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Capacity-Constrained Monopoly

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Abstract

Capacity constraints on production have major effects on a standard monopoly, a monopoly that price discriminates between two submarkets, and a monopoly that sells in two submarkets and faces a price control in only one.

KEYWORDS: capacity constraint, monopoly, price discrimination, price controls, two-sector model

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Introduction

Capacity constraints on production are common, particularly in the short run, and have major effects on the behavior of firms and even the nature of the equilibrium in some markets. (Slide 1) We show how a capacity constraint affects the equilibrium for a standard monopoly, a monopoly that price discriminates between two submarkets, and a monopoly that sells in two submarkets and faces a price control in only one. (Slide 2)

This lecture discusses how to

- model a capacity constraint on a monopoly,
- model a monopoly that sells in two markets,
- use Kuhn-Tucker techniques to analyze the effects of a capacity constraint in a two-sector model (for graduate students),
- and use a two-sector model to analyze the effects of a capacity constraint and a price control in only one sector.

Instructor's Note: This lecture can be presented to undergraduates using only graphs. The second application is also analyzed using a Kuhn-Tucker (calculus) approach for graduate students.

Although the issues discussed in this lecture apply in many markets, we focus on monopolies for simplicity and because many monopolies face capacity constraints. For example, a pharmaceutical covered by a patent may have a key ingredient that is in short supply, or an electric utility has limited production capacity.

Moreover, monopolies frequently price discriminate by charging various groups of consumers different prices. Often these groups of consumers are located in different government jurisdictions. If a monopoly can prevent resale between two countries, its two pricing decisions are independent. However, as we show, if the monopoly faces a binding capacity constraint and sells more in one country, then it must sell less in the other. Thus, a capacity constraint makes a monopoly's pricing decisions in two countries interdependent.

Governments are often tempted to impose price controls to keep prices low for consumers. However, as we know from introductory economics, a price ceiling usually leads to a shortage in a competitive market. A stronger case can be made for imposing a price ceiling when firms have market power. For example, a properly set price control on a monopoly can produce the competitive outcome. Governments impose price ceilings as a means of regulating utilities and other monopolies. They also impose ceilings in other markets where they believe firms have market power, such as the U.S. retail gasoline market in the 1970s, railway freight in Canada in the late 1990s, and the California wholesale electricity market in 2000. A price control in one jurisdiction may not have a spillover effect on other markets if the monopoly can produce as much as it wants

at a constant marginal cost. However, if it faces a capacity constraint, then the price control may have complex effects on other markets and create shortages in the regulated market.

Monopoly

The effect of a capacity constraint is relatively straight forward with the traditional, single-price monopoly that sells in a single market. We go through this exercise primarily to prepare the ground work for the next two analyses.

Suppose that the monopoly faces a standard, downward sloping demand curve and must set a single price (it cannot price discriminate). For simplicity, suppose that it can produce as many units as it wants at a constant marginal cost, MC , of m up to its capacity constraint. (Slide 3) Thus, if the capacity constraint does not bind—the constrained quantity exceeds the number of units that the firm wants to produce—it effectively faces a MC curve that is horizontal at m . At the quantity, $\bar{Q}$, where the constraint binds, the MC becomes infinite: the firm cannot produce more than that quantity. [Note: the traditional upward sloping marginal cost curve is a less dramatic version of this story: See Weber and Pasche (2008).]

In Figure 1 (Slide 4), if the monopoly does not face a constraint, it maximizes its profit by setting its output at Q_1 where its marginal revenue curve, MR , intersects the horizontal $MC_1 = m$ curve at point a . It charges a price of p_1 .

Now suppose that the monopoly faces a binding constraint at $\bar{Q} < Q_1$. Its marginal cost curve is MC_2 , which is horizontal at m for $Q < \bar{Q}$ and vertical at $\bar{Q}$, as Figure 1 shows. As always, the monopoly maximizes its profit by operating where its marginal cost and marginal revenue curve intersect, which is now at point b on the vertical portion of MC_2 . Thus, the monopoly sells $Q_2 = \bar{Q}$ units at p_2 . That is, a binding constraint causes the monopoly output to fall, the price to rise, and the profit to fall. We know that the profit must fall because the monopoly could have produced at Q_2 in the absence of the constraint and it chose to produce more.

Figure 1: Single-Price Monopoly

Multimarket Price Discrimination

Next, suppose that the monopoly sells in two (or more) markets. If the firm must charge the same price everywhere, then we face the same problem as in the traditional monopoly analysis. (Slide 5) The only difference from our previous analysis is that we must start by horizontally summing the demand curves in the two sectors to get an overall market demand curve. Thereafter, the analysis is the same.

In contrast, if the monopoly can charge different prices in the two submarkets, the demand curves differ in the two submarkets, and the monopoly can prevent resales between the two submarkets, then it can price discriminate (Slide 6). The monopoly sells Q_1 units at price p_1 in the first submarket, and Q_2 units at price p_2 in the second submarket.

Traditional Price Discrimination: Capacity Constraint Does Not Bind

Again, we assume that the monopoly can produce units at a constant marginal cost of m until it hits the capacity constraint, where the marginal cost becomes infinite. We start by assuming that the capacity constraint is not binding in the sense that the monopoly wants to produce fewer units than the constraint quantity $Q_1 + Q_2 \leq \bar{Q}$.

We can use a graph, such as Figure 2 (Slide 7), to analyze a monopoly with two submarkets. We take the traditional monopoly diagram for each submarket,

flip one of them and draw them in the same figure as Figure 2 shows. The length of the horizontal axis in the figure is $\bar{Q}$. We measure Q_1 from left to right (as the arrow on the horizontal axis indicates) and Q_2 from right to left.

Figure 2: Standard Price Discrimination

In Figure 2, the capacity is large enough that the marginal revenue curves from the two submarkets do not intersect. In the first submarket, the monopoly produces Q_1 (where the arrow below the axis that is pointing right ends) as determined by the intersection of its marginal revenue curve in that market, MR_1 , with its marginal cost curve, which is horizontal at m . It sells these units at p_1 . Similarly, the intersection of MR_2 and m determines Q_2 in the second submarket.

Although the diagram is slightly unusual in that it shows both markets at once, this analysis is the standard price discrimination analysis. Because the capacity constraint does not bind so that the marginal cost curve is horizontal, the amount the firm sells in one market does not affect its cost in the other market. Moreover, because it can prevent resales, the monopoly does not have to worry about sales in one market affecting the demand curve in the other. That is, the monopoly acts to maximize its profit separately in each submarket.

[Mujumdar and Pal (2005) discuss how, if a monopoly's marginal cost curve is upward or downward sloping, its pricing decision in the two markets is interdependent. The upward sloping case is similar to the situation we study below with a constant marginal cost and a capacity constraint.]

Price Discrimination with a Binding Capacity Constraint

In contrast, suppose that the quantities that the monopoly wants to produce exceeds the capacity constraint: $Q_1 + Q_2 > \bar{Q}$. The monopoly can no longer set its output or price in each submarket independently because its actions in one submarket affect its profit in the other.

Figure 3 shows an example of where the capacity constraint binds. (Slide 8) With these demand curves D_1 and D_2 , in the absence of a capacity constraint, the monopoly would set output in the first submarket at point a , where MR_1 intersects the horizontal line at m . Similarly, it would set output in the second submarket at point b , where MR_2 intersects m . However, to do that, it would have to produce more than $\bar{Q}$, the length of the horizontal axis.

The reason that setting marginal revenue equal to marginal cost does not give us the “right” answer if we do what we just described is that we are using the wrong marginal cost. We need to take into account the capacity constraint that prevents the firm from producing more than $\bar{Q}$.

To determine the correct approach, let’s return to the monopoly’s decision in the first submarket. The monopoly wants to produce at point a , where its marginal revenue in the first submarket equals m . However, at the corresponding quantity, the marginal revenue in the second submarket is at point d . That is, the marginal revenue in the second submarket exceeds that in the first submarket. Consequently, if the monopoly were to shift a unit of output from the first submarket to the second, it would lose $MR_1 = m$ in the first submarket, but it would gain $MR_2 > m$ in the second submarket, so its profit would increase. By repeating this type of argument, we see that because the monopoly is constrained to produce no more than $\bar{Q}$, it should operate where the marginal revenues are equal in the two submarkets at point c . At that pair of quantities, $Q_1 + Q_2 = \bar{Q}$, and the monopoly sets different prices in the two submarkets: $p_1 > p_2$, as the figure shows.

One way to think about this analysis is to say that the real marginal cost is greater than m . As the figure shows, if the marginal cost were $m + \lambda$, the monopoly could use the “usual” approach to determine how to behave in each submarket. The amount λ is the “shadow price” of the constraint. It represents how much more the firm could earn if the capacity increased by one unit. That extra profit is the difference between $MR_1 = MR_2$ and the cost of producing that last unit, m .

Thus, a binding capacity constraint complicates the monopoly’s analysis. It can no longer determine its profit-maximizing behavior separately or independently in each submarket. Rather, it has to consider the tradeoffs between the two submarkets as determined by the different marginal revenue curves in the two

submarkets. Because the capacity constraint binds, total output falls compared to the unconstrained solution. Moreover, the prices will differ between the constrained and unconstrained solutions.

Figure 3: Price Discrimination with a Binding Constraint

Kuhn-Tucker Approach

The monopoly's problem is to maximize the sum of its profits in the two submarkets subject to the constraint that it can produce no more than its capacity: $\bar{Q} - Q_1 - Q_2 \leq 0$. In each jurisdiction $j = 1, 2$, the inverse demand curve is $p_j(Q_j)$ and the revenue is $R_j(Q_j) = p_j(Q_j)Q_j$. The corresponding Kuhn-Tucker problem is

$$\max_{Q_1, Q_2, \lambda} \mathcal{L} = R_1(Q_1) + R_2(Q_2) - m[Q_1 + Q_2] + \lambda[\bar{Q} - Q_1 - Q_2], \quad (1)$$

where λ is the Lagrangean multiplier associated with the capacity constraint. The Kuhn-Tucker first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial Q_1} = \frac{dR_1}{dQ_1} - m - \lambda = 0, \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial Q_2} = \frac{dR_2}{dQ_2} - m - \lambda = 0, \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{Q} - Q_1 - Q_2 \geq 0, \quad (4)$$

$$\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \quad (5)$$

$$\lambda \geq 0. \quad (6)$$

As usual, the interpretation of the Lagrangean multiplier, λ , is the shadow-price of relaxing the constraint. If the capacity constraint binds, Equation (6) holds with a strict inequality, $\lambda > 0$. From Equation (5), we know that $d\mathcal{L}/d\lambda = 0$, so that Equation (4) holds with equality. That is, the sum of the output of the monopoly in the two submarkets exactly equals the capacity, $\bar{Q}$.

By equating Equations (2) and (3), we find that the marginal revenues in both jurisdictions are equal to each other. Moreover, they equal the marginal opportunity cost:

$$MR_1 = MR_2 = m + \lambda, \quad (7)$$

where $MR_j \equiv dR_j \equiv dR_j/dQ_j$, λ is the shadow price of extra capacity, and $m + \lambda$ is the marginal cost of an extra unit sold in one jurisdiction. Figure 3 illustrates the equilibrium in Equation (7). The intersection of the two marginal revenue curves determines the amount of output sold in each jurisdiction and the shadow price of capacity: The height of the intersection point is $m + \lambda$.

Price Control

A capacity constrained monopoly that sells in two submarkets—located in two different jurisdictions—faces a price control in only the first submarkets: $p_1 \leq \bar{p}$. (Slide 9) For example, an electric utility that sells in two states may face a price control in only one of them. (In 2000, California imposed wholesale price controls on electricity unlike the surrounding states. However, there were several producers in the relevant states.)

Price Control without a Binding Capacity Constraint

If the monopoly's capacity is large, then the price control in the first submarket has no effect on the second submarket. (Slide 10) In Figure 4 (Slide 11), the demand curve facing the monopoly is D_1 (a blue line) and the marginal revenue curve is MR_1 (a purple line). In the absence of a price control, the monopoly sets

its output by equating its marginal revenue and its marginal cost, m , at point a . The output is Q_1 , and the price is p_1 .

Now, suppose that the government sets a binding price control: the maximum price that the firm can set, $\bar{p}$, is less than p_1 . If $\bar{p} < m$, the firm shuts down. We assume that $\bar{p}$ is strictly between p_1 and m and high enough that the firm does not shut down (which, in the short run, depends on the size of its fixed cost). As Figure 4 shows, the monopoly now faces a kinked demand curve (green line) that is horizontal at $\bar{p}$ until it hits the original demand curve D_1 , and then is identical to D_1 . Consequently, the new marginal revenue curve (yellow line) is horizontal where the demand curve is horizontal (remember the competitive firm's marginal revenue curve is horizontal), then jumps down to the original marginal revenue curve when the new demand curve is the same as D_1 . The monopoly now charges $\bar{p}$ and produces at Q_2 , determined by where the new marginal revenue curve intersects the marginal cost curve at point b . Thus, the price control induces the monopoly to increase its output and lower its price. The price control does not create a shortage.

Figure 4: Price Control in a Single Market

Price Control with a Binding Capacity Constraint

If the capacity constraint binds, then the effect of the price control in the first submarket will spillover into the second submarket because the sum of the outputs in the two submarkets are limited by capacity. (Slide 12) What do you expect to happen? Your first instinct might be that if the firm produces more in the first market, it must reduce its output in the second market, causing the price there to

rise. But another possibility is that the price control is so low that the firm decides to shift output from the first submarket to the second submarket, expanding output there.

When both the price control and the capacity constraint bind, the solution is of the same form as in the price discrimination example, except that the marginal revenue in the first jurisdiction is now $\bar{p}$, so the condition that $MR_1 = MR_2$ becomes $\bar{p} = MR_2$.¹ Figure 5 (Slide 13) shows that a binding price control in the first submarket may either raise or lower the price in the other. The figure shows the demand in the first submarket but not in the second one. It shows three possible marginal revenue curves in the second submarket.

The figure illustrates that the impact of the price control depends on where the MR_2 curve intersects the price control line at $\bar{p}$ and the MR_1 curve. Again, when the price control binds, the relevant MR_1 curve is horizontal at $\bar{p}$ until it hits the demand curve, it then jumps down to the original MR_1 curve (thin blue line).

Figure 5: Price Control with a Capacity Constraint

Suppose that the marginal revenue curve in the second submarket is relatively high: MR_2^A . Without the price control, MR_2^A intersects MR_1 at point a . With the price control, output is determined at point b . Thus, the effect of the

¹ If the price controls binds, the new problem is

$$\max_{Q_1, Q_2, \lambda} \mathcal{L} = \bar{p}Q_1 + R_2(Q_2) - m[Q_1 + Q_2] + \lambda[\bar{Q} - Q_1 - Q_2].$$

control is to reduce the amount of Q_1 from a to b and to create a shortage equal to the difference between the quantity of Q_1 at c and b . Meanwhile, the quantity of Q_2 expands from that at a to that at b . Consequently, the price control causes prices to drop in both jurisdictions and creates a shortage in one.

Given the MR_2^B curve, imposing the price control in the first submarket causes Q_1 to increase from d to e , a shortage equal to the difference in Q_1 at c and e , Q_2 to decrease from d to e , and hence p_2 to rise. The analysis is very similar with MR_2^C , however there is no shortage in Jurisdiction 1 because the quantity of Q_1 is the same at g and c .

Thus, a price control in one jurisdiction can decrease output in that jurisdiction and thereby increase output and lower price in the other jurisdiction. However, the opposite quantity effect is also possible. Moreover, a shortage may occur in the first submarket with a capacity constraint, but not without one.

Summary

In this lecture, we considered the effect of binding capacity constraints on a monopoly's price and output decisions in several environments. We showed that when a binding capacity constraint is imposed on our base-case of a standard one-price, one-market monopoly, the firm reduces output and raises price.

In the absence of a capacity constraint and with a horizontal marginal cost curve, a monopoly can set prices independently in the two submarkets if it can prevent resales. With a capacity constraint, the monopoly has to consider both submarkets simultaneously. If it sells more output in one submarket, it has to sell less in the other. Consequently, a binding capacity constraint affects the quantity and prices in both submarkets.

Finally, we considered the situation where the two submarkets are in different jurisdictions, and a price control is imposed in only the first submarket. In the absence of a capacity constraint, if the government sets a maximum price that lies between the unregulated monopoly price and the competitive price (marginal cost), the price control causes output to rise and price to fall in this submarket, but it does not create a shortage. The price control has no effect in the second submarket.

With a binding capacity constraint, a binding price control in the first submarket affects output and price in both submarkets. It may lead to shortages in the regulated submarket. It may cause output to rise in the first submarket and fall in the second or vice versa. Thus, the capacity constraint changes the qualitative and quantitative nature of the regulated monopoly solution substantially.

Possible Extensions

Capacity constraints may have unusual effects in other types of markets as well. As an assignment, redo this last analysis for a competitive market. That is, analyze the effect of a price control in only one submarket with and without a capacity constraint. You should also consider the intermediate case where the industry has an upward sloping supply curve (explain why the constant marginal cost case is problematic). Discuss whether only one or both the possible outcomes in the monopoly case can occur in a competitive market. Use your results to explain what happened in Zimbabwe. The price controls on food in Zimbabwe from 2002 on caused shortages in that country and the prices of those goods to fall in neighboring countries.

Capacity constraints have even more dramatic effects in oligopolistic markets. For example, in a price-setting duopoly, without a constraint, we may get a pure-strategy, Bertrand-Nash equilibrium. However with a capacity constraint, there may be no pure strategy equilibrium, as Edgeworth showed (see e.g., Carlton and Perloff, 2005). In a slightly more complex analysis, Kreps and Sheinkman (1983) suppose that firms chose their capacity in the first stage and then play a price game in the second stage. They show that this quantity precommitment—effectively a capacity constraint—may lead to a Cournot-Nash rather than a Bertrand-Nash equilibrium (or the outcome analyzed by Edgeworth).

References

- Carlton, Dennis W., and Jeffrey M. Perloff, *Modern Industrial Organization*, 4th ed., 2005, Boston: Addison Wesley.
- Kreps, David M., and Jose A. Scheinkman, “Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes,” *Bell Journal of Economics*, 1983, 14, 326-37.
- Mujumdar, Sudesh, and Debashis Pal, “Do Price Ceilings Abroad Increase U.S. Drug Prices?” *Economics Letters*, 2005, 87, 9-13.
- Weber, Sylvain and Pasche, Cyril, “Price Discrimination,” *Journal of Industrial Organization Education*, 2008, 2(1, 3):