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Experimental Duopoly in the Classroom

Charles F. Mason

Abstract

This in-class duopoly game experiment allows students to learn about strategies and equilibria. It is appropriate for use in undergraduate and graduate classes and takes about two hours to complete.

KEYWORDS: experiment, duopoly, classroom

1. Theory

A key component of economic analysis of oligopoly has to do with collusion. While firms are concerned with maximizing their own profits, there is a concern that they may jointly act so as to maximize industry profits. This can be an important concept to convey to students, and a natural way to get the basic ideas across is to involve them in an oligopoly experiment.

An important factor in the evaluation of collusive tendencies is repeated play. There is a tension between cooperating with a cartel, and thereby earning a stream of large profits, and defecting from the cartel, thereby raising profits in the short term and lowering them later on. Without repeated play, the incentive to defect is overwhelming; with repeated play, firms may be more inclined to collude. Accordingly, any experiment designed to convey the basic tension between competition and collusion needs to allow students the opportunity to interact over several periods. The series of experiments described below is intended to illustrate the importance of repeated play. Two designs are used: one with a series of one-shot interactions, facilitated by re-matching participants after each period, and one where students interact against the same rival for several periods.

In many applications, it is common to observe very little cooperation in the first design, but a fair bit of cooperation (at least in early periods) in the second design. Since the idea is to focus on the role of repeated play, the games used in these experiments are simple 2x2 Prisoner's dilemmas. Instructors interested in further experiments might want to use larger payoff matrices (as in Holt, 1985); consider the importance of similarity of agents' payoff functions (as in Mason, Nowell, and Phillips, 1992); or the role of information regarding the rival's payoffs (as in Cason and Mason, 1999).

2. Theory

Many models of collusion envision firms using trigger strategies. These strategies are comprised of two parts: a cooperative-looking choice that players select so long as there have been no departures from the strategy (by either player), and a "punishment phase", consisting of a punishment output and a period over which punishment will occur, in the event of defection. Consider a two-person infinitely repeated game with payoffs identical to a quantity choosing, duopoly market. Two players simultaneously make choices q_{1t} and q_{2t} in each period $t = 1, 2, \dots$. In any period t , player i 's payoffs are $\pi_i(q_{it}, q_{jt})$, for $i, j = 1, 2$. The weight placed upon next period's payoffs, i.e. the discount factor, is δ for each player. For simplicity, let's restrict the model to stationary, symmetric equilibria of the repeated game-combinations of strategies that are the same for

each player at each point in time. It can be argued that such strategies are easily identified by agents and more likely to represent “self-evident patterns of play,” so that they are most likely to be selected from the set of subgame perfect equilibria (Kreps, 1990).

Perhaps the best-known example of a trigger strategy is the “grim reaper strategy.” Under this strategy, the cooperative-looking action, q^c , is selected so long as both players have chosen this action in all previous periods. The punishment action is the one-shot Nash equilibrium choice q^n and the punishment phase is forever. That is, if either party defects in some period τ , both play q^n in all future periods $t = \tau + 1, \tau + 2, \dots$. Let $q^d(q^c)$ be the optimal one-period defection to the cooperative choice, q^c , i.e. the Nash best response. Then, we may write symmetric cooperative payoffs as $\pi^c \equiv \pi(q^c, q^c)$ and defection profits as $\pi^d \equiv \pi(q^d(q^c), q^c)$. It is evident that both payoffs are uniquely determined by q^c , and so henceforth we shall write them only as functions of q^c . We assume that π^c is a concave function of q^c . Finally, we write the one-shot Cournot/Nash equilibrium payoffs as π^n . For the choice q^c to be supported as part of a subgame perfect equilibrium path, these payoffs must satisfy the incentive constraint:

$$(1) \quad \pi^d(q^c) + \delta\pi^n/(1 - \delta) \leq \pi^c(q^c)/(1-\delta).$$

The interpretation of this constraint is that players can realize the cooperative payoff perennally if no one defects. Following any defection, however, play reverts to an infinite sequence of one-shot Cournot/Nash equilibrium choices. Thus, each player must compare the present discounted value of continued cooperation, the right side of (1), against the present discounted value of defection, the left side of (1), which included the one-shot gain from defection and the discounted flow of one-shot Cournot/Nash profits.

It is well-known that there are many values of q^c that satisfy this constraint, but we shall consider the most profitable of these. This choice yields strategies that payoff dominate all alternative (stationary symmetric) subgame perfect equilibria. Let us write this value as q^* ; it solves the constrained optimization problem

$$\begin{aligned} &\text{Max } \pi^c \\ &q^c \quad \text{s.t. } (1-\delta)\pi^d(q^c) + \delta\pi^n \leq \pi^c(q^c). \end{aligned}$$

The unconstrained optimum choice q^m will have at the margin $\pi^{c'}(q^m) = 0$. Since π^c is concave, $\pi^{c'}(q^c) < 0$ for $q^c > q^m$. If complete collusion is not feasible then $q^c > q^m$ in the relevant range, and so it follows that q^* is the smallest value of

q^c that satisfies the incentive constraint. At this choice the incentive constraint is just binding and:

$$(2) \quad \pi^c(q^*) - (1-\delta)\pi^d(q^*) - \delta\pi^n = 0.$$

If several players are randomly assigned into pairs in each of several periods, there are two potential outcomes. In one, each player might view this as a super-game involving the entire set of players, in which case there could be a cooperative-looking outcome in the event that players are sufficiently optimistic about the entire set of potential rivals, and in the event that players are particularly patient. Otherwise, the equilibrium outcome is for each player to choose the one-shot Cournot/Nash equilibrium. The more plausible of these two possibilities is the latter, i.e. Cournot competition.

3. Experimental Design

Most modern oligopoly experiments are run on computers, in large part because this facilitates data collection. But for instructional purposes, running the experiment by hand within the classroom will work; this approach has the added benefit of flexibility: it can be conducted just about anywhere, whether the room has computers available or not.

To start off, the instructor determines the number of participants. If the class is of modest size, say 30 students or less, it is feasible to use all the students at some point. If the class is larger, a subset of students will have to be recruited. In either event, it can help speed things up if a student serves as helper during the experiment. If an even number of students has been recruited, one can use two helpers (one for each phase). If an odd number of students, has been recruited, it is best to use one helper for the entire experiment. The first phase of the experiment will go more smoothly if at least ten, but no more than sixteen, students participate in the first part. With ten or more students it is relatively simple to ensure there are no repeat matchings; with more than sixteen students, the logistics become troublesome. Any remaining students can be allocated to the second design. It is best to have similar numbers in each part.

Each participant gets a packet including the instructions, a record sheet, and the relevant payoff tables. (Appendix A contains a set of instructions, with a copy of the sample payoff table; Appendix B contains a record sheet and a copy of the payoff table). The record sheet shows the subject's choices, his or her rival's choices, and his or her points received in each period. The two stacks of small pieces of paper will be used by the students to write down their choices. These slips should be labeled in advance by subject number (A1, A2, and so on; B1, B2, and so on) and "choice period" (1, 2, 3, 4, 5 for the first part, and 1, 2... 10 for the second part).

There are two designs in the experiment. Each part will take between 15 and 30 minutes to run, and reading the instructions should take about five minutes. The instructor can use any remaining time to contrast the results, and to point out the importance of repeated play in facilitating cooperation. Some time should be spent drawing an analogy between the abstract payoffs in the prisoner's dilemma games used here and firms' payoffs in a duopoly.

The sequencing of the experiment is as follows:

1. Recruit volunteers from the class, bearing in mind the points in subsection 6.E. Designate the assistant(s) and separate the others into two groups, one for each part of experiment.
2. Ask the helper and those you have selected for the first phase to come to the front of the room. Make sure there is enough space for half the participants on one side of the room and for the other half on the other side of the room. As the participants come down front, label each student as either an "A" or a "B." Have the A participants go to one side, and the B students go to the other side of the room.
3. Pass out enough slips of paper for every student to make five choices. Use one color for every subject on one side, and a different color for the slips on the other side of the room. Each slip should have two numbers on it: the student's ID number, and the period number. Also pass out the packets. Read the instructions aloud, while the students read their own copies. Answer any questions.
4. When you get to the last paragraph of the instructions, put the example payoff matrix from the instructions on the overhead or on the chalkboard. Explain that subjects will be arranged in pairs. Each person is matched with someone from the other side of the room (termed "the other person," below). In the first part of the experiment, the groupings are done in a random fashion; pairings are rearranged before each choice period. In every period each subject will choose a "row" value. The other person's row choice becomes that person's column choice, with payoffs determined by the intersection of row and column. Note that the other person's payoffs can also be found at this intersection.
5. Propose the combination where the student picks "blue" and the other person picks "green." Ask the class what the payoffs are for each player. Get two different people to answer, and make sure that everyone understands exactly how the payoffs are determined.
6. Start the experiment for design 1. Have all participants write a choice on the slip of paper corresponding to period 1. After everyone has done so, ask the helper to collect all the slips from one side, while you collect all the slips from the other side. Bring the slips to the front of the room.

- Match each slip from side A with a slip from side B (see *random matching*, below). Order the slips from the B side in terms of the A person with whom they are matched (e.g., if B5 is matched with A1, put B5's slip on top; if B7 is matched with A2, B7's slip goes next, and so on). Similarly, order the slips from the A side in terms of the B person with whom they are matched (e.g., if A6 is matched with B1, put B1's slip on top; if A4 is matched with B2, A4's slip goes next, and so on).
7. Have your helper take the B slips to the A people, while you take the A slips to the B people. Each subject should have his or her record sheet in plain sight, so you can walk amongst them handing out the slips to the appropriate person. *Do not* ask for subjects to identify themselves as you distribute the slips. After the slips are handed out, each participant is to record the other person's choice and their own payoffs on their record sheet. Wander among the students, checking to see that they are entering this information correctly. Answer any questions that arise here.
 8. Repeat steps 6 and 7 for period 2. Make it absolutely clear that you are using a different pairing arrangement than the one you used in period 1, so that nobody is matched with the same person.
 9. Repeat step 8 for periods 3, 4, and 5.
 10. Collect the record sheets from the participants, and ask them to return to their seats. Ask the class to predict the number of students who picked each action. Go through the record sheets, mentally tabulating the number in each category for periods 1 and 5. Keep track of these numbers, so that you can write them on the board at the end of the experiment. See *Presentation Example* below.
 11. Ask the helper and those you have selected for the second part of the experiment to come to the front of the room. Make sure there is a place for half the participants on one side of the room, and enough seats for the other half on the other side of the room. As the participants come down front, label each student as either an "A" or a "B." Have the A participants go to one side, and the B students go to the other side of the room.
 12. Pass out enough slips of paper for every student to make ten choices. Use one color for every subject on one side, and a different color for the slips on the other side of the room. Each slip should have two numbers on it: the student's ID number, and the period number. Make sure that there are only two differences between the first and second phases: the number of periods, and the fact that participants in the second phase always play against the same opponent. Also pass out the packets. Because the instructions are much the same as for the first part, you can summarize them rather quickly. Be sure to tell the students that, unlike the first part,

- they will play against the same person for the entire length of the second part. Make sure the new participants clearly understand how to read the payoff table; you may want to go over through a second hypothetical combination to reinforce this point. Answer any questions.
13. Have all participants write a choice on the slip of paper corresponding to period 1. After everyone has done so, ask the helper to collect all the slips from one side, while you collect all the slips from the other side. Bring the slips to the front of the room. Match each slip from the A side with a slip from the B side. Order the slips from the B side in terms of the A person with whom they are matched (e.g., if B5 is matched with A1, put B5's slip on top; if B7 is matched with A2, B7's slip goes next, and so on). Similarly, order the slips from the A side in terms of the B person with whom they are matched.
 14. Have your helper take the B slips to the A people, while you take the A slips to the B people. Each subject should have his or her record sheet in plain sight, so you can walk amongst them handing out the slips to the appropriate person. *Do not* ask for subjects to identify themselves as you distribute the slips. After the slips are handed out, each participant is to record the other person's choice and their own payoffs on their record sheet. Wander among the students, checking to see that they are entering this information correctly. Answer any questions that arise here.
 15. Repeat steps 13 and 14 for period 2. Make it absolutely clear that you are using the same pairing arrangement as you used in period 1, so that each participant is matched with the same person.
 16. Repeat steps 13 and 14 for periods 3, 4, 5, 6, 7, 8, 9, and 10.
 17. Collect the record sheets from the participants, and ask them to return to their seats. Ask the class to predict the number of students who picked each action. Go through the record sheets, mentally tabulating the number in each category for periods 1 and 5. Write these numbers on the board. See *Presentation Example* below.
 18. Contrast the values from the two designs. There should be a larger number of students who chose "cooperate" in period 5 of the second design than in period 5 of the first design. Use the results to illustrate the importance of repeated play in facilitating cooperation. Ask the students to think about how this example might relate to firms in an oligopoly, interpreting "left" as "collude" and "right" as "compete."

An illustration of how one might use this design is contained in Appendix C.

It is a good idea to have a plan for matching students *before* arriving at the classroom. The first phase will go far more smoothly, with less chance for awkward pauses, if this has been done in advance. Likewise, it is important to

have an idea of the maximum number of students you will allow to participate in each phase, and bring enough slips for that number. In this way, one can ensure having enough slips for any configurations that arise. The slips can be cut from regular note paper, so long as there are two colors. Using different colors is an easy way to keep track of A and B students (so as to avoid the embarrassment of handing out an A person's slip to a different A person, rather than to the associated B person).

Try to create significant physical space between the A and B groups. The idea is to prevent someone in A figuring out who a certain B player is, for example by seeing the slip that B player receives. On the other hand, it is best to keep all the subjects in one area of the room. This greatly simplifies the collection and distribution of slips. It also allows the audience to watch the entire operation, which can create a sense of performance that may help keep non-participants interested in the goings-on.

The design has to have enough periods to allow for some experimentation, but not so many that students become bored. Because of the possibility of subtle strategic interaction in the second phase, it will almost surely take more periods to get a stable pattern there than in the first phase. That said, it is best to compare the two treatments in terms of the subjects' history (how many periods have passed), rather than the future (how many periods to go).

Two final points: First, it is important that students clearly understand how participants are to be rewarded (e.g., their average score becomes the number of bonus points they will receive on the next homework). The idea is to ensure that they feel motivated, and hence will put some thought into their decisions. Also, the helper needs to know how many bonus points he or she will receive. One idea is to give them the Nash equilibrium outcome as their score, on the grounds that they are being asking them to do a fair bit of footwork, and their contribution is quite important to the entire operation. Second, the instructor should not assume students will immediately grasp the workings of a payoff matrix. It is not at all uncommon for students to get the role of rows and columns mixed up, or for them to transpose payoffs. (Hence the practice round, with questions directed at the audience, and the insistence on taking questions.)

Presentation of the results can be as easy or as hard as one wants to make it. Since there may be lots of information that can be drawn from, it makes sense to think about how to summarize the results in advance. The idea to present simple sample statistics for early and late periods is an easy way to give a flavor of the differences between the two phases. One should probably avoid presenting results from the last period, since behavior here could easily be influenced by end-game effects, and it is best to avoid such subtleties at this point. A graph that shows the proportion of students selecting each action over time is probably a better way to summarize the information, but it will be virtually impossible to

construct such a graph in class during the experiment. One idea would be to bring such a graph to the next class session.

Let me close by offering up a few ideas for items to discuss, and some caveats. There are a variety of potential topics for discussion, including these:

- a) In what ways are the results from the two designs different?
- b) What might have caused these differences? Why?
- c) Would behavior in the first part would have been more like behavior in the second part if the first part had lasted ten periods instead of five? Why?
- d) Would behavior in the first part would have been more like behavior in the second part if subjects in the first part had faced the same other person throughout? Why?
- e) Based on the results from this experiment, how likely is it that firms would collude in markets that are constantly changing? Why?
- f) Would the results from the second part would have been different if players faced different matrices (as opposed to the same matrix, as in this design)? How? Why?

Try to get students to describe the importance of repeated contact. The idea behind the fifth question is that constantly changing markets offer less scope for continued repeated contact. The idea behind the sixth question is that different matrices make it harder to see what the other person is likely to do, making it more difficult to work towards a cooperative outcome.

As with any experiment, there are no guarantees that the results will work out as expected. Nevertheless, by paying attention to the points above one can limit the possibility that matters will go awry. But if in spite of that attention the students' behavior seems wildly inconsistent with predicted behavior, it is best to simply report the outcomes as they are, compare them to the predicted results, and see if the students can come up with an explanation for the departures.

Appendix A: Instructions for Students

During this experiment you will be matched with one other person. The points you earn in each period depend on the actions you take and those taken by the person you are matched with. The bonus points you get from participating will equal your average score over the phase.

The points you get are determined by a “payoff matrix.” This matrix shows the rewards to each person for each of the possible combinations. An example of such a matrix is provided at the bottom of this page. In each cell of the payoff matrix, your payoff is the first number, and the other person’s payoff is the second number. For example, in the matrix below, the combination where you select “green” and the other person selects “blue” gives you 6 points, while the other person gets 25 point. Each participant is provided with the same payoff matrix for the experiment (found on the record sheet). The payoff matrix for the experiment is different from the example given below. The possible actions available to each participant are “right” and “left”.

Each participant is provided with a stack of colored slips of paper. Each slip has your ID number on it, as well as the period. Use that slip for the corresponding period (the slip marked “period 1” for the first period, “period 2” for the second period, and so on). You will be asked to select an action in each period. To make a choice, write the word “left” or the word “right” on the slip of paper. Then, write your choice for that period on your record sheet. After all participants have made a choice, my assistant and I will collect the slips for that period, bring them to the front of the room, and match them up. We will then distribute your slip to the person you are matched with, and you will get their slip. To help us find you as we distribute these choices, please have your record sheet in plain sight, with your participant number easy to see. After you get the slip with the other person’s choice, write it down on the record sheet in the space below yours. Then, use the combination of choices to determine your payoff. Write this payoff in the cell at the bottom of the column for that period. We will check everyone’s recording and calculation after the first period, to make sure that you all are finding the correct payoff.

First part of the experiment: Those who are participating in the first part will face a series of “other players”. The person you are matched with is randomly determined in every period. This part of the experiment consists of five periods.

Second part of the experiment: Those who are participating in the second part are matched with one “other player” for the whole experiment. This part of the experiment consists of ten periods.

Before we start, I want you to imagine that you pick “blue” and the other person picks “green”. Using the payoff matrix below, determine each player’s payoff.

	Green	Blue
Green	20,20	6,25
Blue	25,6	8,8

Are there any questions?

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Appendix B: Record Sheet

Your name _____

Your ID / subject number _____

Period	1	2	3	4	5
Your choice					
Other's choice					
Your payoff					

Period	6	7	8	9	10
Your choice					
Other's choice					
Your payoff					

PAYOFF MATRIX

		The other person's choice	
		Left	Right
Your Choice	Left	10,10	25,5
	Right	5,25	20,20

Appendix C: Presentation Example

The following table is based on the results from the last time I ran this experiment:

Part 1 (5 pairs)

Period	1	2	3	4	5
Number students picking left	4	3	3	2	1
Number pairs picking left	1	1	1	0	0

Part 2 (5 pairs)

Period	1	2	3	4	5	6	7	8	9	10
Number students picking left	8	9	8	8	8	8	7	6	3	0
Number pairs picking left	3	4	4	4	4	4	3	3	1	0

Random Matching

For period 1 in phase 1, pair A1 with B2, A2 with B3, and so on. Pair the last A player (call him or her AN) with the first B player (B1). In period 2, match A1 with B3, A2 with B4, etc. Proceed along these lines through period 5. A somewhat terse description of this procedure is:

Visualize a pair of column vectors, one for each cohort. Call the initial vectors **A** and **B**. The element in the first row of **A** is A1, the element in the second row is A2, and so on, and likewise for **B**. Define the permuted vector **B_t** for period $t = 1, \dots, 5$, by taking the bottom t elements of **B** and inserting them at the top. Thus, **B₃** is the column vector (BN-2, BN-1, BN, B1, B2, ...), where BN-2 is the N-2th B player, etc. Match **A** with **B_t** in round t .

This approach can help with the distribution of slips to players. When you collect the slips, order them from player 1 to player N (i.e., physically construct the initial vectors). Then take R slips from the bottom of the B vector, where R is the period, and put them on top of the B stack. At the same time, take R slips from the *top* of the A stack and put them on the bottom. For example, if you have 5 pairs, in the third round your B stack should be: B3,B4,B5,B1,B2; your A stack should be A4,A5,A1,A2,A3. Pass the A stack back to B players in order (so B1 gets A4, and so on), and pass the B stack back to the A players in order (A1 gets B3).

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