

# Erratum to “On homological classification of pomonoids by regular weak injectivity properties of $S$ -posets”

Erratum

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**Abstract:** The original version of the article was published in Central European Journal of Mathematics, 2007, 5(1), 181–200, DOI: 10.2478/s11533-006-0036-3. Unfortunately, the original version of this article contains a mistake: in Theorem 5.2 only conditions (i) and (ii) (and not (iii)) are equivalent. We correct the theorem and its proof.

**MSC:** 06F05, 20M30

**Keywords:** Ordered monoid •  $S$ -poset • Weak injectivity

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Theorem 5.2 of [X. Zhang, V. Laan, On homological classification of pomonoids by regular weak injectivity properties of  $S$ -posets, Cent. Eur. J. Math., 2007, 5(1), 181–200, DOI: 10.2478/s11533-006-0036-3] and its proof should be the following.

## Theorem 5.2.

The following conditions are equivalent for a pomonoid  $S$ :

(i) all regularly divisible right  $S$ -posets are regularly principally weakly injective,

(ii) for every element  $s \in S$  there exist  $r, r_1, \dots, r_n, s_1, \dots, s_n, s'_1, \dots, s'_n \in S$  and left po-cancellable elements  $c_1, \dots, c_n \in S$  such that

$$\begin{aligned} c_1 s_1 &\leq r_1 s \leq c_1 s'_1 \\ c_2 s_2 &\leq r_2 s_1 \leq r_2 s'_1 \leq c_2 s'_2 \\ c_3 s_3 &\leq r_3 s_2 \leq r_3 s'_2 \leq c_3 s'_3 \\ &\dots \\ c_n s_n &\leq r_n s_{n-1} \leq r_n s'_{n-1} \leq c_n s'_n \\ s &= s s_n = s s'_n. \end{aligned} \tag{4}$$

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**Proof.** (i)  $\Rightarrow$  (ii) follows as in the original paper.

(ii)  $\Rightarrow$  (i). Assume that (ii) holds. Let  $A_S$  be a regularly divisible right  $S$ -poset,  $s \in S$ , and  $f : sS \rightarrow A$  an  $S$ -poset morphism. Then for  $s$  we have inequalities and equalities as in (4). Hence  $f(s) = f(s)s_n = f(s)s'_n$ . Using regular divisibility of  $A$ , there exists  $a_1 \in A$  such that  $f(s) = a_1c_n$ . Consequently,

$$f(s) = a_1c_ns_n \leq a_1r_ns_{n-1} \leq a_1r_ns'_{n-1} \leq a_1c_ns'_n = f(s),$$

and so  $f(s) = a_1r_ns_{n-1} = a_1r_ns'_{n-1}$ . Again, by regular divisibility of  $A$ ,  $a_1r_n = a_2c_{n-1}$  for some  $a_2 \in A$ . Thus

$$f(s) = a_2c_{n-1}s_{n-1} \leq a_2r_{n-1}s_{n-2} \leq a_2r_{n-1}s'_{n-2} \leq a_2c_{n-1}s'_{n-1} = a_1r_ns'_{n-1} = f(s)$$

and  $f(s) = a_2r_{n-1}s_{n-2} = a_2r_{n-1}s'_{n-2}$ . In this way we finally arrive at  $f(s) = a_nr_1s$  for some  $a_n \in A$ , i.e.  $f = \lambda_{a_nr_1}$ . So  $A$  is regularly principally weakly injective by Proposition 3.3.  $\square$

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We would like to thank Azam Saberi for pointing out a gap in our original proof of implication (iii)  $\Rightarrow$  (i) of Theorem 5.2.