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# Incorporation of the GPS satellite ephemeris covariance matrix into the precise point positioning

*Research article*

Masoud Shirazian*

*Division of Geodesy and Geoinformatics, Royal institute of Technology, 100 44 Stockholm, Sweden*

**Abstract:**
In GPS positioning, usually the satellite ephemeris are fixed in the observation equations using broadcast or published values. Therefore, to have a realistic covariance matrix for the observations one must incorporate a well-defined covariance matrix of the satellite ephemeris into the observations covariance matrix. Contributions so far have discussed only the variance and covariance of the observations. Precise Point Positioning (PPP) is a technique aimed at processing of measurements from a single (stand-alone) GPS receiver to compute high-accurate position. In this paper, the covariance matrix of the satellite ephemeris and its impact on the position estimates through the PPP are discussed.

**Keywords:**
covariance matrix • PPP • satellite ephemeris


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## 1. Introduction

The use of a diagonal covariance matrix of the observations is widely accepted by GPS users. The diagonal arrangement of the covariance matrix is based on the assumption that the noise of the GPS observables is normally distributed and independent (Tiberius and Borre 1999). Moreover, Euler and Goad (1999); Gerdan (1995); Jin (1996); Tiberius (1999) and Barnes (2000) represented that the standard deviation of the GPS observations is elevation dependent.

Since 2005 the International GNSS Service (IGS) has been publishing orbit and clock files, which contain uncertainty of satellite coordinates and clocks (see Hilla 2010). Shirazian (2006) showed that incorporating these uncertainties into the PPP processing may improve the results. Here, an elaborate study is conducted on this issue.

*E-mail: masoudsh@kth.se

Satellite ephemeris covariances are not publicly available in any of the GPS related publications yet. In this paper, we will discuss different ways of finding cross-covariance between the satellite position components. The influence of incorporating these cross-covariances is tested on real data. Moreover, time-correlation of these components is studied and discussed in this paper.

## 2. Phase-only precise point positioning

Traditionally, PPP employs the ionosphere-free code and phase observations while the satellite coordinates and clocks are fixed to their values from the published precise ephemeris to estimate the receiver position. However, due to the fact that some of the systematic effects (e.g. the hardware delays and the multipath), affect the code and phase observations differently, there are some unknown biases in the system of observation equations when using phase and code observations together. To avoid such biases we decided to use only phase observations for the PPP purpose.

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### 2.1. Observations

Undifferenced ionosphere-free linear combinations of L1 and L2 phase data (L3) are used as raw observations. The observation equation for this observation reads:

$$\phi_r^s = \rho_r^s + c\left(dt_r - dt^s\right) + T_r^s + \lambda a_r^s + \eta + \epsilon_\phi \qquad (1)$$

where $\phi_r^s$ is the L3 carrier phase (see Hofmann-Wellenhof et al. 2008) from satellite $s$ to receiver $r$, $\lambda$ is the L3 wave length (10.3 cm), $a_r^s$ is the non-integer phase ambiguity, and $\epsilon_\phi$ is measurement noise including multipath. The term $\eta$ contains corrections for various systematic effects that are to be computed from known models.

Systematic effects with impact of more than 1 cm must be included in $\eta$ in Eq. (1). Sorted with respect to their impact, these are: relativistic effect, tropospheric refraction, satellite and receiver antenna offsets, solid Earth tide and phase wind-up effect. In-depth explanations about the relativistic and solid Earth tide effects can be found in the IERS Conventions (2010) (Petit and Luzum 2010). The phase wind-up effect is completely explained in Wu et al. (1993). The tropospheric delay is more complicated. There are many ways to treat it. Among them, we have chosen the method described in Krueger et al. (2004). Following this approach we have modeled the ZHD (Zenith Hydrostatic Delay), corrected the observations for it and estimated the ZWD (Zenith Wet Delay) on an epoch-by-epoch basis. To be consistent with the IGS, we used Niell's mapping function (see Niell 1996). The IGS ANTEX files (Beutler et al. 1999) are used for absolute receiver antenna phase center correction. The satellite antenna phase centre correction is applied just on the radial component of the observation vector in the body frame of the satellite to be consistent with the IGS process to estimate orbits and clocks. The size of the correction is 1.023 m.

The satellite coordinates and clocks are computed from the IGS sp3c files (final orbits and clocks). Together with the approximate values for the receiver position and receiver clock, the a-priori troposphere delay, and the corrections mentioned in the previous paragraph, we obtain the computed observation from Eq. (1).

### 2.2. The system of observation equations

The linearized observation equations, in the form of the Gauss-Markov model, for each epoch are:

$$E\left\{\underbrace{\begin{pmatrix}\Delta\phi_r^1\\ \Delta\phi_r^2\\ \vdots\\ \Delta\phi_r^m\end{pmatrix}}_{\Delta y_k}\right\} = \underbrace{\begin{pmatrix} -e_r^{1\,T} & \lambda_j & 0 & \dots & 0 & c & M_w\left(z_r^1\right)\\ -e_r^{2\,T} & 0 & \lambda_j & \dots & \vdots & c & M_w\left(z_r^2\right)\\ \vdots & \vdots & \vdots & \ddots & 0 & \vdots & \vdots\\ -e_r^{m\,T} & 0 & 0 & \dots & \lambda_j & c & M_w\left(z_r^m\right)\end{pmatrix}}_{A_k} \underbrace{\begin{pmatrix}\Delta x_r\\ a_{rj}^1\\ a_{rj}^2\\ \vdots\\ a_{rj}^m\\ dt_{rk}\\ D_{wk}\end{pmatrix}}_{\Delta x};\ D\left\{\Delta y_k\right\} = q_{y_k} \qquad (2)$$

The vector $\Delta y_k$ consists of the original L3 observations minus computed observations from Eq. (1), $q_{y_k}$ is the diagonal covariance matrix of the observations, of which the diagonal entries are the inverse of cosine of the zenith angle of satellites.

$$q_{y_k} = \begin{pmatrix} \frac{1}{\cos^2 z_r^1} & 0 & \cdots & 0\\ 0 & \frac{1}{\cos^2 z_r^2} & \cdots & \vdots\\ \vdots & \vdots & \ddots & 0\\ 0 & 0 & \cdots & \frac{1}{\cos^2 z_r^m}\end{pmatrix}_k \qquad (3)$$

### 2.3. Covariance matrix for the observations

As one can use the satellite orbits and clocks in the form of weighted known parameters for positioning purposes, they must be incorporated into the system of observation equations. Then Eq. (2) can be converted to the following equation:

$$E\left\{\begin{pmatrix}\Delta y_k\\ \Delta x^s\end{pmatrix}\right\} = \begin{pmatrix} A_k & A_k^s\\ 0 & I\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta x^s\end{pmatrix};$$
$$D\left\{\begin{pmatrix}\Delta y_k\\ \Delta x^s\end{pmatrix}\right\} = \begin{pmatrix} q_{y_k} & 0\\ 0 & Q_k^s\end{pmatrix} \qquad (4)$$

where $A_k^s$ is the relevant design matrix of orbits and clock at epoch $k$, converting orbit and clock errors into the range domain and

$Q_k^s$ is the relevant covariance matrix of the orbits and clocks at epoch $k$ (obtained from the IGS final orbit and clock files). By pre-elimination technique, we can simplify Eq. (4) so that $\Delta x^s$ disappears from the right hand side of Eq. (4) and we avoid estimating it. Multiplying the second row of the equation by $-A_k^s$ and adding the two rows leads to:

$$E\{\Delta y_k - A_k^s \Delta x^s\} = A_k \Delta x;$$
$$D\{\Delta y_k - A_k^s \Delta x^s\} = q_{y_k} + A_k^s Q_k^s A_k^{s\,T}. \quad (5)$$

Then the covariance matrix of observations for each epoch is:

$$Q_{y_k} = \underbrace{\sigma_0^2 q_{y_k}}_{L3\ measurements} + \underbrace{A_k^s Q_k^s A_k^{s\,T}}_{orbits\ \&\ clocks} \quad (6)$$

The covariance matrix consists of two parts. The first part describes the measurement accuracy with $\sigma_0^2 q_{y_k}$, the elevation dependent covariance matrix of raw observations at epoch $k$ and the scale $\sigma_0 = 0.01$ is chosen so that the standard deviation of the ionosphere free linear combination in the zenith direction is 1 cm. The second part describes the effect of orbit and clock errors on the observations.

Finally, the covariance matrix for all epochs reads:

$$Q_y = blkdiag\left(Q_{y_1}, Q_{y_2}, \cdots, Q_n\right) \quad (7)$$

where $n$ is number of epochs. This covariance matrix is diagonal. If the correlations between orbits and clocks were available, we would have a non-diagonal covariance matrix for the observations and consequently different estimates of the unknown parameters. Now everything is ready for the least squares adjustment of the data. After the least squares adjustment, the overall model test (see Teunissen 2000 and Appendix) will be performed and, if necessary, data snooping will be applied to the data.

### 2.4. Data used for the numerical study

24-hour sets of observations at five IGS permanent stations are selected to be processed. 4$^{th}$ of January 2006 is the date for Algonquin (ALGO), Graz (GRAZ) and Tehran (TEHN) stations, 19$^{th}$ of June 2005 and 15$^{th}$ of September 2006 for Brussels (BRUS) station (Brussels1 and 2) and 7$^{th}$ of November 2006 for Saskatoon (SASK) station. To fix the satellite coordinates and clocks, we used the IGS sp3c files (final orbits and clocks) of the aforementioned days. IGS ANTEX files are used for the absolute receiver antenna phase center correction.

For validation of the estimates of station coordinates, they are compared with the station coordinates obtained from the ITRF (International Terrestrial Reference Frame) solution at the date of observation.

It is necessary to mention that all satellites below 15 degrees of elevation are dismissed (i.e. cut-off angle is 15 degrees).

### 2.5. Processing strategy

As mentioned before, we decided to use the undifferenced ionosphere-free linear combination of L1 and L2 phase data (L3) as our observations. There are the following reasons for this decision. First, due to unavailability of the IGS final orbits and clocks, we are not able to do real-time precise point positioning at cm level of accuracy. Since the code observations are necessary for real-time positioning and PPP is done in post-process, we decided not to use code data except for time synchronization. Second, due to the low accuracy of the code observations, they do not play any significant role in the adjustment stage. This means that they do not improve the estimates significantly. Third, satellite clocks estimated using code observations and those estimated using phase observations are different (because of different observation noise and different systematic effects, affecting code and phase observations differently). Fixing satellite clocks to the same values for both code and phase observations introduces a bias to the process. Since there is no guarantee that the bias remains constant during the whole observation time span, it could not be absorbed by the phase ambiguities. This means, if we lump this bias to the phase ambiguities, we might lose the epoch-independence of the phase ambiguities, and it results in less reliable estimates of the parameters.

To find a more realistic covariance matrix of the observations we restricted ourselves to the sampling rate of 15 minutes, because the uncertainties of the satellite coordinates and clocks are available only at every 15 minutes in the sp3c files. This time interval allows us to avoid interpolation of the satellite clocks, which degrades their quality (see Montenbruck et al. 2005).

The unknown parameter vector consists of station coordinates (static solution- one set of X, Y, Z coordinates per station, based on 96 epochs), phase ambiguities, receiver clocks and wet part of tropospheric delay. The last two parameters are estimated for each epoch.

Since the time interval is 15 minutes, the number of epochs in a 24-hour time span is 96. Therefore, the number of observations is not so large. These conditions allow us to process all the data in one batch.

To verify the influence of the covariance matrix of satellite orbits and clocks, all the data is processed once with the weight matrix consisting of the inverse of the covariance matrix of the observations only, and next time the covariance matrix of satellite orbits and clocks are incorporated (see Shirazian 2006).

### 2.6. Results and discussion

In this part the results of the numerical computations are discussed. In Tables 1 and 2 the discrepancies of the station position components from their ITRF values in the geocentric and local geodetic (topocentric) systems are given, together with the standard deviations for the position components computed by the MATLAB code (written by the author) from the inverse normal matrix. For the computations in Tables 1 and 2 different weight matrices for the observations were used. The weight matrix used in the computations for Table 1 is computed just by using the first part of the Eq. (6). This means that only the elevation dependent measurement errors are taken into account. Table 2 corresponds to the same data, but with a different weight matrix, in which the covariance matrix of the orbits and clocks is incorporated (Shirazian 2006).

An important statistical test, the overall model test (see Appendix), should be done after a least-squares adjustment. According to Teunissen (2000) and Amiri-Simkooei (2007), this test helps us study whether the functional and the stochastic models are properly selected. In this section, the latter (stochastic model) is focused on. We conduct this test twice; once before incorporation of the satellite orbits and clocks uncertainties and another time after incorporating them into the covariance matrix of the observations. The results are listed in Table 3.

As can be seen in the table, the test for all stations failed when the satellite orbits and clocks uncertainties are not incorporated. After incorporating them into the covariance matrix of the observations, the test passes for all cases. This reveals that the choice to incorporate the satellite orbits and clock uncertainties is a correct assumption and leads us to a more realistic covariance matrix.

## 3. Covariances of the satellite orbits and clocks

To have a full covariance matrix of the satellite coordinates and clocks, one needs to have their cross-covariances as well as their auto-covariances.

### 3.1. Cross-covariances from different published satellite ephemeris

There are more than 10 analysis centers (ACs) working with the International GNSS Service (IGS). They determine satellite orbits and clocks and publish them on the internet. Having the products of these analysis centers, one can form a vector from the values of every satellite position component at each epoch. We chose the EMR, ESA, GFZ, JPL, MIT, NGS and SIO analysis centers to obtain such vectors of the satellite coordinates (because only data from these analysis centers were available for the dates of our observations). This means that we have a vector for each component X, Y and Z for which the seven entries are the values from the seven ACs above. Then one can proceed to obtaining mean, variance, covariance and correlation of the satellite position components at each epoch as:

$$\mu_{X_i} = \frac{1}{N} X_i^T U \qquad (8)$$

where:

$\mu_{X_i}$ : is the mean of the sample vector $X_i$

$X_i$ : is the sample vector $X$ at epoch $i$

$N$ : is the length of the sample vector (7 in our case)

$U$ : is a $N \times 1$ column vector of which all entries are equal to one

$$\sigma^2_{X_i} = \frac{1}{N-1} \left(X_i - \mu_{X_i} U\right)^T \left(X_i - \mu_{X_i} U\right) \qquad (9)$$

where:

$\sigma^2_{X_i}$ : is the variance of the sample vector $X$ at epoch $i$

$\mu_{Y_i}$, $\sigma^2_{Y_i}$, $\mu_{Z_i}$ and $\sigma^2_{Z_i}$ can be computed analogous to $\mu_{X_i}$ and $\sigma^2_{X_i}$.

$$\sigma_{X_i Y_i} = \frac{1}{N-1} \left(X_i - \mu_{X_i} U\right)^T \left(Y_i - \mu_{Y_i} U\right) \qquad (10)$$

where:

$\sigma_{X_i Y_i}$ : is the covariance between vectors $X$ and $Y$ at epoch $i$

$$\varrho_{X_i Y_i} = \frac{\sigma_{X_i Y_i}}{\sigma_{X_i} \sigma_{Y_i}} \qquad (11)$$

where:

$\varrho_{X_i Y_i}$ : is the correlation coefficient of $X$ and $Y$ at epoch $i$

Analogously, we can write this equation for $XZ$ and $YZ$ at each epoch.

Computing the necessary estimates of the satellite coordinates and their full covariance matrix at each epoch (as can be used in Eq. (6)) from the data published by the above-mentioned ACs using Eqs. (8) to (10) is an attempt to find a more realistic covariance matrix of the GPS observations in PPP.

However, performing the PPP process with this full covariance matrix of the observations results in a system of observation equations of which the normal (cofactor) matrix ($A^T Q_y^{-1} A$) is not invertible. This is because of the large covariances as the off-diagonal entries of the covariance matrix of the observations (many of them are bigger than the diagonal entries which are variances).

This large size of the covariances returns to the networks that different analysis centers use for the purpose of orbit determination. As these networks of the permanent GPS stations are not so different from each other (compared to the satellite constellation), the whole configurations (ground network and the satellites) selected by different analysis centers do not change significantly in geometry. Therefore, this closeness of the geometry of the configurations results in highly correlated estimates of the satellite coordinates and clocks.

### 3.2. Cross-covariances from a time series of satellite ephemeris

If one needs a full covariance matrix for the ephemeris a reasonable way is to compute the required covariances from a data series of some days of the ephemeris using spectral analysis techniques (e.g. Fourier spectral analysis). Fig. 1 shows the covariances obtained from processing of three time series of 3, 5 and 7 days of the ephemeris of a satellite (PRN#1) as an example. The time difference between two consecutive epochs is 15 minutes (15 minutes time interval).

Looking at the Fig. 1, one can infer that there is no significant improvement in using longer data series than 3 days. Thus, we decided to use a 3-day data series of the ephemeris to compute the covariances.

The MATLAB function "*xcov*" is employed to compute the covariances from the data series. This function works based on the Fast

Table 1. Station coordinate discrepancies and their standard deviations in cm (at 95% level of confidence).

| Stations | Geocentric coordinate system (ITRF) | | | | | | Topocentric coordinate system | | | 3 Dimensional discrepancy | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta x$ | $\sigma_X$ | $\Delta y$ | $\sigma_Y$ | $\Delta z$ | $\sigma_Z$ | $\Delta N$ | $\Delta E$ | $\Delta U$ | 3D | $\sigma_{3D}$ |
| Algoquin | -1.12 | 2.88 | -4.33 | 4.89 | -3.23 | 4.81 | -5.12 | -1.99 | 0.46 | 5.51 | 7.48 |
| Brussels1 | 6.86 | 3.26 | 1.94 | 1.97 | -0.92 | 3.83 | -5.99 | 1.41 | 3.70 | 7.18 | 6.12 |
| Brussels2 | 9.81 | 3.68 | -4.64 | 2.11 | -2.90 | 4.39 | -9.14 | -5.37 | 3.72 | 11.23 | 6.07 |
| Graz | 7.73 | 4.29 | 1.95 | 2.71 | -0.98 | 4.44 | -6.50 | -0.18 | 4.71 | 8.03 | 6.78 |
| Saskatoon | -3.01 | 2.61 | -3.47 | 3.99 | 0.69 | 4.70 | -2.88 | -1.91 | 3.11 | 4.65 | 6.81 |
| Tehran | 2.63 | 3.60 | 5.97 | 4.16 | -0.35 | 3.49 | -3.96 | 1.67 | 4.91 | 6.53 | 6.54 |

Table 2. Station coordinate discrepancies and their standard deviations in cm (at 95% level of confidence) when covariance matrix of orbits and clocks is incorporated.

| Stations | Geocentric coordinate system (ITRF) | | | | | | Topocentric coordinate system | | | 3 Dimensional discrepancy | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta x$ | $\sigma_X$ | $\Delta y$ | $\sigma_Y$ | $\Delta z$ | $\sigma_Z$ | $\Delta N$ | $\Delta E$ | $\Delta U$ | 3D | $\sigma_{3D}$ |
| Algoquin | 0.93 | 2.62 | -8.29 | 4.79 | -0.15 | 4.70 | -6.07 | -0.80 | 5.66 | 8.34 | 7.24 |
| Brussels1 | 5.95 | 3.13 | 1.70 | 1.75 | -2.04 | 3.66 | -5.98 | 1.24 | 2.25 | 6.51 | 5.95 |
| Brussels2 | 8.91 | 3.53 | -5.03 | 1.99 | -3.38 | 4.18 | -8.72 | -5.69 | 2.75 | 10.77 | 5.44 |
| Graz | 5.17 | 3.94 | 2.08 | 2.43 | -3.55 | 4.01 | -6.47 | 0.63 | 1.17 | 6.60 | 6.16 |
| Saskatoon | -2.97 | 2.24 | -2.40 | 3.62 | 0.40 | 4.22 | -2.24 | -2.18 | 2.24 | 3.84 | 6.11 |
| Tehran | 2.83 | 3.26 | 4.28 | 3.75 | -1.48 | 3.19 | -4.18 | 0.46 | 3.29 | 5.34 | 5.94 |

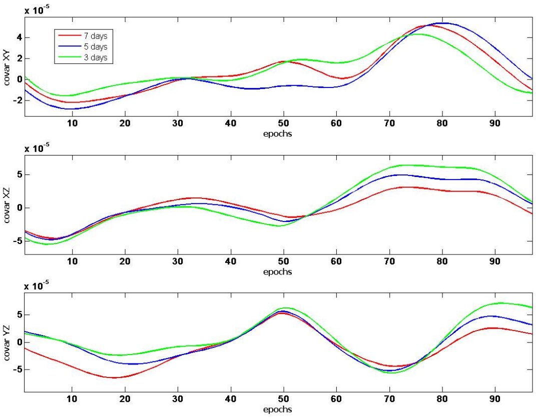


Figure 1. Cross-covariances computed from 3, 5 and 7 days of the ephemeris of PRN#1.

Fourier Transform (FFT). This requires the data to be stationary (see Orfanidis 2007 for more details). To obtain stationary data one should remove the deterministic part (trend) from the data series and just the stochastic part remains in the data after detrending. Thus, the selection of the trend is of high importance.

Usually, for the GPS satellite orbits, one can fit either polynomials or trigonometric functions to the data. These fitted functions can be used as trend for the original data. Schenewerk (2003) compared the performance of different orders of the above-mentioned functions for satellite orbit interpolation. He concluded that among all orders, trigonometric functions of order 9 and polynomials of order 11 give the best results. Thus, we chose to fit an 11-order polynomial to 17 points from the published ephemeris. To avoid the edge-effect error, one can use this interpolating polynomial as a moving piecewise tool. This means that two consecutive 17-point arcs have 16 common points and the interpolated orbits between the 3 central points of the arc are accurate enough to be used in a positioning process.

The next step is to incorporate these covariances into the system of the observation equations. For this, we used the same data as in Section 2. Table 4 shows the discrepancies between the station coordinate estimates when the full covariance matrix of the orbits and clocks is used.

### 3.3. Autocorrelation of satellite ephemeris

Using Eq. (7), one can form the covariance matrix of the observations. An important question here is why the off-diagonal entries of the matrix are zero. To answer this question, autocorrelation of

Table 3. Overall model test (at 95% level of confidence) before and after incorporation of the satellite orbits and clocks uncertainties.

| Stations | Critical value ($k_\alpha$) | Before incorporation | | After incorporation | |
|---|---|---|---|---|---|
| | | $\hat{\sigma}_0^2$ | *Test result* | $\hat{\sigma}_0^2$ | *Test result* |
| Algoquin | 1.118 | 7.620 | Fail | 0.892 | Pass |
| Brussels1 | 1.120 | 5.272 | Fail | 0.611 | Pass |
| Brussels2 | 1.116 | 5.199 | Fail | 0.548 | Pass |
| Graz | 1.120 | 6.292 | Fail | 0.725 | Pass |
| Saskatoon | 1.125 | 5.315 | Fail | 0.487 | Pass |
| Tehran | 1.111 | 6.731 | Fail | 0.766 | Pass |

Table 4. Station coordinate discrepancies and their standard deviations in cm (at 95% level of confidence) when the full covariance matrix of orbits and clocks is incorporated.

| Stations | Geocentric coordinate system (ITRF) | | | | | | Topocentric coordinate system | | | 3 Dimensional discrepancy |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta x$ | $\sigma_X$ | $\Delta y$ | $\sigma_Y$ | $\Delta z$ | $\sigma_Z$ | $\Delta N$ | $\Delta E$ | $\Delta U$ | 3D |
| Algoquin | 1.81 | 1.95 | -6.00 | 3.24 | -2.55 | 3.09 | -6.26 | 0.53 | 2.51 | 6.77 |
| Brussels1 | 5.68 | 3.15 | 1.49 | 1.77 | -3.42 | 3.81 | -6.64 | 1.05 | 1.00 | 6.79 |
| Graz | 5.35 | 3.29 | 2.38 | 2.08 | -3.62 | 3.40 | -6.71 | 0.86 | 1.29 | 6.88 |
| Tehran | 2.73 | 2.64 | 3.78 | 3.06 | -1.65 | 2.60 | -4.06 | 0.24 | 2.82 | 4.95 |

the satellite coordinates must be studied (clocks are expected to be intrinsically random).

The autocovariance between $x(t)$ and $x(t+\tau)$ for stationary process $x(t)$ is (Priestley 1981):

$$C_{x_t x_{t+\tau}} = C(\tau) = E\{(x(t)-\mu)(x(t+\tau)-\mu)\} \quad (12)$$

where $\mu$ is the mean; the first moment. The autocorrelation then reads:

$$\varrho(\tau) = \frac{C(\tau)}{C(0)} \quad (13)$$

The reader should note that the autocovariance at zero lag is $C(0) = \sigma^2$ (the variance of $x$).

Let $x_1, \ldots, x_N$ be a series of a stationary process $x(t)$. Then, one can form $(N-\tau)$ pairs of observations $(x_t, x_{t+\tau})$, where the observations are separated by lag $\tau$. The autocovariance is then:

$$C(\tau) = \frac{1}{N}\sum_{t=1}^{N-\tau}(x_t - m_1)(x_{t+\tau} - m_1) \quad (14)$$

and then the autocorrelation $\varrho(\tau)$ can be computed by Eq. (13).

Now, we can implement the above theory on a time series of the satellite coordinates. To study the autocorrelation of the satellite position components, we downloaded hi-rate sp3 files from the JPL analysis center web site. The time interval between two consecutive epochs is 30s. The reason we chose this type of the published ephemeris is that it enables us to have minor lags. To detrend the data and have a stationary time-series, we fitted a polynomial to the data and removed the trend. A sample of results is illustrated in Fig. 2 below:

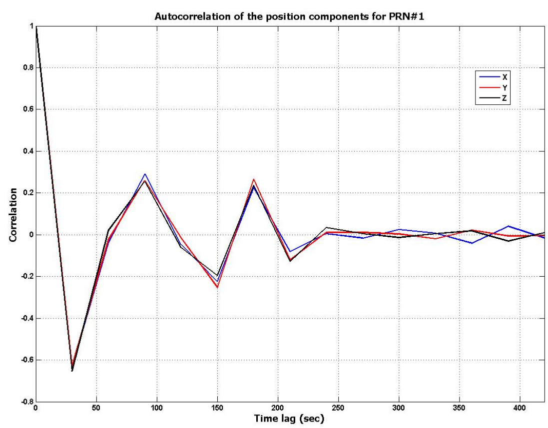


Figure 2. Autocorrelation of the position components for PRN#1. (The time interval between two consecutive epochs is 30 s).

As one can see in Fig. 2, the autocorrelation of each of the position components converges to zero after about 270 s. Therefore, if one selects the time interval between the epochs for PPP, equal to 5 minutes, the assumption of having uncorrelated satellite coordinates seems to be correct.

## 4. Conclusions

Looking at the results of the numerical studies conducted in this paper one can infer that incorporation of the satellite ephemeris uncertainties into the system of observation equations improves the station coordinates estimates. Although all the discrepancies did not decrease, all overall model tests passed after incorporation of the uncertainties.

Moreover, to find the covariance of the satellite ephemeris one can use data series of the IGS orbits and clocks. Higher accuracy classes of orbits could be used as trend for the lower accuracy ones. However, the incorporation of these covariances into the system of observation equations does not show significant improvement in the position estimates.

Studying the time correlations of the satellite coordinates reveals that five-minute time interval for observation epochs is long enough to prevent having time-correlation in the PPP observations, induced by the auto-correlations of the satellite coordinates.

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## References

Amiri-Simkooei A.R., 2007, Least-squares variance component estimation, Ph.D. dissertation, Delft University of Technology.

Barnes J.B., 2000, Real time kinematic GPS and multipath: Characterisation and improved least squares modelling. PhD dissertation, University of Newcastle upon Tyne.

Beutler G., Rothacher M., Schaer S, Springer T.A., Kouba J., Neilan R.E., 1999, The International GPS Service (IGS): an interdisciplinary service in support of Earth sciences. Advances in Space Research, 23, 4, 631–635.

Euler H.-J., Goad C.C., 1991, On optimal filtering of GPS dual frequency observations without using orbit information. Bulletin Géodésique (1991) 65: 130-143, Springer-Verlag Berlin.

Gerdan G.P., 1995, A comparison of weighting double-difference pseudorange measurements. Trans. Tasman Surveyor 1, 1, 60-66.

Hilla S., 2010, The Extended Standard Product 3 Orbit Format (SP3-c). http://igscb.jpl.nasa.gov/igscb/data/format/sp3c.txt

Hofmann-Wellenhof B., Lichtenegger H., and Wasle E., 2008, Gnss: Global Navigation Satellite Systems: Gps, Glonass, Galileo, and More. Springer, pp.127

IERS Conventions, 2010, Gérard Petit and Brian Luzum (Eds.). (IERS Technical Note ; 36) Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2010. 179 pp., ISBN 3-89888-989-6.

Jin, X.X., 1996, Theory of carrier adjusted DGPS positioning approach and some experimental results, Delft University Press, 163 pp.

Krueger E., Schueler T., Hein G.W., Martellucci A. and Blarzino G., 2004, Galileo Tropospheric Correction Approaches developed within GSTB-V1, Proc. of GNSS 2004 - European Navigation Conference, 17-19 May 2004, Rotterdam, The Netherlands.

Montenbruck O., Gill E., and Kroes R., 2005, Rapid orbit determination of LEO satellites using IGS clock and ephemeris products, GPS Solutions, DOI:10.1007/s10291-005-0131-0.

Niell A.E., 1996, Global mapping functions for the atmosphere delay at radio wavelengths. Journal of Geophysical Research 101 (B), 3227-3246.

Odijk D., 2002, Fast precise GPS positioning in the presence of ionospheric delays, Ph.D. dissertation, Delft University of Technology.

Orfanidis S., 2007, Optimum Signal Processing, 2nd edn, McGraw-Hill.

Priestley M.B., 1981, Spectral Analysis and Time Series. Academic Press, Orlando.

Schenewerk M., 2003, A brief review of basic GPS orbit interpolation strategies, DOI 10.1007/s10291-002-0036-0, GPS Solutions (2003) 6, 265-267

Shirazian M., 2006, An attempt on more realistic view of GPS precise point positioning. Proceeding of the 3rd ESA workshop on satellite navigation user equipment technologies (NAVITEC 2006), 11-13 December 2006, ESA/ESTEC, Noordwijk, The Netherlands.

Teunissen P.J.G., 2000, Testing theory; an introduction, Delft University Press, 90-93.

Tiberius C.C.J.M., 1999, The GPS data weight matrix: What are the issues? Proceedings of the National Technical Meeting and the 19th Biennal Guidance Test Symposium, San Diego, USA, January 25-27, 219-227.

Tiberius C.C.J.M. and Borre K., 1999. Probability distribution of GPS code and phase data. Zeitschrift für Vermessungswesen 124, 8, 264-273.

Wu, J.T., Wu S.C., Hajj G.A., Bertiger W.I., and Lichten S.M., 1993, Effects of antenna orientation on GPS carrier phase, Man. Geodetica 18, 91-98.

## Appendix A: THE OVERALL MODEL TEST

A test on the above-mentioned $\hat{\sigma}_0^2$, called an *overall model test*, is done to determine if the selected weight matrix is acceptable or there are blunders in the observation vector. An overview of the test is given below (Teunissen 2000):

*Hypotheses*

We require to test the null hypothesis $H_0 : E\{y\} = Ax$ versus the alternative hypothesis $H_A : E\{y\} \in \mathbb{R}^m$. This alternative hypothesis means that the redundancy $df$ equals zero and then $\hat{V} = 0$.

*Test statistic*

The test statistic is $\hat{\sigma}_0^2 = \dfrac{\hat{V}^T Q_y^{-1} \hat{V}}{df}$.

*Distribution of* $\hat{\sigma}_0^2$

$\hat{\sigma}_0^2 \sim F(df, \infty, 0)$, where $F$ denotes Fisher distribution.

*Overall model test*

reject $H_0$ if $\hat{\sigma}_0^2 > k_\alpha$ where $k_\alpha$ is the critical value at the confidence level $1-\alpha$ and accept $H_0$ otherwise.