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Third-Degree Price Discrimination

Qihong Liu and Konstantinos Serfes

Abstract

This lecture deals with third-degree price discrimination in both monopolistic and oligopolistic markets. The classical monopoly paradigm serves as a benchmark. Next, we move to an oligopoly setting, first with best-response symmetry, then with best-response asymmetry. We end with behavior-based price discrimination. This lecture targets advanced undergraduate and graduate students.

KEYWORDS: third-degree price discrimination, best-response asymmetry, price discrimination by purchase history

1 Introduction

Our aim in this lecture is to offer an overview of the main theories of third-degree price discrimination.¹ We start from the classical monopoly paradigm and then we relax assumptions, one at a time, to arrive at recent theories of oligopolistic price discrimination.² We start with the case of monopoly third-degree price discrimination in Section 2. We then introduce competition and analyze oligopolistic price discrimination, which is split into two sections. The case of best-response symmetry is investigated in Section 3, and Section 4 considers the opposite case of best-response asymmetry. In Section 5, we look at price discrimination by purchasing history (behavior-based price discrimination), where the role of long-term contracts is also examined.³

2 Monopolistic third-degree price discrimination

Consider a monopolist selling to two separate markets or groups (e.g., students vs. general population, or business travelers vs. leisure travelers). Let $q_i(p_i)$ denote the demand function in market $i = 1, 2$, with p_i being the market price. The monopolist has a constant marginal cost $c \geq 0$. Assuming that both markets are served, the aggregate demand is $Q(p_1, p_2) = q_1(p_1) + q_2(p_2)$, and the monopolist's total profit is

$$\Pi = \sum_{i=1}^2 \pi_i(p_i) = \sum_{i=1}^2 (p_i - c)q_i(p_i).$$

It is assumed (throughout this lecture) that π_i is smooth and strictly concave.

¹Third-degree price discrimination is when a seller targets different groups of consumers with different prices for the same product.

²For definitions of the different forms of price discrimination, including third-degree, and more examples, we refer the reader to the lecture by Weber and Pasche (2008). Our lecture should be viewed as complementary to theirs. We differ in that we focus only on third-degree price discrimination, while Weber and Pasche also examine first- and second-degree. But we go deeper into third-degree price discrimination, while Weber and Pasche do not go beyond the monopoly case. Finally, our approach is more mathematical and hence it targets more advanced students.

³This lecture has benefited tremendously from the recent surveys on price discrimination by Armstrong (2006) and Stole (2007).

Next, we will consider two cases. In the first case, the monopolist cannot price discriminate, and has to set the same (uniform) price in both markets. In the second case, he can price discriminate across the markets he serves. Consumer arbitrage is infeasible. We will start with the uniform pricing case.

2.1 Uniform pricing

Let p denote the uniform price. The monopolist's problem is

$$\max_p \Pi = \sum_{i=1}^2 (p - c)q_i(p).$$

The optimal price p^* is defined by the first order condition $\Pi'(p^*) = 0$, i.e.,

$$\sum_{i=1}^2 [(p^* - c)q'_i(p^*) + q_i(p^*)] = 0.$$

The marginal valuations of the consumers in both markets are equal, since they are both equal to the common market price p^* . Moreover, it can be shown that p^* satisfies the standard elasticity condition

$$\frac{p^* - c}{p^*} = \frac{1}{\varepsilon(p^*)}$$

where,

$$\varepsilon(p) = -\frac{p}{Q(p)}Q'(p)$$

is the elasticity of demand.

2.2 Price discrimination

We now consider the case of price discrimination, where the monopolist can charge two prices: p_1 and p_2 for market 1 and 2 respectively. With constant marginal cost, the two markets can be treated separately and the monopolist simply maximizes its profit in each market,

$$\max_{p_i} \pi_i = (p_i - c)q_i(p_i), \quad i = 1, 2.$$

The optimal price p_i satisfies the following condition

$$\frac{p_i - c}{p_i} = \frac{1}{\varepsilon_i(p_i)}$$

where,

$$\varepsilon_i(p_i) = -\frac{p}{q_i(p_i)}q'_i(p_i)$$

is now the elasticity of demand of market i , $i = 1, 2$.

We can see that in each market the optimal price is inversely related to the elasticity of demand. Therefore, relative to the optimal uniform price p^* , the monopolist will increase his price in the low elasticity market (e.g., business travelers, general population) and decrease his price in the high elasticity market (e.g., leisure travelers, students).

2.3 Output effects of price discrimination

Clearly, the monopolist can do no worse under price discrimination than under uniform pricing, since by setting $p_1 = p_2$ he can mimic the behavior under uniform pricing. However, it is unclear how price discrimination affects social welfare. Because prices in the two markets, in general, differ from each other ($p_1 \neq p_2$), marginal valuations across the two markets are not equalized.⁴ Hence, price discrimination creates a welfare loss due to this *output misallocation effect*. If, on top of that, aggregate output decreases, then we can conclude that social welfare (and thus consumer welfare) must decrease, since the monopolist already produces too little under uniform pricing. Therefore, a necessary condition for social welfare to increase is that output increases.

Next, we investigate how output changes when price discrimination is possible. To do so, we introduce a hypothetical constrained optimization problem, by imposing a constraint that the price differential between the two markets cannot be more than r . Without loss of generality, assume that the demand elasticity in market 2 is lower than that in market 1, thus $p_2 > p^* > p_1$. We call market 2 the strong (low elasticity) market (e.g., business travelers, general population) and market 1 the weak (high elasticity) market (e.g., leisure

⁴Under price discrimination, output is not allocated to the consumers who value it the most. If the two groups were allowed to trade, after production and sales have taken place, the high elasticity group (who is paying a lower price) would sell some units to the low elasticity group (who is paying a high price). Such trade does not affect aggregate output but makes both groups of consumers better off, leading to higher social welfare.

eral population) and market 1 the weak (high elasticity) market (e.g., leisure travelers, students). Then the monopolist's problem is

$$\max_{p_1, p_2} \Pi(r) = (p_1 - c)q_1(p_1) + (p_2 - c)q_2(p_2),$$

$$\text{subject to } p_2 \leq p_1 + r.$$

When r is small, the constraint is binding, thus $p_2 = p_1 + r$.⁵ The monopolist's problem becomes

$$\max_{p_1} \Pi(r) = \pi_1(p_1) + \pi_2(p_1 + r).$$

The monopolist will choose p_1 to satisfy

$$\frac{d\Pi(r)}{dp_1} = (p_1 - c)q'_1 + q_1 + (p_2 - c)q'_2 + q_2 = 0.$$

Let $p_1^*(r)$ denote the solution. Next, differentiating the above expression with respect to r , we can obtain

$$\sum_{i=1}^2 [2q'_i + (p_i - c)q''_i]p'_i(r) = 0.$$

This implies that

$$\sum_{i=1}^2 q'_i p'_i(r) = -\frac{1}{2} \sum_{i=1}^2 (p_i - c)q''_i p'_i(r).$$

Aggregate output as a function of r is given by

$$Q(r) = q_1(p_1^*(r)) + q_2(p_1^*(r) + r).$$

Then

$$\frac{dQ}{dr} = \sum_{i=1}^2 q'_i p'_i(r) = -\frac{1}{2} \sum_{i=1}^2 (p_i - c)q''_i p'_i(r). \quad (1)$$

⁵When r is sufficiently large and the constraint is not binding, increasing r will not affect Q , i.e., $Q'(r) = 0$.

This is Robinson's "adjusted-concavity" condition and compares the relative curvature of demand in the two markets, Robinson (1933). Recall that in weak markets $p'(r) < 0$, while in strong markets $p'(r) > 0$. Thus $p_1'(r) < 0$ and $p_2'(r) > 0$.

If demands are linear, then $q_i'' = 0$, which further implies that $dQ/dr = 0$. Therefore, price discrimination leaves aggregate output unchanged relative to output under uniform pricing. Social welfare, in this case, decreases, because of the misallocation of output effect.

If the demand in the weak market is convex ($q_1'' > 0$) and the demand in the strong market is concave ($q_2'' < 0$), then $dQ/dr > 0$. Therefore, aggregate output under price discrimination increases. The intuition is as follows. A convex demand in the weak market implies a large output increase following a price decrease in that market, while a concave demand in the strong market implies a small output decrease following a price increase in that market. The overall effect is for output to increase.

With demand curves that have similar curvature in both groups (i.e., both strictly convex or both strictly concave), we cannot make general statements.

2.4 Summary of main results

- **Prices:** Due to price discrimination, the price in the strong (inelastic) market increases and in the weak (elastic) market decreases.
- **Profits:** The monopolist becomes better off when he can price discriminate.
- **Aggregate output:** With linear demands aggregate output remains unchanged. For more general demands, it depends on the adjusted-concavity condition. It can either increase or decrease.
- **Welfare:** The misallocation of output effect reduces social welfare. Hence, social welfare (and consumer welfare) increases only if aggregate output increases. When demands are linear, price discrimination hurts social welfare.⁶

In this section, we analyzed price discrimination when there is no competition. Next, we move away from monopoly, and investigate how oligopolistic

⁶We maintain the assumption that both markets are served when the monopolist charges one price. Otherwise, price discrimination can lead to the opening of new markets which boosts social welfare.

price discrimination affects profits and welfare. We will distinguish between two cases: the case of best-response symmetry and the case of best-response asymmetry. We will start with the former.

3 Oligopolistic third-degree price discrimination with best-response symmetry

As in the previous section, there are two markets $i = 1, 2$, but now there are two firms $j = a, b$. We maintain the assumption of constant marginal cost c . Let $BR_i^j(p)$ denote firm j 's best-response to the other firm's price p in market i . We say that market 1 is weak (and 2 strong) for firm j if and only if $BR_1^j(p) < BR_2^j(p)$ for all p . That is, for any single price p charged by the rival, firm j charges a higher price in its strong market. The market exhibits *best-response symmetry* if the weak and strong markets of each firm coincide, Corts (1998). For example, both firms agree that business travelers represent the low elasticity (strong) group and leisure travelers represent the high elasticity (weak) group. Otherwise, there is *best-response asymmetry*, where one firm's strong market is the other firm's weak market and vice versa. A European automobile manufacturer, for instance, may view Europe as its strong market and the US as its weak market, while the opposite is true for a US auto manufacturer. This distinction has important implications about the direction of price changes when firms are allowed to price discriminate. As we mentioned above, we assume best-response symmetry in the remaining of this section.

Let $q_i^j(p_i^a, p_i^b)$ denote the demand for firm j 's output in market i . We assume the following symmetry condition

$$\frac{\partial q_i^a(p, p)}{\partial p_i^b} = \frac{\partial q_i^b(p, p)}{\partial p_i^a}.$$

The market elasticity of demand in market i (at $p_i^a = p_i^b = p$) is

$$\varepsilon_i^m(p) = -\frac{p}{q_i(p)} q_i'(p).$$

Firm j 's own-price elasticity of demand in market i is

$$\varepsilon_{i,j}^f(p_i^a, p_i^b) = -\frac{p_i^j}{q_i^j(p_i^a, p_i^b)} \frac{\partial q_i^j(p_i^a, p_i^b)}{\partial p_i^j}.$$

Due to the symmetry assumption, at symmetric prices we have

$$\begin{aligned}
 \frac{\partial q_i^a(p, p)}{\partial p_i^a} &= \frac{\partial q_i^a(p, p)}{\partial p_i^a} + \left[\frac{\partial q_i^a(p, p)}{\partial p_i^b} - \frac{\partial q_i^b(p, p)}{\partial p_i^a} \right] \\
 &= \left[\frac{\partial q_i^a(p, p)}{\partial p_i^a} + \frac{\partial q_i^a(p, p)}{\partial p_i^b} \right] - \frac{\partial q_i^b(p, p)}{\partial p_i^a} \\
 &= q_i'(p) - \frac{\partial q_i^b(p, p)}{\partial p_i^a}.
 \end{aligned}$$

When firm a raises its price in market i , its demand in market i goes down. The decrease in demand can be decomposed into two parts: (1) some consumers choose not to buy (outside option), represented by $q'(p)$ and (2) some consumers switch firms and buy from firm b , represented by the second term $-\partial q_i^b(p, p)/\partial p_i^a$.

Using the above expression, at equal prices, the own-price elasticity in market i becomes:

$$\varepsilon_i^f(p) = -\frac{p}{q_i(p)} q_i'(p) + \frac{p}{q_i(p)} \frac{\partial q_i^b(p, p)}{\partial p_i^a} = \varepsilon_i^m(p) + \varepsilon_i^c(p),$$

where $\varepsilon_i^c(p) > 0$ is the cross-price elasticity of demand.

The market elasticity $\varepsilon_i^m(p)$ measures the sensitivity of a consumer to taking the outside option, i.e., not consuming either good. The cross-price elasticity $\varepsilon_i^c(p)$ measures the consumer's sensitivity to switching to the rival's product. Non-cooperative duopolists (in a symmetric price equilibrium) will set prices across markets such that

$$\frac{p_i - c}{p_i} = \frac{1}{\varepsilon_i^f(p)} \Rightarrow \frac{p_i - c}{p_i} = \frac{1}{\varepsilon_i^m(p) + \varepsilon_i^c(p)}, \quad i = 1, 2.$$

3.1 Price effects of price discrimination

Under monopoly, the firm's own price elasticity is equivalent to the market price elasticity, $\varepsilon_i^f(p) = \varepsilon_i^m(p)$. Since price is inversely related to $\varepsilon_i^f(p)$ in each market, optimal price in each market is also inversely related to $\varepsilon_i^m(p)$. That is, as we showed in the previous section, the monopolist charges a high price in the market with low $\varepsilon_i^m(p)$ (strong market) and a low price in the market with high $\varepsilon_i^m(p)$ (weak market). In an oligopolistic environment, optimal price is still inversely related to $\varepsilon_i^f(p)$ in each market, just as under monopoly. However, $\varepsilon_i^f(p) \neq \varepsilon_i^m(p)$, since now there is the cross-price elasticity, $\varepsilon_i^c(p)$, which comes

with competition. Therefore, the pattern that price is low (high) in market with high (low) market elasticity, may not be true in an oligopolistic industry. In particular, the strong market (i.e., the one with low firm-level elasticity $\varepsilon_i^f(p)$) may very well be the one with high market elasticity $\varepsilon_i^m(p)$. Therefore, the price may be higher in the market with higher $\varepsilon_i^m(p)$. This is the case when

$$\varepsilon_w^f(p) > \varepsilon_s^f(p) \text{ and } \varepsilon_w^m(p) < \varepsilon_s^m(p),$$

where the subscripts w and s denote weak and strong market respectively. For the above conditions to hold, it must be that

$$\varepsilon_w^c(p) > \varepsilon_s^c(p),$$

i.e., cross-price elasticity is lower in the weak market. For example, students may be the group with the high market elasticity, $\varepsilon_i^m(p)$, as in the monopoly case, but they may also exhibit high brand loyalty, i.e., low $\varepsilon_i^c(p)$. As a result, and contrary to the monopoly case, students may end up paying a higher price than the general population, because firms compete more vigorously for the group with the higher cross-price elasticity (the general population).

While average price under duopoly is lower than that under monopoly (and hence more efficient; closer to marginal cost), the pattern of relative prices may be more inefficient under a duopoly. This implies that competition, while effective at controlling average prices, is ineffective in generating the **correct pattern** of relative prices, *pattern of relative prices effect*. From the industry's perspective, the correct pattern of relative prices should be for firms to set a high price in the low market elasticity group and a low price in the high market elasticity group. In the above example, based on the market elasticities, $\varepsilon_i^m(p)$, it is more profitable for the industry to charge a lower price to the students and a higher price to the general population.

Finally, it can be shown that the uniform price is between the two discriminatory prices, as in the monopoly case. This result, as we will see in the next section, does not hold under best-response asymmetry.

3.2 Output effects of price discrimination

Without loss of generality, assume that market 2 is the strong market so that $p_2 > p_1$ at the symmetric equilibrium. Starting at symmetric (across firms) prices, consider the following marginal profit to firm $j = a, b$ if it changes its price in market $i = 1, 2$

$$d\pi_i(p) \equiv q_i^j(p) + (p - c) \frac{\partial q_i^j(p, p)}{\partial p_i^j}.$$

Similar to our approach in the case of price discrimination under monopoly, we impose a hypothetical constraint that the difference between the prices in the two markets for each firm cannot exceed r . Each firm will choose p_1 to satisfy

$$d\pi_1(p_1) + d\pi_2(p_1 + r) = 0,$$

the solution of which is denoted as $p_1^*(r)$.

The effect on aggregate output from a fixed r is given by

$$Q(r) = q_1(p_1^*(r)) + q_2(p_1^*(r) + r).$$

It can be shown that $Q'(r) > 0$ if and only if the following is positive (Holmes (1989)),

$$\left[\frac{(p_2 - c)}{2q_2'(p_2)} \frac{d}{dp_2} \left(\frac{\partial q_2^a(p_2, p_2)}{\partial p_2^a} \right) - \frac{(p_1 - c)}{2q_1'(p_1)} \frac{d}{dp_1} \left(\frac{\partial q_1^a(p_1, p_1)}{\partial p_1^a} \right) \right] + \left[\frac{\varepsilon_2^c(p_2)}{\varepsilon_2^m(p_2)} - \frac{\varepsilon_1^c(p_1)}{\varepsilon_1^m(p_1)} \right].$$

The first term is similar to adjusted-concavity condition given by (1). The second term is the elasticity-ratio condition. If the cross-price elasticities are zero, then the second term is zero and the above condition is reduced to the adjusted-concavity condition. On the other hand, when demands are linear, the adjusted concavity condition is zero, and the output effect depends solely on the elasticity-ratio condition. In particular, with linear demands, output increases due to price discrimination if and only if the elasticity ratio condition holds

$$\frac{\varepsilon_2^c(p_2)}{\varepsilon_2^m(p_2)} - \frac{\varepsilon_1^c(p_1)}{\varepsilon_1^m(p_1)} > 0. \quad (2)$$

For this to hold and thus for aggregate output to increase with price discrimination, we want the ratio $\varepsilon_i^c/\varepsilon_i^m$ in the strong market to be higher than that in the weak market. The intuition is as follows. In the strong market the price rises and output falls, while the opposite is true in the weak market. For aggregate output to increase, we want the quantity reduction in the strong market to be smaller than the quantity increase in the weak market. When

the above elasticity ratio is high in the strong market, it implies that there is relatively more reshuffling of consumers between the two rival firms than losing consumers to the outside option. Consequently, output reduction in the strong market is not large. On the other hand, a low elasticity ratio in the weak market suggests that a price reduction will be effective in attracting new consumers into the market, implying much higher output.

3.3 Profit effects of price discrimination

An individual firm's profit always increases when allowed to price discriminate, holding the rival's responses constant. However, when both firms price discriminate, the equilibrium industry profit may rise or fall. In the case of linear demands, profits, as well as output, increase if the elasticity-ratio condition (2) is satisfied. But if the elasticity-ratio condition is violated, then discrimination may reduce profits, Holmes (1989). This can be seen with the aid of the following example.

3.3.1 An example with linear demands

Demand in market $i = 1, 2$ (Market 2 is the strong market and market 1 is the weak market) for firm $j = a, b$ is linear

$$q_i^j = 1 - \alpha_i p_i^j - \beta_i (p_i^j - p_i^{-j}).$$

The market and cross-price (absolute) elasticities of demand in market i are

$$\varepsilon_i^m(p) = \frac{\alpha_i p}{1 - \alpha_i p} \text{ and } \varepsilon_i^c(p) = \frac{\beta_i p}{1 - \alpha_i p}.$$

The firm-level elasticity is

$$\varepsilon_i^f(p) = \varepsilon_i^m(p) + \varepsilon_i^c(p) = \frac{(\alpha_i + \beta_i) p}{1 - \alpha_i p}.$$

The elasticity ratio condition (2) holds if $\beta_2/\alpha_2 > \beta_1/\alpha_1$. Assume that marginal cost is zero.

Uniform price The equilibrium prices and profits are

$$p^a = p^b = \frac{2}{2\alpha_1 + 2\alpha_2 + \beta_1 + \beta_2}$$

and

$$\pi^a = \pi^b = \frac{4(\alpha_1 + \alpha_2 + \beta_1 + \beta_2)}{(2\alpha_1 + 2\alpha_2 + \beta_1 + \beta_2)^2}.$$

It can be shown that $\varepsilon_1^f(p) > \varepsilon_2^f(p)$, evaluated at the uniform price, if and only if $2\alpha_1 + \beta_1 > 2\alpha_2 + \beta_2$. In the rest of this example, we assume that this inequality is satisfied.

Price discrimination The equilibrium prices and profits are

$$p_1^a = p_1^b = \frac{1}{2\alpha_1 + \beta_1} \text{ and } p_2^a = p_2^b = \frac{1}{2\alpha_2 + \beta_2}$$

and

$$\pi^a = \pi_1^a + \pi_2^a = \frac{\alpha_1 + \beta_1}{(2\alpha_1 + \beta_1)^2} + \frac{\alpha_2 + \beta_2}{(2\alpha_2 + \beta_2)^2} = \pi^b = \pi_1^b + \pi_2^b.$$

Given the assumption that $2\alpha_1 + \beta_1 > 2\alpha_2 + \beta_2$, the discriminatory price in market 1 (the weak market) is lower than the discriminatory price in market 2 (the strong market). Next, we consider two examples depending on whether the elasticity ratio condition holds. Both examples are provided in Table 1. Note that p^a and p^b denote uniform prices, while p_i^a and p_i^b denote the discriminatory price in market $i = 1, 2$. Elasticities are evaluated at the uniform price levels.

	market 1	market 2
(α_i, β_i)	(1, 4)	(0.5, 4.5)
ε_i^f	1.053	0.952
ε_i^c	0.842	0.857
ε_i^m	0.211	0.095
$\frac{\varepsilon_i^c}{\varepsilon_i^m}$	4	9
$p^a = p^b$	0.174	0.174
$q_i(p^a, p^b)$	0.826	0.913
$\pi_i(p^a, p^b)$	0.144	0.159
$p_i^a = p_i^b$	0.167	0.182
$q_i(p_i^a, p_i^b)$	0.833	0.909
$\pi_i(p_i^a, p_i^b)$	0.139	0.165

	market 1	market 2
(α_i, β_i)	(0.1, 4)	(1, 1)
ε_i^f	1.172	0.770
ε_i^c	1.143	0.385
ε_i^m	0.029	0.385
$\frac{\varepsilon_i^c}{\varepsilon_i^m}$	40	1
$p^a = p^b$	0.278	0.278
$q_i(p^a, p^b)$	0.972	0.722
$\pi_i(p^a, p^b)$	0.270	0.201
$p_i^a = p_i^b$	0.238	0.333
$q_i(p_i^a, p_i^b)$	0.976	0.667
$\pi_i(p_i^a, p_i^b)$	0.232	0.222

Table 1: Examples with linear demands

In the first example, $\alpha_1 = 1$, $\beta_1 = 4$, $\alpha_2 = 0.5$ and $\beta_2 = 4.5$. Under these parameters, the elasticity condition holds ($\beta_2/\alpha_2 > \beta_1/\alpha_1$). Consequently, output increases due to price discrimination ($0.833 + 0.909 > 0.826 + 0.913$), and profit for each firm also increase ($0.139 + 0.165 > 0.144 + 0.159$). In the second example, $\alpha_1 = 0.1$, $\beta_1 = 4$, $\alpha_2 = 1$ and $\beta_2 = 1$. In this case, the elasticity ratio condition is violated. Although the weak group has a low market elasticity, its cross-price elasticity is high (weak brand loyalty), resulting in a high firm-level elasticity.⁷ Price discrimination raises the price in market 2 (strong market) and lowers it in group 1 (weak market). Output ($0.976 + 0.667 < 0.972 + 0.722$) and profits ($0.232 + 0.222 < 0.270 + 0.201$) decrease under price discrimination. This is because of the higher cross-price elasticity in the weak group. A price increase would be more effective in increasing profit in the low market elasticity group (group 1) than in group 2. However, price discrimination increases the price in the ‘wrong’ market, i.e., the strong market, from the perspective of firm profits.

3.4 Summary of main results

- **Prices:**

- As in the monopoly case, the uniform price is lower than the discriminatory price in the strong market and higher than the discriminatory price in the weak market.
- However, because of the cross-price elasticity, the high price may be in the group with the high market elasticity.

- **Profits:** Unlike the monopoly case, price discrimination can make oligopolists worse-off.

- **Aggregate output:** Even with linear demands, aggregate output can increase or decrease. It depends on the elasticity ratio condition.

- **Welfare:** There is an additional effect here, relative to monopoly, which comes from the possibility that the pattern of relative prices may not be socially efficient under price discrimination, i.e., high price in the high market elasticity group.

⁷As we mentioned earlier, students, or more generally younger consumers, because they want to develop a certain ‘image’ for themselves, may exhibit strong brand loyalty (i.e., low cross-price elasticity), and despite the fact that their market elasticity is high, from a firm’s perspective the elasticity is low.

4 Oligopolistic third-degree price discrimination with best-response asymmetry

In this section, we consider the case where the firms do not rank the two markets symmetrically, see Corts (1998) for more details. That is, one firm's strong market is the other firm's weak market and vice versa (best-response asymmetry). As in the previous section, there are two markets $i = 1, 2$ and two firms $j = a, b$. Market 1 is firm a 's strong market but firm b 's weak market. This implies that $BR_1^a(p) > BR_2^a(p)$ and $BR_1^b(p) < BR_2^b(p)$ for all p .

4.1 Price asymmetry

Price symmetry refers to the case where both firms charge low prices in the same market, i.e., $p_i^j < p_{-i}^j \Rightarrow p_i^{-j} < p_{-i}^{-j}$. For example, the market for senior citizens or students. When this condition is not satisfied, we say that there is price asymmetry. For example, a supermarket may send deeper discounts to shoppers who live farther away from it, but closer to its rival. Figure 1 depicts the best-response curves of the two firms in the two markets. E_1 and E_2 represent the equilibrium discriminatory prices. The situation depicted is characterized by best-response asymmetry and price asymmetry. Firm a charges a higher price in market 1 and firm b charges a higher price in market 2.

The best-response curve of firm j under uniform pricing will fall somewhere in between BR_1^j and BR_2^j , $j = a, b$. If the equilibrium under uniform pricing falls in the upper-right shaded area, then price discrimination leads to an all-out competition with lower prices in both markets. This happens when each firm puts more weight on its strong market. In this case, firm a 's best-response curve under uniform pricing will be closer to $BR_1^a(p_1^b)$, i.e., firm a 's strong market, than to $BR_2^a(p_2^b)$ which is firm a 's weak market. Similarly, firm b 's best-response is closer to $BR_2^b(p_2^a)$, i.e., firm b 's strong market. If the uniform pricing equilibrium falls in the lower-left shaded area, then the opposite happens and price discrimination leads to higher prices in both markets.

4.1.1 Example: A Hotelling-type model

A Hotelling-type model is an example which exhibits best-response asymmetry, since consumers closer to one firm (its strong market) will be farther away from the other firm (its weak market). Consumers are uniformly distributed on the

Figure 1: Best-response asymmetry

$[0, 1]$ interval. Firm a , who is located at 0, views the consumers in the $[0, 1/2]$ interval (market 1) as its strong market and the consumers in the $[1/2, 1]$ interval (market 2) as its weak market. The opposite is true for firm b , which is located at 1. Transportation cost is linear in the distance traveled from a consumer's location to a firm, and the unit transportation cost is t .

Under price discrimination, the best-response functions of firm a in market 1 and for firm b in market 1 are given by

$$p_1^a = \frac{p_1^b + t}{2} \text{ and } p_1^b = \frac{p_1^a}{2}.$$

Similarly, we can write down the best-response functions in market 2.

Under uniform pricing, the best-response functions of the two firms are given by

$$p^a = \frac{p^b + t}{2} \text{ and } p^b = \frac{p^a + t}{2}.$$

These best-response functions are plotted in Figure 2, where $BR_i^j(p_i^{-j})$ represents firm j 's best-response curve in market i when firms can price discriminate. Moreover, $BR_2^b(p_2^a)$ and $BR_1^a(p_1^b)$ are also the best-response curves

under uniform pricing, which yields equilibrium prices equal to (t, t) . Price discrimination makes each firm more aggressive in the strong market of the rival firm. For example, the best-response curve of firm b in market 1 (the strong market of firm a) shifts to the left, relative to the uniform price best-response curve, to $BR_1^b(p_1^a)$. This implies that firm b prices more aggressively in firm a 's strong market, because it does not worry that it will lose revenue from its own loyal customers. The optimal response of firm a is to lower its price in its strong market to 'defend' its loyal consumers. As a result all prices fall relative to the uniform price equilibrium. The discriminatory prices are $(2t/3, t/3)$ in market 1 and $(t/3, 2t/3)$ in market 2. Note that this model also exhibits price asymmetry, since $p_1^a > p_2^a$, but $p_1^b < p_2^b$. Firms are worse-off when they price discriminate.⁸

4.2 Price symmetry

While a Hotelling-type model exhibits both best-response asymmetry and price asymmetry, best-response asymmetry does not necessarily imply price asymmetry. An example of best-response asymmetry but price symmetry is shown in Figure 3. There is best-response asymmetry, since market 1 is firm a 's strong market, but it's firm b 's weak market. However, there is price symmetry, since both firms charge lower prices in market 1 and higher prices in group 2. To see this compare the prices at E_1 (market 1) with the prices at E_2 (market 2) on Figure 3.

4.3 Summary of main results

- **Prices:** Unlike the monopoly and oligopoly under best-response symmetry, the uniform price can be either higher or lower than the discriminatory prices in both markets. Best-response asymmetry does not necessarily imply price asymmetry.

⁸Here, we assumed that firms can segment consumers into two groups. Thisse and Vives (1988) assumed that firms can identify the location of each consumer on the Hotelling interval perfectly. More generally, how fine segmentation is depends on the 'quality' of information firms have about the preferences of the consumers. The growth of the Internet and the development of sophisticated software have allowed firms to collect a vast amount of data about consumers and hence to improve the quality of information they have about consumer preferences. Liu and Serfes (2004) show that equilibrium profits are non-monotonic (i.e., exhibit a U-shape pattern) with respect to information precision (quality).

Figure 2: Hotelling best-response curves

Figure 3: Best-response asymmetry with price symmetry

- **Welfare:** Consumers can become unambiguously better off when all prices decrease due to price discrimination.

5 Price discrimination by purchase history

So far, we have assumed that firms are endowed with information about consumer preferences and can use it to segment the market. But often this information is revealed to the firms by past purchase decisions. Under such an assumption, a firm knows whether a consumer bought its own product or the rival's product, and it can condition its price accordingly. Consumers who bought the product of the rival will receive a discount.⁹ Initial purchases in this context become important because they reveal information. Hence, in this section we examine how these information revelation considerations affect the firms' pricing strategies in a dynamic model, in the spirit of Fudenberg and Tirole (2000), with and without long-term contracts.

A new question that arises in this dynamic environment, relative to the previous sections, is how the possibility of price discrimination in the future affects today's prices.

5.1 No long-term contracts

Consider a two-period Hotelling model. Two firms— A and B —are located at 0 and 1 respectively. A continuum of consumers of measure 1 is uniformly distributed on the interval $[0, 1]$. Transportation cost is linear in the distance traveled to the firm, and the unit transportation cost is t . All agents have a common discount factor denoted by δ .

Let x^* denote the marginal consumer in period 1. All consumers in $[0, x^*]$ buy from firm A in period 1 and those in $[x^*, 1]$ buy from firm B in period 1. The second period prices in firm A 's territory, i.e., in $[0, x^*]$, can be shown to be

$$p_{Ao}^2 = \frac{t}{3} + \frac{2tx^*}{3}, \quad p_{Br}^2 = -\frac{t}{3} + \frac{4tx^*}{3}, \quad (3)$$

where the subscript “ o ” means own and “ r ” means rival firm, and the superscripts mean that these are second period prices.

⁹This is called “paying customers to switch,” as in Chen (1997) and Shaffer and Zhang (2000). There is another type of price discrimination by purchase history, where firms reward its own loyalty customers (e.g. Banerjee and Summers (1987), Fong and Liu (2009)). Esteves (2009) presents a survey on behavior-based price discrimination.

Note that both firms' prices are increasing in the first period market share of firm A . Firm B becomes less aggressive as x^* increases because it poaches consumers with more intermediate preferences (i.e., consumers that are farther away from A). These consumers do not need a significant price discount from the rival to switch firms. Consequently, given that prices are strategic complements, firm A becomes less aggressive as well.

Similarly, the second period prices in firm B 's territory $[x^*, 1]$ are

$$p_{Ar}^2 = t - \frac{4tx^*}{3}, \quad p_{Bo}^2 = t - \frac{2tx^*}{3}. \quad (4)$$

Prices in firm B 's territory increase with the market share of firm B , for the same reason why prices in firm A 's territory increase with firm A 's market share.

In period 1 the marginal consumer is indifferent between: i) buying from A in period 1 and in the second period being poached by firm B and ii) buying from firm B in period 1 and in the second period being poached by firm A . This implies

$$\begin{aligned} u(p_A^1) + \delta u(p_{Br}^2) &= u(p_B^1) + \delta u(p_{Ar}^2) \\ \Leftrightarrow (p_A^1 + tx^*) + \delta(p_{Br}^2 + t(1 - x^*)) &= (p_B^1 + t(1 - x^*)) + \delta(p_{Ar}^2 + tx^*). \end{aligned}$$

Plugging in the expressions for p_{Ar}^2 , p_{Br}^2 , from (3) and (4), and solving for x^* , we can obtain

$$x^* = \frac{3p_B^1 - 3p_A^1 + 3t + \delta t}{2t(\delta + 3)}.$$

Note that when firms can price discriminate based on purchase history, firm demand in period 1 is less elastic when δ increases. The intuition is as follows. Suppose that p_A^1 decreases. Then, some consumers will switch from B to A . Now let's look at the second period. As a result of A having higher market share, firm B will price less aggressively in period 2 in firm A 's territory, while firm A will price more aggressively in firm B 's territory. The benefit a consumer receives by switching from B to A in period 1 diminishes when the second period is taken into account. When δ increases, the second period becomes more important, thus fewer consumers will switch, indicating lower demand elasticity in the first period.

It can be shown that the equilibrium prices in period 1 are

$$p_A^1 = p_B^1 = t + \frac{\delta t}{3}.$$

Price discrimination decreases the first period elasticity and consequently increases prices and profits in the first period, relative to the prices when $\delta = 0$. The consumers in $[0, 1/3]$ buy from firm A in both periods and the ones in $[1/3, 1/2]$ switch to B in period 2. Note that switching is inefficient, since a social planner would want each consumer to purchase from the closest firm. The $[1/2, 1]$ interval is symmetric to the $[0, 1/2]$ interval. However, price discrimination lowers prices in the second period, since firms pay customers to switch. This leads to lower profit in the second period. The overall equilibrium profits are

$$\Pi_A = \Pi_B = \frac{t(8\delta + 9)}{18}. \quad (5)$$

Firms' overall profits are lower when firms have the ability to price discriminate than when they charge uniform prices in both periods. In the latter case, each firm's profit is simply $t(1 + \delta)/2$, which is higher than $t(8\delta + 9)/18$.

5.2 Long-term contracts

Suppose now that in the beginning of first period, each firm can offer a long-term contract to lock consumers for two periods so that poaching is avoided or reduced. Firms can still offer short-term contracts in each period.

Denote the long-term contract price by p_A^{LTC} and p_B^{LTC} respectively. It must then be that the cost of a long-term contract is equal to the cost of two short-term contracts from the same firm, i.e., $p_A^{LTC} = p_A^1 + \delta p_{Ao}^2$ and $p_B^{LTC} = p_B^1 + \delta p_{Bo}^2$. To see this, suppose that $p_A^{LTC} < p_A^1 + \delta p_{Ao}^2$. Then, firm A can raise p_A^{LTC} slightly. This will not affect the demand structure and a higher p_A^{LTC} improves its profit. On the contrary, if $p_A^{LTC} > p_A^1 + \delta p_{Ao}^2$, then no consumer will sign firm A 's long-term contract.

Let $\tilde{x}_A$ and $\tilde{x}_B$ denote the consumers who are indifferent between a long-term and a series of short-term contracts (not necessarily from the same firm) respectively. The demand structure is the following

- Period 1:
 - Each firm's most loyal customers buy its long-term contract. Consumers in $[0, \tilde{x}_A]$ buy firm A 's long-term contract and consumers in $[\tilde{x}_B, 1]$ buy firm B 's long-term contract.
 - Less loyal customers buy short-term contracts. Consumers in $[\tilde{x}_A, x^*]$ buy from firm A and consumers in $[x^*, \tilde{x}_B]$ buy from firm B .

- Period 2: Some consumers (the relatively less loyal) who bought a short-term contract switch firms.

Because each firm has locked its most loyal consumers through its long-term contracts, it becomes more aggressive in its own turf. It can be shown that the second period equilibrium prices in $[\tilde{x}_A, x^*]$ are

$$p_{Ao}^2 = \frac{t}{3} + \frac{2tx^*}{3} - \frac{4t\tilde{x}_A}{3}, \quad p_{Br}^2 = -\frac{t}{3} + \frac{4tx^*}{3} - \frac{2t\tilde{x}_A}{3}. \quad (6)$$

Similarly, the second period equilibrium prices in $[x^*, \tilde{x}_B]$ are

$$p_{Ar}^2 = \frac{t}{3} - \frac{4tx^*}{3} + \frac{2t\tilde{x}_B}{3}, \quad p_{Bo}^2 = -\frac{t}{3} - \frac{2tx^*}{3} + \frac{4t\tilde{x}_B}{3}. \quad (7)$$

Note that prices are lower than when long-term contracts are not feasible (compare (6) and (7) with (3) and (4)).

It can be shown that the first period short-term equilibrium prices are

$$p_A^1 = p_B^1 = t.$$

The equilibrium prices of the long-term contracts are

$$p_A^{LTC} = p_B^{LTC} = t + \frac{\delta t}{2}.$$

Since the equilibrium is symmetric we focus on $[0, 1/2]$. From $[0, 1/8]$ consumers buy long-term contracts from firm *A*. From $[1/8, 1/2]$ they buy a short-term contract from firm *A* in period 1. From $[1/8, 3/8]$ they also buy from *A* in period 2, while from $[3/8, 1/2]$ they switch to firm *B*.

When long-term contracts are not feasible, $1/3$ of the consumers switches. However, only $1/4$ of the consumers switches when long-term contracts are feasible. Since there is less switching, which is inefficient, long-term contracts improve social welfare.

The equilibrium profits are

$$\Pi_A = \Pi_B = \frac{t(7\delta + 16)}{32}. \quad (8)$$

Firms are worse off when long-term contracts are offered (compare (8) with (5)). Why, then, do firms offer long-term contracts? By offering a long-term contract a firm, say firm *A*, commits to more aggressive second period pricing (for the reasons we explained above). Firm *B*, in turn, lowers its poaching price which makes the product of firm *A* more attractive in the first period. This allows firm *A* to raise its first period price. The game has features of prisoners' dilemma.

5.3 Summary of main results

- Firms learn consumer preferences by observing past purchase behavior. This allows firms to segment consumers and price discriminate.
- When long-term contracts are not available, the possibility of this kind of price discrimination in the future increases today's prices, but lowers the future prices.
- When long-term contracts are feasible, consumers become better off and firms worse off.
- Long-term contracts improve social welfare because they reduce inefficient switching.

6 Concluding remarks

In this lecture, we have examined the effects of third-degree price discrimination. We begin with a monopolist who can segment the market into two groups: weak (high elasticity) group and strong (low elasticity) group. This flexibility allows the seller to raise the price in the strong market and lower it in the weak market. The seller's profit increase. A negative effect on welfare due to price discrimination is that the marginal valuations across consumers are not equalized. Output and social welfare can increase or decrease depending on the curvatures of the demand functions. When competition is introduced, under the assumption of best-response symmetry, some of the key results change. Price discrimination can lead to lower profits. An additional negative effect due to price discrimination is that the price can increase in the wrong group, i.e., the group with the high market elasticity. This can happen because under competition the cross-price elasticity also matters in determining the firm-level elasticity. When the assumption of best-response symmetry is relaxed new results emerge. Most notably, under best-response asymmetry, price discrimination can lower the prices for all groups of consumers. Finally, in a dynamic (two-period) framework, firms can segment consumers according to past purchase behavior. This possibility makes demand in period 1 less elastic which results in higher first period prices. Long-term contracts can lock consumers and mitigate wasteful switching between firms. Nevertheless, the ability to offer long-term contracts can hurt firm profits.

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