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Reply to the comments on: "Series solution of hydromagnetic flow and heat transfer with Hall effect in a second grade fluid over a stretching sheet"

Reply

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Abstract: In this reply to comment on "Series solution of hydromagnetic flow and heat transfer with Hall effect in a second grade fluid over a stretching sheet" by R. A. Van Gorder and K. Vajravelu manuscript [R. A. Van Gorder, K. Vajravelu, Cent. Eur. J. Phys., DOI:10.2478/s11534-009-0145-2], we once again claim that the governing similarity equations of Vajravelu and Roper [K. Vajravelu, T. Roper, Int. J. Nonlin. Mech. 34, 1031 (1999)] are incorrect and our claim in [M. Ayub, H. Zaman, M. Ahmad, Cent. Eur. J. Phys. 8, 135 (2010)] is true. For the literature providing justification regarding this issue is discussed in detail.

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1. Reply to comments

The Eq. (2)

$$\left(f'\right)^2 - f\, f'' = f''' + \lambda_1 \left(2f'\, f''' - \left(f''\right)^2 - f\, f^{(iv)}\right), \quad (1)$$

presented in comments of R. A. Van Gorder and K. Vajravelu [1], is again not similar to the Eq. (9) of Vajravelu and Roper [2] which in their paper [2] is quoted as

$$\left(f'\right)^2 - f\, f'' = f'' + \lambda_1 \left(2f'\, f''' - \left(f''\right)^2 - f\, f^{(iv)}\right). \quad (2)$$

The error occurs in the first term on the right hand side, in their paper, there is f'' (f double prime) instead of f''' (f triple prime).

The momentum equation which is Eq. (3) in the comments [1] and Eq. (5) in Vajravelu and Roper [2] is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2} + \lambda\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right], \quad (3)$$

has an error on the right hand side in the second term enclosed in the square brackets, which is $\partial u/\partial y\; \partial^2 v/\partial y^2$, in our derivation it is $\partial u/\partial y\; \partial^2 u/\partial x\partial y$.

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This means that the correct form of this Eq. (3) is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}+\lambda\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right]. \tag{4}$$

For its justification a great deal of literature is available. The justification is as follows:

1. The Eq. (5) of Vajravelu and Rollins [3] is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_0 B_0^2}{\rho}u+\lambda\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right], \tag{5}$$

   and if we neglect the magnetic effects by taking $B_0=0$ then the above Eq. (5) reduces to our corrected Eq. (4). If they are claiming Eq. (3) is correct then they are wrong in paper [3] and if they are claiming Eq. (5) of [3] is correct then they are wrong in paper [2]. Both the papers [2, 3] are written by the same authors and in their own papers there is a contradiction. At one place [3] they are reporting the same resluts like us and at the other place they are contradicting the same results in [1, 2].

2. Consider Eq. (6) of Sajid *et al.* [4] (Ref. [4] has a complete derivation of the equation)

$$\begin{aligned}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+3\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right]\\ +\frac{2\alpha_2}{\rho}\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+\frac{6\left(\beta_2+\beta_3\right)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u.\end{aligned} \tag{6}$$

   If this model is required to be compatible with thermodynamics [5], then we may take $\alpha_2=-\alpha_1$. If we set in the above Eq. (6), $\alpha_2=-\alpha_1$, $\beta_2=0$, $\beta_3=0$ and $B_0=0$ then the above Eq. (6) reduces to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+3\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right]-\frac{2\alpha_1}{\rho}\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}, \tag{7}$$

   which after simplification becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right], \tag{8}$$

   which can be written as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\,\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right]. \tag{9}$$

   This equation also justifies the existence of the term $\partial u/\partial y\ \partial^2 u/\partial x\partial y$, to be correct.

3. In Hayat *et al.* [6], Eq. (2) contains the term $\partial u/\partial y\ \partial^2 u/\partial x\partial y$ and not $\partial u/\partial y\ \partial^2 v/\partial y^2$ which also justifies our claim.

4. In [7] see Eq. (6), in [8] see Eq. (5), in [9] see Eq. (10) and in [10] see Eq. (1.1) these all justify our claim.

2. Justification for the positive sign in the second term on the right-hand-side of Eq. (1) of the comments [1]

If we are correct in the above justification then we may insert the self similar solution

$$u=Bxf'\left(\eta\right),\quad v=-\left(B\nu\right)^{1/2}f\left(\eta\right),\quad \eta=\left(B/\nu\right)^{1/2}y, \tag{10}$$

in corrected Eq. (4) and one may obtain the Eq. (1) of comments [1],

$$f'^2-f\ f''=f'''+\lambda_1\left(2f'\ f'''+f''^2-f\ f^{(iv)}\right). \tag{11}$$

2.1. Complete proof

Consider the expression on right hand side (enclosed in the square brackets) of Eq. (4)

$$\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}=u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}. \tag{12}$$

Substituting the values from Eqs. $(5a-5h)$ of comments [1] in Eq. (12), we get

$$\begin{aligned}\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}&=u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\\ &=Bxf'\left(\frac{B}{\nu}\right)Bf'''+Bf'\left(\frac{B}{\nu}\right)Bxf'''+\left(\frac{B}{\nu}\right)^{1/2}Bxf''\left(\frac{B}{\nu}\right)^{1/2}Bf''+(-1)(B\nu)^{1/2}f\left(\frac{B}{\nu}\right)^{3/2}Bxf^{(iv)}\\ &=\frac{B^3x}{\nu}\left[f'f'''+f'f'''+\left(f''\right)^2-ff^{(iv)}\right]=\frac{B^3x}{\nu}\left[2f'f'''+\left(f''\right)^2-ff^{(iv)}\right]\end{aligned} \tag{13}$$

which is exactly the same result as reported by us in [11]. If we study Eq. (6) of the comments [1] there is again an error in the factor $B^3x/\sqrt{\nu}$, here in the denominator instead of $\sqrt{\nu}$, we note that only ν appears.

3. Final remarks

All this explains that the term $(f'')^2$ multiplying λ_1 in Eq. (1) must have a positive sign. All this also justifies our claim that the governing similarity equation for the boundary layer in Vajravelu and Roper [2] is incorrect. Moreover if one inspects Eq. (14) of [2] there is also an error in the second term on the left hand side, the term $v\ \partial T/\partial x$ must be replaced by $v\ \partial T/\partial y$.

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