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Nudging Students Forward Towards Backward Induction

Utteeyo Dasgupta

Abstract

This paper uses variations in a popular parlor game to provide useful instructional benefits. The paper builds a classroom activity to nudge students towards thinking in a backward-inductive manner. The pedagogic innovation is in introducing the game repeatedly with progressively smaller action spaces in every repetition. As the numbers of available actions become smaller and smaller in the reduced games, students stumble on to the backwardly inductive winning strategy. The described procedures are also useful in laying down the intuitive foundations of the concepts of dominant strategies, and first- and second-mover advantages.

KEYWORDS: classroom experiment, game theory, backward induction

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Introduction

Introducing backward induction to students is a challenging idea. Even adults can have a hard time grasping the logic of starting at the very end of a sequential decision-making problem, then folding their way back to arrive at an optimal solution! An exception probably will be a weathered city traveler using a public-transport system. To attend a 10:30a.m. meeting she will probably argue as follows: If I need to be at the department meeting at 10:30 a.m., I must take the 9:00 a.m. train - which means I must get out of the house at 8:50a.m. to take the bus to the station; this means I must wake up by 7:00a.m. to make my lunch...and so on! Even if she has never heard of a backward-induction solution, she practices it every day!

The backward-induction logic is one of the key principles used in the critical solution-concept of the sub-game perfect equilibrium.¹ Watson (2002) describes it as "...a process of analyzing a game from back to front (from information sets at the end of the tree to information sets at the beginning)." Intuitively, the solution procedure involves solving for optimal actions from the very last stage of a game and iteratively folding the game tree back after incorporating optimal decisions for players at every stage. Learning, understanding, and finally using the backward induction logic in solving games do not come naturally. When introducing the topic for the first time, instructors often face reactions ranging from stoicism to complete disbelief from the students. Pedagogic research suggests that using classroom games (Holt 1999) can help promote student engagement, active learning, and better learning outcomes (Emerson and Taylor 2004). There has been a plethora of classroom experiments to aid in teaching economics. Experinomics and Econport are two prominent repositories which provide descriptions of many classroom games and experiments that require very little preparation.² This paper is similar to many of the experiments found in the two repositories mentioned above. It is a trivial extension to declare a prize (money or class room credits) for the winners or the winning team in the exercises described below.

The basic game described here is quite flexible in accommodating slight changes in the rules of the game (see Dixit 2005, Gneezy, Rustichini and Vostroknutov 2009, Dufwenberg, Sundaram and Butler 2010). One exciting demonstration of the game is found in a popular reality show called *Survivor* (Dixit 2005). In this reality show, the game was played between two competing teams as a part of the immunity challenge, where the losing team had to vote out

¹ The backward induction process is often given intuitive labels for student understanding. For example, Dixit, Skeath and Reiley (2009) call it the "roll back" process while Watson (2002) calls it the "fold back" process.

² See experinomics.com and econport.org.

one of its members.³ In the game there were 21 flags, and two players took alternating turns to remove a specified number of flags. A player, at his or her turn had the choice of removing 1, 2, or at most, 3 flags. The player who removed the last flag (whether as the sole remaining flag or one of the last surviving set of 2 or 3 flags) was declared the winner.

The way to think about the solution for the *Survivor* challenge is as follows: Suppose players A and B are in the situation where there are only 4 flags left, and it is player B's turn. By the rules of the game, B is restricted to removing 1, 2, or all 3 flags at his turn, which means A at her turn can accordingly pick the last 3, 2 or the sole remaining flag, to claim victory. Now to ensure that B is left facing four remaining flags at his turn, A, at her previous turn, must leave B facing 8 flags. Using the same logic iteratively would imply B leaving 12, 16, and 20 flags in previous moves, respectively. Therefore, in the beginning with all 21 flags remaining, the first player should remove the first flag, and then proceed to take four, minus whatever the other takes at his immediately preceding turn.

Dixit (2005) suggested laying out coins on the glass of the overheard projector as a convenient way of playing this game in-class where students could easily see what was going on. He suggested repetitive play of the above game can help the instructor to successfully initiate the discussion of the backward-induction technique. Results from a recent experiment by Dufwenberg, Sundaram and Butler (2010) suggest that one can go even further in nudging students towards the logic of backward induction. Dufwenberg et al. (2010) introduced students successively to two games, the full game of 21 and a reduced version, with only 6 numbers. The experiments were run in search of evidence for two types of epiphany (as defined by the authors). Epiphany 1 was defined to be the realization of the analytical structure of the problem, and Epiphany 2 was defined as the realization that a dominant strategy exists. They found that playing a smaller version of the Game of 21 first, promotes understanding that the bigger game might have an analytical solution and consequently establish better performance in the bigger game.

This paper uses insights from the above literature to build a classroom activity to introduce and promote better understanding of the backward-induction logic. The primary goal of the classroom exercise is to nudge students towards thinking in a backwardly inductive manner. In addition, the described procedures are useful in laying down the intuitive foundations of the concepts of dominant strategies, and first- and second-mover advantages. The pedagogic innovation is in introducing the game repeatedly with progressively smaller action spaces in every repetition. I start with the full-blown game, and in every round introduce a slightly smaller version of the game with the same rules. At the beginning with 21

³ A video clip of the episode is available for viewing at <http://www.youtube.com/watch?v=gf4FGeeXovU>.

numbers, the backward-induction solution is rarely evident to any student. However, as the game becomes smaller and smaller in available player actions after each repetition, many students start realizing the winning strategy in the smaller game. I follow up with a discussion of the fold-back strategy for the full version of the game to point out the general principle obvious in the smaller games apply equally well to the full version. I subsequently use another game of 21, with slightly modified rules to lay the basis for discussion on first- and second-mover advantages.

Description of the Game

Two players take turns to play this game. There are 21 numbers, starting from 1 and ending in 21, presented in an increasing order. Each player at his or her turn is required to pick or remove numbers sequentially. The rules of the game specify how many numbers a player can pick or remove at each turn. The player, who reaches a position to pick number 21 first, wins the game.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21

Figure 1: The Game of 21.

There are two versions of the game. In one version, each player is restricted to choose, at most, three consecutive numbers at every turn. That is, a player at her turn, can pick one, two or three consecutive numbers at a time. Let us call it the “3-hop game of 21.” In the other version, a player is restricted to choose one or, at most two, consecutive numbers at a time. Let us call it the “2-hop game of 21.” A nice characteristic about the game is that one of the players has a dominant strategy, and although the games are nearly identical, a slight change in the rule, brings about strategic advantages in moving first or moving second. If the rules of the game specifies that a player can remove, at most, n numbers, then a player should leave his opponent facing the last $n+1$ numbers, and before that $2(n+1)$ numbers, and before that $3(n+1)$ numbers and so on. It is straightforward to verify that if one uses numbers 1 through 21, the 3-hop game has a first-mover advantage, and the 2-hop game has a second-mover advantage.

Description of the Classroom Activity

Typically one can spend about thirty minutes on the activities proposed here. An ideal time to play this classroom game is before starting with the topics of sequential games.

In the beginning of the lecture, numbers 1 through 21, are written on the blackboard, arranged in three rows containing seven numbers in each.⁴ Next, the rules of the game are introduced with some examples of player moves. It is important to provide examples of player moves because students often think that the numbers can be chosen in random order. One needs to make sure that students understand that, from the beginning, players can choose numbers *only* in a sequential manner without skipping any numbers in between. I usually start with the 3-hop game of 21. Two students are asked to volunteer.⁵ Once two students volunteer, they are given the option to be the first or the second mover. I have found students to be cautious at this point, preferring to be the second mover, and relegating the possibility of any misplay to their opponents! As pointed out earlier, this means that the opponent, if adept at using the concept of backward induction, can guarantee herself a win in this version of the game. This seldom happens, if ever. After the first mover has been decided, the two players take turns to choose numbers, and the instructor strikes them off of the blackboard. Figures 2 and 3 describe an example of how the game might proceed.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21

Figure 2: **Player 1** picked number 1 in his turn, and *Player 2* picked numbers 2 and 3.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21

Figure 3: **Player 1** in his subsequent move has chosen 4, 5 and 6, and *Player 2* has chosen 7, 8 and 9.

In my experience, students in the beginning randomly choose one or two numbers at a time until the second row is almost complete. As the players enter the third row, the two contestants, along with many of their classmates, realize who the winner will be. After the game ends, two new students (or leaders of two other groups) play again resulting in further familiarity with the process of the game. However, the majority are still clueless as to how to proceed in the game,

⁴ It is not necessary to arrange them in this particular order. One can instead write them all in one row. I have found it to be useful to express them in three rows as it increases the anticipation and the excitement as players approach the final row!

⁵ Alternatively, it is also possible to divide the class into groups of two or three, and assign a leader for each group to have a tournament between two groups at a time. The leader is then responsible for taking final decisions at each turn after consulting with the group members. (This can work especially well in upper level classes with smaller enrollments.)

and at least through the first two rows pick one, two or three numbers without a conscious thought.

The next step is to reduce the number of available actions in the game. At the end of the first two repetitions, the numbers 18 through 21, are erased to construct a game with a smaller action space. Figure 4 shows the new game – the game of 17!

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17				

Figure 4: The Game of 17.

Two new volunteers are chosen to play the game. From this point on, the instructor should continue reducing the action space of the game in every subsequent run of the game. Hence, the very next run will start with 13 numbers, followed by 9 numbers in the next round, and finally the last run with 5 numbers on the board. Every time a new set of volunteers should be picked. By the time the game is reduced to the game of 5 (with only the numbers 1 through 5) many students start getting an intuitive understanding of the way to proceed in the game, and, at the very least, the rest of the students sense how not to proceed in the game. At the end of the game, the instructor goes over the backward-induction solution of the smaller game first, and the full version next. It is also useful to explain, at this point, the advantage of being a first mover in this game.⁶

Next, one can challenge the students for a final game of 21 - the 2-hop version. The new rules are introduced where a player has the option to remove, at most, two numbers at a time from the set of 21 numbers. The contestants are again offered the option of playing first or playing second. Often I have found students rush to be the first mover at this juncture, the conclusions of the previous version still fresh in their minds! However, I have also noticed students choosing to be the second mover after a deliberate and conscious pause. Once the new game is over, the backward-induction solution of the game and the advantages of being a second mover in this game, can be explained.

At the end of the exercise I find that a majority of students are convinced of the usefulness of the backward-induction technique. Once students have such an intuitive flavor for the procedure, one can start the discussion on extensive form games, the solution concept of backward induction and first and second mover advantages. The instructor can end the lecture with assignments involving the game of Nim and Marienbad, both of which share with the Game of 21, the

⁶ Note that the sequence of reduction suggested here ensures that the first mover advantage in each new (smaller) game is maintained.

feature of finite two-player zero-sum two-outcome games with perfect information.⁷

Conclusion

This paper presents simple variations of the Game of 21, that provide useful instructional benefits with little material and preparation. The classroom exercise systematically reduces the available actions in the game as it is played multiple times. As the action space becomes progressively small, many students stumble on to the logic of backward induction. My experience suggests that introducing the backward induction in this manner does a remarkable job in making students familiar with the logic which otherwise looks strange to a student new to a game theory course.

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⁷ Dutta (1999) offers some nice exercises involving the two games that are constructive and entertaining (See pp. 8-10, 175-176).