ISPs have multiple interconnection points, it is a common practice for each ISP to employ hot-potato routing: handing off traffic to the neighboring ISP as quickly as it can. This may cause delays in packet transmission affecting the neighboring ISPs,⁴¹ and capacity constraints should aggravate the problem. Then, it is an interesting issue who should

³⁸In fact, a graph G in which $i_1j_1, i_2j_2, j_1j_2 \in G$ is also efficient in this model. However, it becomes less efficient than LS network if we take transmission costs into account as in Section 8. This is because in any connected network, the information request between i_1 and i_2 is more than the request between j_1 and j_2 , and thus the heavier traffic should be roundabout in G than in LS network.

³⁹Detailed computations for the stability conditions are available from the authors upon request.

⁴⁰Besen *et al.* also argue that bargaining powers of core ISPs are reduced as a result of secondary peering or multi-homing, although they use the Nash solution for the bargaining outcome instead of the Myerson value.

⁴¹"A 2003 measurement study of Internet routes found that, between pairs of neighboring ISPs, more than 30% of paths have inflated latency due to hot potato routing, with 5% of paths being delayed by at least 12 milliseconds" (excerpted from "Routing" in Wikipedia).

bear the cost of increasing the capacity due to inefficient routing.⁴² Thus, we may consider capacity constraint by endogenizing the investment in the connection cost which will determine the capacity of resulting links. Such a model could provide an answer for the choice problem between allowing or disallowing transit traffic, although the analysis seems extremely complicated.

Resilience An ISP's decision to form links is usually affected by the resilience consideration of its network. In other words, ISPs prepare against the breakdown of its link or its router. Bala and Goyal (2000b) or Jun and Kim (2007) may provide a useful reference for the efficiency and the stability of a network based on the consideration.

Endogenous Market Shares One serious restriction of our models is that the market share of each ISP is given. In the real world, however, this is true only for a short run and the market share of an ISP is endogenously determined by consumer prices which in turn rely on interconnection fees. In other words, interconnection charges of an ISP are not only affected by the market shares of ISPs, but also affect their market shares.⁴³

Market Shares for Content Providers We implicitly assumed that the ratio of the number of Internet users to the number of content providers affiliated with an ISP is the same for all ISPs. However, this ratio is generally not uniform across ISP. Some ISPs called IDC (Internet Data Center)-type ISPs have more contents relative to consumers, while others have relatively more users. It will be interesting to see which market share of the two is the more effective determinant of the bargaining power in Internet interconnection agreements.

Route Filtering We have stood mute on the possibility of route filtering. The rationale was that no ISP has an incentive for route filtering as long as the network structure is determined in the first stage. However, if an ISP can commit to route filtering, in other words, if it can precommit that it will provide connectivity only to some specific ISPs, it can affect ISPs' decision to form links. In this three stage game, ISPs may find filtering profitable.⁴⁴

⁴²In a simplified cooperative game, Ma *et al.* (2007) showed that ISP will maximize its profit by following optimal routing strategy, which is different from hot potato routing.

⁴³See Laffont *et al.* (2003) for a Internet interconnection model with flexible market shares.

⁴⁴Route filtering between interconnecting ISPs can be related to the issue of network neutrality if the act of filtering is (unduly) discriminatory. Though the network neutrality

Possibility of Mergers Finally, one potentially interesting study would be to address the possibility of mergers. It is clear that as long as the merger decision follows link formation decisions, a merger cannot improve efficiency. However, if ISPs can commit to a merger before their connection decisions, the analysis may give a guideline on whether a merger should be encouraged or discouraged from the efficiency point of view.

Appendix

Lemma 1 $x(k)$ is increasing in k .

Proof. It is easy to see that

$$x(2) - x(1) = \alpha v(2\alpha) - \alpha v(\alpha) > 0,$$

$$x(3) - x(2) = \alpha \left[\frac{3}{2}v(3\alpha) - v(2\alpha) \right] + \alpha (v(2\alpha) - v(\alpha)) > 0.$$

Suppose that $x(k) > x(k-1)$. Then, we have

$$\begin{aligned} x(k+1) &= \frac{1}{2}\alpha(k+1)v((k+1)\alpha) + \frac{1}{2}k[x(k) - \alpha v(\alpha)] \\ &> \frac{1}{2}\alpha kv(k\alpha) + \frac{1}{2}(k-1)[x(k-1) - \alpha v(\alpha)] \\ &= x(k). \end{aligned}$$

Therefore, by induction, the proof is completed. $\parallel$

debate usually involves an ISP and bandwidth-hogging applications (or its providers) such as Vonage or Bit Torrent, two interconnecting ISPs can get involved in such a dispute once one of them enters into content businesses. In 2006, a Korean broadband ISP, PowerComm, blocked the entire traffic (route) from Hanaro, another broadband ISP, on the first day Hanaro started VOD services. For more on economic approaches to the network neutrality, refer to Schuett (2010).

Proof of Proposition 3:

The total interconnection charge the centered ISP j receives is determined by

$$\begin{aligned} a_j &= x(n) - \alpha v(1) \\ &= \frac{1}{2}[v(1) + (n-1)(x(n-1) - \alpha v(\alpha))] - \alpha v(1) \\ &= \left(\frac{1}{2} - \alpha\right)v(1) + \frac{1}{2}(n-1)(x(n-1) - \alpha v(\alpha)) > 0, \end{aligned}$$

since $x(n-1) > \alpha v(\alpha)$ for all $n \geq 3$ from Lemma 1. This implies that the centered ISP receives the interconnection charge from other peripheral ISPs. $\parallel$

Proof of Proposition 4:

(i) is trivial, because $\Delta_l(S; G') \geq \Delta_l(S; G)$ for all S and with strict inequality for $S = \{j\}$ if $l = i$ and $S = \{i\}$ if $l = j$.

(ii) Suppose one adds a link $ij \notin G'$ to G . Let S^k be a coalition of size k such that $i \notin S^k$. If we consider $P_i(G|_{S^k})$ and $P_j(G|_{S^k})$, we must have either $P_i(G|_{S^k}) = P_j(G|_{S^k})$ or $P_i(G|_{S^k}) \cap P_j(G|_{S^k}) = \emptyset$. If adding ij to G increases the Myerson value of i in G , the following two conditions must be met; (1) $j \in S^k$ and (2) $S^k \cup \{i\}$ must not contain a path from i to j except the link ij . Therefore, we have $\Delta_i(S^k; G + ij) = \Delta_i(S^k; G)$ if $P_i = P_j$ and $\Delta_i(S^k; G + ij) > \Delta_i(S^k; G)$ if $P_i \cap P_j = \emptyset$.

For a given S^k , define $\delta_i(ij; S^k, G) = \Delta_i(S^k; G + ij) - \Delta_i(S^k; G)$. Then, we have

$$\delta_i(ij; S^k, G) = (\beta + \gamma)v(\beta + \gamma) - \beta v(\beta) - \gamma v(\gamma),$$

where $\beta(G) \equiv \alpha(1 + \#P_i(G|_{S^k}))$ and $\gamma(G) \equiv \alpha(1 + \#P_j(G|_{S^k}))$. We can see that $\delta_i(ij; S^k, G)$ increases in β and γ if $w''(\cdot) \geq 0$. Meanwhile, we know that $\beta(G) \geq \beta(G')$ and $\gamma(G) \geq \gamma(G')$ if $G \subset G'$, since $\#P_l(G'|_{S^k}) \leq \#P_l(G|_{S^k})$ for all $l = i, j$ and for all S . This completes the proof of (ii). $\parallel$

Proof of Proposition 5:

(i) Since the profits in bill-and-keep agreements are the same for all ISPs, we only need to show that $y_i^M(G^*) > y_i^M(G^*)$.

We know that $\Delta_i(\emptyset) = \Delta_j(\emptyset)$ for the empty coalition. Let S_i^k be a coalition of size $k (\geq 1)$ such that $i \notin S_i^k$. Then, it is easy to see that $\Delta_j(S_j^k) = \Delta_i(S_i^k)$ for all S_j^k and S_i^k such that $j \notin S_i^k$ and that $\Delta_j(S_j^k) > \Delta_i(S_i^k)$ for all S_j^k and S_i^k such that $j \in S_i^k$. Therefore, the result is immediate. $\parallel$

(ii) By symmetry of G^L , it suffices to consider ISP i and ISP $i + 1$ for $i \leq m - 1$. We know that $\Delta_i(\emptyset) = \Delta_{i+1}(\emptyset) = \alpha v(\alpha)$. Consider a coalition of size k . The number of S_i^k , $\binom{n-1}{k}$, is the same for all i . It is easy to see that $\Delta_i(S_i^k)$ is larger as $S_i^k \cup \{i\}$ contains more consecutive numbers. Let $\aleph(l; S_i^k) = \#\{S_i^k \mid S_i^k \cup \{i\} \text{ contains } l \text{ consecutive numbers}\}$. If $i \leq l < n - 2i$, $\aleph(l; S_i^k) = i < i + 1 = \aleph(l; S_{i+1}^k) = i + 1$. If $l < i$ or $l \geq n - 2i$, $\aleph(l; S_i^k) = \aleph(l; S_{i+1}^k)$. Thus, $y_i^M(G^L) < y_{i+1}^M(G^L)$ for $i \leq m - 1$. $\parallel$

Proof of Proposition 6:

(i) Suppose a connected component C in G contains a cycle. A deletion of any link within the cycle would not affect $\sum_{i \in N(C)} \tilde{\pi}_i(C)$ only with a save of $2f > 0$. This implies that a deletion of any link within the cycle will improve efficiency by $2f$. $\parallel$

(ii) We will characterize the most efficient one G^k among forests with k components, where $1 \leq k \leq n$. It is trivial that G^1 is a tree and $E(G^1) = v(1) - 2(n - 1)c$. Let $\#C_k = m_k$. For $k = 2$,

$$\begin{aligned} E(G^2) &= \max_{m_1+m_2=n} m_1 \alpha v(m_1 \alpha) + m_2 \alpha v(m_2 \alpha) - 2(n - 2)f \\ &= \alpha v(\alpha) + (n - 1) \alpha v((n - 1) \alpha) - 2(n - 2)f. \end{aligned}$$

Similarly, we have

$$\begin{aligned} E(G^k) &= \max_{\sum_{i=1}^k m_i = n} \sum_{i=1}^k m_i \alpha v(m_i \alpha) - 2(n - k)f \\ &= (k - 1) \alpha v(\alpha) + (n - k + 1) \alpha v((n - k + 1) \alpha) - 2(n - k)f. \end{aligned}$$

For $k = n$, $E(G^n) = n \alpha v(\alpha)$. We will show that $E(G^1) = \max_{1 \leq k \leq n} E(G^k)$ if $f < \alpha(v(2\alpha) - v(\alpha))$. We have

$$\begin{aligned} E(G^k) - E(G^{k-1}) &= \alpha v(\alpha) + (n - k + 1) \alpha v((n - k + 1) \alpha) - (n - k + 2) \alpha v((n - k + 2) \alpha) + 2f. \end{aligned}$$

Thus, $E(G^k) < E(G^{k-1})$ if $f < \hat{f}(k)$ where

$$\hat{f}(k) = \frac{\alpha}{2} [(n - k + 2) v((n - k + 2) \alpha) - (n - k + 1) v((n - k + 1) \alpha) - v(\alpha)].$$

Since $w''(\theta) \geq 0$, $\hat{f}(k)$ is decreasing in k . Therefore, $E(G^1) \geq E(G^k)$ for all k if $f < \hat{f}(n) = \alpha(v(2\alpha) - v(\alpha))$. $\parallel$

Proof of characterization of stable networks without friction:

(i) Consider $\tilde{G} = G^N - ij$. For any subset S of $I \setminus \{i\}$, $\Delta_i(S; G^N) = \Delta_i(S; \tilde{G})$ except for $S = \{j\}$, because any pair of nodes in S are indirectly linked. Note that the probability that $S = \{j\}$ is $\frac{1}{n(n-1)}$. Thus, ISP i will have no incentive to sever a link ij if $f < y_i^M(G^N) - y_i^M(\tilde{G}) = \frac{2}{n(n-1)}\alpha(v(2\alpha) - v(\alpha))$, and neither will ISP j by the equal bargaining power property of the Myerson value. Thus, G^N is stable. Now, suppose $G \neq G^N$ is stable. Then, there must be a link $ij \notin G$. Consider $G + ij$. Then, by Proposition 4(ii), $y_i^M(G + ij) - y_i^M(G) \geq y_i^M(G^N) - y_i^M(G^N - ij) > f$. This implies that G is not stable. Contradiction.

(ii) It is clear that no pair of peripheries have an incentive to form an additional link if $f > y_i^M(\tilde{G}) - y_i^M(G^*)$. Consider $\tilde{G}^* = G^* - ij$ where j is the center. By Property 3 of the Myerson value, it must hold that $y_i^M(G^*) - y_i^M(\tilde{G}^*) = y_j^M(G^*) - y_j^M(\tilde{G}^*)$. Since $y_i^M(\tilde{G}^*) = \alpha v(\alpha)$ for all $i \neq j$, no ISP would have an incentive to delete a link if $f < y_i^M(G^*) - \alpha v(\alpha)$.

Lemma 2 $y(n)$ increases in n .

Proof of Lemma 2:

Consider a link $(i-1)i$ for $i \leq (n+1)/2$ for an odd n . By the property of equal bargaining power, we have

$$y_i^M(G^L) - y(n-i+1) = y_{i-1}^M(G^L) - y(i-1).$$

Rearranging terms yields

$$y_i^M(G^L) - y_{i-1}^M(G^L) = y(n-i+1) - y(i-1).$$

Since $y_i^M(G^L) > y_{i-1}^M(G^L)$ from Proposition 5, we have

$$y(n-i+1) > y(i-1). \quad (8)$$

By plugging $i = (n+1)/2$ into equation (8), we get

$$y\left(\frac{n+1}{2}\right) > y\left(\frac{n-1}{2}\right). \quad (9)$$

Since equation (9) holds for any odd n , the proof is completed. $\parallel$

(iii) The proof is completed by Claim 1 and Claim 2 below.

Claim 1 *The end ISPs are most willing to form a link between them. In particular, no pair of ISPs in G^L will add a link between them if $f > \alpha v(1) - y(n)$.*

Proof. Consider an additional link ij for $i < j$. Let S^k be a coalition of size k such that $i \notin S^k$. Consider $P_i(G^L|_{S^k})$ and $P_j(G^L|_{S^k})$. Then, either $P_i(G^L|_{S^k}) = P_j(G^L|_{S^k})$ or $P_i(G^L|_{S^k}) \cap P_j(G^L|_{S^k}) = \emptyset$. If adding ij to a graph increases the Myerson value of i , it must be that $j \in S^k$ and that $S^k \cup \{i\}$ must not contain a path from i to j except the link ij . Thus, $\Delta_i(S^k; G^L + ij) = \Delta_i(S^k; G^L)$ if $P_i = P_j$ and $\Delta_i(S^k; G^L + ij) > \Delta_i(S^k; G^L)$ if $P_i \cup P_j = \emptyset$. Meanwhile, $P_i = P_j$ is possible when S^k contains all nodes between i and j in G^L . If $k < t(ij; G^L)$, this event cannot happen. So, the number of events that S^k satisfy $P_i \cup P_j = \emptyset$ is maximized or the number of events that $P_i = P_j$ is minimized when $t(ij; G^L)$ is maximized, i.e., $i = 1$ and $j = n$. Thus, the result follows from Proposition 5. $\parallel$

Claim 2 *The end ISPs suffer least from the deletion of their link. In particular, if $f < y(n) - y(1)$, no ISP in G^L has an incentive to delete a link.*

Proof. Let $\delta_i(ij) = y_i^M(G^L) - y_i^M(G^L - ij)$. We will abuse notation by writing ISP i 's Myerson value in G^L with n as $y_i^M(n)$. If link $(i-1)i$ is deleted, we have $\delta_i((i-1)i) = y_i^M(n) - y_1^M(n-i+1)$. If the link $i(i+1)$ is deleted, we get $\delta_i(i(i+1)) = y_i^M(n) - y_1^M(i)$. If $n \geq 2i-1$, $y_1^M(n-i+1) \geq y_1^M(i)$ by Lemma 2, yielding $\delta_i((i-1)i) \leq \delta_i(i(i+1))$. Therefore, for an odd n , we have

$$\delta_i((i-1)i) \begin{matrix} \leq \\ > \end{matrix} \delta_i(i(i+1)), \text{ if } i \begin{matrix} \leq \\ > \end{matrix} \frac{n+1}{2}.$$

For an even n , we have

$$\begin{aligned} \delta_i((i-1)i) &< \delta_i(i(i+1)), \text{ if } i \leq \frac{n}{2}, \\ \delta_i((i-1)i) &> \delta_i(i(i+1)), \text{ if } i \geq \frac{n}{2} + 1. \end{aligned}$$

Therefore, if the end ISPs maintain their links, so do all the other ISPs. The end ISPs will maintain their links if $f < \delta_1(12) = y(n) - y(1)$. $\parallel$

Lemma 3 $y_i^M(G^C + ii^*) \geq y_i^M(G^C + ij)$ for any $j \neq i$.

Proof of Lemma 3:

It is clear that $\sum_{S^1} \Delta_i(S^1; G^C + ij)$ is the same for all $j \neq i$, because $\Delta_i(S^1; G^C + ij) = 2\alpha v(2\alpha) - \alpha v(\alpha)$ if $S^1 = \{j\}$ and $\alpha v(\alpha)$ if $S^1 \neq \{j\}$. Now, consider S^k with $k \geq 2$. From the same reason as in the proof of Claim 1, $\Delta_i(S^k; G^C + ij) = \Delta_i(S^k; G^C)$ if $P_i = P_j$ and $\Delta_i(S^k; G^C + ij) > \Delta_i(S^k; G^C)$ if $P_i \cup P_j = \emptyset$. Similarly, given k , the probability that $P_i \cup P_j = \emptyset$ is maximized when $t(ij; G^C)$ is maximized, i.e., $j = i^*$. $\parallel$

(iv) The proof is immediate from Lemma 3. $\parallel$

Proof of Proposition 7:

(i) Suppose a stable network G is not connected, i.e., there is more than one connected component, say $C_0(G)$ and $C_1(G)$. Take any ISP $i \in C_0(G)$. If $C_0(G)$ consists only of ISP i , $y_i^M(G) = \alpha v(\alpha)$. Now, consider a network $G + ij$ for $j \in N(C_1(G))$. Let $N(C_1(G)) = m (\geq 1)$. Then, the gain of ISP i from connecting a link ij is

$$\Delta(G) \equiv y_i^M(G + ij) - \alpha v(\alpha).$$

If $m = 1$, it is clear that $\Delta(G) = \alpha(v(2\alpha) - v(\alpha)) > 0$, which implies that both i and j have an incentive to add the link.

Assume that $m \geq 2$. By the definition of the Myerson value, we have

$$\Delta(G) = \sum_{S \subset I \setminus \{i\}} \nabla_i(S) \frac{\#S!(n - \#S - 1)!}{n!},$$

where $\nabla_i(S) \equiv U(S \cup \{i\}) - U(S) - \alpha v(\alpha) \equiv \Delta_i(S) - \alpha v(\alpha)$. Given S , $\nabla_i(S)$ is affected only by the network structure of C_1 , since V is component additive.

Our goal is to find the minimum of $\Delta(G)$ when the network structure of C_1 is the choice variable. Let $\mathcal{S} = \{S \subset N(C_1) \mid \nabla_i(S) > 0\}$. Then, we know that $j \in S$ for all $S \in \mathcal{S}$. For each S , define $P_j(C_1|_S) = \{l \in S \mid \exists Q_{jl} \text{ in } C_1|_S\}$. Let $k(S; C_1) = \#P_j(C_1|_S)$. Compare C_1 with C'_1 such that $C_1 \subset C'_1$ and $N(C_1) = N(C'_1)$. Then, $k(S; C_1) \leq k(S; C'_1)$ with strict inequality for some $S \in \mathcal{S}$. Note that

$$\Delta_i(S) = \alpha(k(S; G) + 1)v(\alpha(k(S; G))) - \alpha k(S; G)v(\alpha k(S; G)).$$

If $w''(\cdot) \geq 0$, we have $\Delta_i(S; C'_1) \geq \Delta_i(S; C_1)$ with strict inequality for some $S \in \mathcal{S}$. Hence, it is sufficient to compare trees to find the minimum of $\Delta(G)$.

We claim that, given S , $\Delta_i(S; C_1)$ is minimal when $C_1 = G^L \in \mathcal{T}$. The reason is that the number of S such that $k(S; G^L) = k$ for each $k = 1, \dots, m$ is only one in G^L , while it is more than or equal to one in any connected graph. Moreover, by Lemma 2, $\Delta(G)$ is minimized when $m = 1$. Therefore, if $C_0(G) = \{i\}$, G cannot be stable if $f < \alpha(v(2\alpha) - v(\alpha))$, because ISP i and j have an incentive to add the link ij regardless of the structure of $C_1(G)$.

Now, consider the case that $C_0(G)$ contains more than one ISP. For the same reason, the gain of ISP i from connecting ij is maximized when $C_0(G)$ as well as $C_1(G)$ are line networks and $\#N(C_0(G)) = \#N(C_1(G)) = 1$. This completes the proof. $\parallel$

(ii) If $f < \alpha(v(2\alpha) - v(\alpha))$, the efficient network has $(n - 1)$ links by Proposition 6. Since a network with less than $(n - 1)$ links cannot be connected, it cannot be stable by part (i). $\parallel$

Proof of Proposition 8:

(i) Suppose a component C of a stable network G contains a cycle, say, $Q = \{1, 2, \dots, i, 1\}$. Since any pair of ISPs in C remains connected after a deletion of a link $\tau(\tau + 1)$ for all $\tau = 1, \dots, i$, the payoffs of both ISP τ and ISP $\tau + 1$ remain unaffected by the deletion only with saving the connection cost $f > 0$ under the bill-and-keep agreements. This implies that G can never be stable for any $f > 0$. $\parallel$

(ii) Since a tree is connected, clearly no pair in a tree have an incentive to add a link. $\parallel$

Lemma 4 *If G does not contain a cycle, there is at least a node such that $d(i) = 1$. In fact, there must be at least two such nodes.*

Proof. Suppose that $d(i) \geq 2$ for all $i \in I$. Start with node 1. Without loss of generality, we will label an adjacent node to node 1 as node 2. Since $d(2) \geq 2$, node 2 must have an additional adjacent node. Let us call it node 3. Since $n < \infty$, this process cannot continue infinitely. Let $k \leq n$ be the node that no new link can be directly linked to. Since $d(k) \geq 2$, node k must have another link with some node $i = 1, \dots, k - 1$ or their other adjacent nodes. This implies that G contains a cycle, which is a contradiction. Thus, there must be at least one node with $d(i) = 1$. Now, suppose that there is only one node with $d(i) = 1$. Without loss of generality, label the node as node 1. Similarly, start with the node and label the only adjacent node as node 2. If we continue this procedure, it leads to a cycle. Contradiction. $\parallel$

Let i_0 be a node with $d(i_0) = 1$ and its adjacent node be j . The net loss of node j from severing link i_0j is $\alpha(v(1) - v(1 - \alpha))$ which is the minimum loss of a node severing its link. Thus, no ISP in a tree will sever a link if $f < \alpha(v(1) - v(1 - \alpha))$. Now, due to (i), we only need to show that a forest with less than $n - 1$ links, say G , cannot be stable. If node i_0 with $d(i_0) = 1$ adds a link with node j in G , $y_{i_0}^M(G + i_0j) - y_{i_0}^M(G) \geq \alpha v(\alpha) = \alpha^2$. Since $f < \alpha^2$, node i_0 and node j always have an incentive to add the link. This implies that G is not stable. $\parallel$

Proof of Proposition 9:

(i) Let the new efficiency measure under friction be $E_f(G)$. Then, $E_f(G) = E(G) - T(G) = V(G) - W(G) - T(G)$, where $V(G) = \sum_{i \in I} y_i^M(G)$, $W(G) = \sum_{i \in I} d(i)f$. Let the set of connected graphs be $\mathcal{G}_1$. It is clear that $V(G)$ is the same for all $G \in \mathcal{G}_1$. We will show that $G^N = \arg \min_{G \in \mathcal{G}_1} W(G) + T(G)$. Consider $G_1 = G - i_1j_1$ where $G = G^N$. Then, $\Delta W = -2f$ and $\Delta T = 2\alpha^2c$. So, $E_f(G_1) - E_f(G^N) = \Delta W + \Delta T = 2(\alpha^2c - f) < 0$ if $f > \alpha^2c$. Consider a sequence of G_k such that $G_k = G_{k-1} - i_kj_k$ for some $i_kj_k \in G_{k-1}$ until G_{k+1} is disconnected. Similarly, we can see that $E_f(G_i) < E_f(G_{i-1})$ for all $i = 1, \dots, k$ if $f > \alpha^2c$. Thus, $E_f(G^N) > E_f(G)$ for all $G \in \mathcal{G}_1$ if $f < \alpha^2c$.

Now, we will show that $E(G^N) > E(G)$ for all $G \in \mathcal{G} \setminus \mathcal{G}_1$. Let $\mathcal{G}_l$ be the set of graphs which have $l (\geq 2)$ connected components $C_i, i = 2, \dots, l$ and $\#N(C_i) = m_i$ with $\sum_{i=1}^l m_i = n$. The most efficient graph in $\mathcal{G}_l$, denoted by G^l is that each component of it is completely connected. The reason is that a component which is less than completely connected can improve efficiency by adding a link until it becomes completely connected if $f < \alpha^2c$ as we argued above. Then, we have

$$E(G^l) = \sum_{i=1}^l \alpha m_i v(\alpha m_i) - \sum_{i=1}^l m_i(m_i - 1)\alpha^2c - \sum_{i=1}^l m_i(m_i - 1)f.$$

The proof will be completed if we show that efficiency is improved by connecting any two components in G^l . For that, it is sufficient to compare $E(G^l)$ with $E(\hat{G}^l)$ where $\hat{G}^l$ is the graph obtained by completely connecting two components in G^l if $f < \alpha^2c$ by the same argument above. Letting $\Delta E = E(\hat{G}^l) - E(G^l)$, we have

$$\begin{aligned} \Delta E &= \alpha(m_i + m_j)v(\alpha(m_i + m_j)) - (m_i + m_j)(m_i + m_j - 1)\alpha^2c - 2m_im_jf \\ &\quad - (\alpha m_iv(\alpha m_i) + \alpha m_jv(\alpha m_j) - m_i(m_i - 1)\alpha^2c - m_j(m_j - 1)\alpha^2c) \\ &= \alpha(m_i + m_j)v(\alpha(m_i + m_j)) - \alpha m_iv(\alpha m_i) - \alpha m_jv(\alpha m_j) \\ &\quad - 2m_im_i\alpha^2c - 2m_im_jf. \end{aligned}$$

If $v(x) = x$, ΔE is reduced to $2m_j m_i (\alpha^2 (1 - c) - f)$. Hence $E(\hat{G}^l) > E(G^l)$ if $f < \alpha^2 (1 - c)$, for all m_i, m_j . Since this holds for all l , G^N is most efficient if $f < \min\{\alpha^2 c, \alpha^2 (1 - c)\}$. $\parallel$

(ii) We have $V(G) = V(G')$ and $W(G) = W(G')$ for any $G \neq G' \in \mathcal{T}$. Since $\#\{ij \in G \mid t(ij; G) = 1\} = n-1$ for all $G \in \mathcal{T}$ but $\#\{ij \in G \mid t(ij; G^*) = 2\} = \binom{n}{2} - (n-1)$, which is maximal among all $G \in \mathcal{T}$, $T(G^*) < T(G)$ for all $G \in \mathcal{T}$. Thus, $E(G^*) > E(G)$ for all $G \in \mathcal{T}$. $\parallel$

Proof of Proposition 10:

(i) The total value of transmission costs is $T(G^N) = n(n-1)\alpha^2 c$. Thus, we have

$$y_i^C(G^N) = \sum_{k=1}^{n-1} \frac{(n-1)!}{k!(n-k-1)!} ((k+1)k - k(k-1)) \Lambda = (n-1)\alpha^2 c,$$

where $\Lambda = \frac{k!(n-k-1)!}{n!} \alpha^2 c$. If we delete link ij from G^N , the total value of transmission costs is

$$T(G^N - ij) = (n-2)(n+1)\alpha^2 c + 4\alpha^2 c = (n^2 - n + 2)\alpha^2 c \text{ for } n \geq 3.$$

Thus, for ISP $i(j)$, we have

$$\begin{aligned} & y_i^C(G^N - ij, n) \\ &= \sum_{k=2}^{n-1} \frac{(n-2)!}{(k-1)!(n-k-1)!} ((k-1)k + 2(k-1+2) - k(k-1)) \Lambda \\ &+ \sum_{k=2}^{n-2} \left(\frac{(n-1)!}{k!(n-k-1)!} - \frac{(n-2)!}{(k-1)!(n-k-1)!} \right) ((k+1)k - k(k-1)) \Lambda \\ &+ \frac{1}{n(n-1)} (n-2) 2\alpha^2 c \\ &= \frac{(n+n^2+2)(n-2)}{n(n-1)} \alpha^2 c. \end{aligned}$$

If $n = 2$, $T(G^N - ij) = 0$ and $y_i^C(G^N - ij) = 0$. Thus, when link ij is deleted, the change in ISP i 's Myerson value of $\langle I, G, T \rangle$ is

$$\begin{aligned} y_i^C(G^N - ij) - y_i^C(G^N) &= \frac{(n^2 + n + 2)(n - 2)}{n(n - 1)}\alpha^2 c - \alpha^2 c(n - 1) \\ &= \frac{(n^2 - n - 4)}{n(n - 1)}\alpha^2 c, \end{aligned}$$

for all $n \geq 2$. Thus, G^N is stable if

$$\begin{aligned} f &< y_i^M(G^N, \hat{V}) - y_i^M(G^N - ij, \hat{V}) \\ &= y_i^M(G^N, V) - y_i^M(G^N - ij, V) - ((y_i^C(G^N) - y_i^C(G^N - ij))) \\ &= \frac{2\alpha(v(2\alpha) - v(\alpha))}{n(n - 1)} + \frac{(n^2 - n - 4)}{n(n - 1)}\alpha^2 c. \end{aligned}$$

(ii) For each peripheral ISP i , we have

$$\begin{aligned} y_i^C(G^*) &= \sum_{k=2}^{n-1} \frac{(n-2)!}{(k-1)!(n-k-1)!} \left(\frac{k(1+2(k-1))+k}{-(k-1)(1+2(k-2))-(k-1)} \right) \Lambda \\ &\quad + \frac{2}{n(n-1)}\alpha^2 c = \frac{1}{3}(4n-5)\alpha^2 c. \end{aligned}$$

If we add a link ii' to G^* , the total value of transmission costs in $\hat{G}^*$ is

$$\begin{aligned} T(\hat{G}^*) &= ((n-1) + (n-3)(1+2(n-2)) + 2(2+2(n-3)))\alpha^2 c \\ &= 2(n-2)n\alpha^2 c. \end{aligned}$$

Then, we have

$$\begin{aligned} y_i^C(\hat{G}^*) &= \sum_{k=2}^{n-1} \left(\frac{\frac{(n-2)!}{(k-1)!(n-k-1)!}}{-\frac{(n-3)!}{(k-2)!(n-k-1)!}} \right) \left(\frac{k(1+2(k-1))+k}{-(k-1)(1+2(k-2))-(k-1)} \right) \Lambda \\ &\quad + \sum_{k=2}^{n-1} \frac{(n-3)!}{(k-2)!(n-k-1)!} \left(\frac{k(1+2(k-1))+k-2}{-(k-1)(1+2(k-2))-(k-1)} \right) \Lambda \\ &\quad + \sum_{k=1}^{n-2} \frac{(n-3)!}{(k-1)!(n-k-2)!} 2\Lambda + \frac{2}{n(n-1)}\alpha^2 c = \frac{2}{3}(2n-3)\alpha^2 c. \end{aligned}$$

Thus, we have $y_i^C(\hat{G}^*) - y_i^C(G^*) = \frac{2}{3}(2n-3)\alpha^2 c - \frac{1}{3}(4n-5)\alpha^2 c = -\frac{1}{3}\alpha^2 c$. Now, consider $\tilde{G}^* = G^* - ij$. Since $y_i^C(\tilde{G}^*) = 0$, the proof is immediate. $\parallel$

(iii) We have $y_i^C(G^C) = T(G^C)/n$ where $T(G^C) = 2\alpha^2 cn \sum_{k=1}^{(n-1)/2} k = \frac{1}{4}\alpha^2 cn(n-1)(n+1)$ if n is odd and $\frac{1}{4}\alpha^2 cn^3$ if n is even.

We know that the end nodes are most willing to add a link between them from Claim 1 and the simple fact that the added link reduces the total communication cost most. So, no pair of ISPs in G^L will add a link in the game $\langle I, G^L, \hat{V} \rangle$ if $f > \alpha v(1) - y(n) + \Delta y$. Also, from Claim 2 and the continuity of T with respect to c , there exists $\bar{c}(> 0)$ such that for all $c \leq \bar{c}$ end ISPs suffer least from deleting their link. Thus, for any $c \leq \bar{c}$, no ISP will not delete its link if $f < y(n) - \alpha v(\alpha) - y^C(n)$. $\parallel$

(iv) This is immediate from the observation that $y_i^C(\hat{G}^C) \leq y_i^C(G^C + ij)$ for any $j \neq i$. $\parallel$

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