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A comment on the analogous confinement of a neutral particle to a quantum dot via noninertial effects

Research Article

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Abstract: In this contribution, we discuss the nonrelativistic limit of the Dirac equation for a neutral particle with a permanent electric dipole moment interacting with external fields in a noninertial frame. We show a case where the geometry of the manifold can play the role of a hard-wall confining potential due to noninertial effects, and can yield bound states analogous to a confinement of the spin-half neutral particle interacting with external fields to a quantum dot described by a hard-wall confining potential [33].

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1. Introduction

Studies of noninertial effects in experiments of interferometry have discovered important quantum effects, for instance, the Sagnac effect [1, 2] and the Mashhoon effect [3]. These quantum effects have opened interesting discussions about the influence of the noninertial effects in nonrelativistic quantum systems [4–12]. New studies of noninertial effects have been made in the relativistic regime [13, 14], and in the presence of a weak gravitational field [15]. In recent years, the study of noninertial effects have been extended to the quantum dynamics of neutral particles with permanent magnetic and electric dipole moments. For instance, by considering a rotating frame, the analogue of the Aharonov-Casher effect has been studied in [16]. By considering a Fermi-Walker reference frame, the Anandan quantum phase for a neutral particle with permanent magnetic dipole moment has been discussed in [17]. Non-inertial effects of the Fermi-Walker reference frame have also been studied in the context of holonomies in curved spacetime background [18]. The relativistic Einstein-Podolsky-Rosen correlations have been studied via the Fermi-Walker transport in the cosmic string spacetime background [19].

The influence of noninertial effects has also been studied in bound states, where an interesting coupling between the angular momentum and the angular velocity of the rotating frame has been observed. This coupling is called the Page-Werner *et al.* coupling [11–13]. More studies of noninertial effects have been done in analogous systems of the Landau quantization [20]. In Ref. [21], it has been shown that the Landau-Aharonov-Casher quan-

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tization [22, 23] can be achieved in a rotating frame. In Ref. [24–26], it has been shown that the Landau-He-McKellar-Wilkens quantization [27–29] can be obtained without assuming the existence of magnetic charges. Recently, bound states for a neutral particle with permanent magnetic dipole moment analogous to a quantum dot have been obtained via the noninertial effects of the Fermi-Walker reference frame [30–32].

The aim of this paper is to discuss the quantum dynamics of a nonrelativistic neutral particle with a permanent electric dipole moment interacting with external fields in a noninertial frame, by exploring the restricted physical region of the spacetime imposed by noninertial effects. We show that the restriction of the physical region of the spacetime imposed by noninertial effects can yield bound states analogous to having a neutral particle interacting with external fields confined to a quantum dot with a hard-wall confining potential [33–37]. We show that the geometry of the spacetime plays the role of a hard-wall confining potential [33–37] due to the presence of noninertial effects. Therefore, this contribution fills a lack in the study of the influence of noninertial effects on the quantum dynamics of a neutral particle with a permanent electric dipole moment interacting with external fields.

This paper is organized as follows: in Section 2, we make a brief review of the mathematical formulation of the spinor theory in curved spacetime [38], and introduce a noninertial frame called a Fermi-Walker reference frame [39]. In the following, we discuss the analogous confinement of the neutral particle with a permanent electric dipole moment to a quantum dot with a hard-wall confining potential induced by noninertial effects; in Section 3, we present our conclusions.

2. Analogous confinement of a neutral particle to a quantum dot via noninertial effects

We begin this section by making a brief review of the mathematical formulation of the spinor theory in curved spacetime [38], and by introducing the relativistic description of a quantum dynamics of a neutral particle with permanent electric dipole moment. In the following, we discuss the nonrelativistic limit of the Dirac equation and the analogous confinement of the neutral particle to a quantum dot with a hard-wall confining potential [33–37]. Recently, the Fermi-Walker reference frame has been used in several distinct studies such as holonomies in curved spacetime background [18], relativistic Einstein-Podolsky-Rosen correlations [19], geometric quantum phases for neutral particles [17], Landau quantization for a neutral particle [24–26], and two-dimensional quantum dots [30, 31]. We consider a system with cylindrical symmetry, where we can write the line element in the form: $ds^2 = -d\mathcal{T}^2 + d\mathcal{R}^2 + \mathcal{R}^2\, d\Phi^2 + d\mathcal{Z}^2$. To study the influence of non-inertial effects on the quantum dynamics of a neutral particle, we make a coordinate transformation: $\mathcal{T} = t$; $\mathcal{R} = \rho$; $\Phi = \varphi + \omega\, t$; $\mathcal{Z} = z$, where ω is the constant angular velocity of the rotating frame. Thus, we can write the line element in the form:

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 \left(d\varphi + \omega\, dt\right)^2 + dz^2. \tag{1}$$

We can note that the line element (1) is defined in the range $0 < \rho < \frac{1}{\omega}$. It is easy to check that for values of $\rho \geq 1/\omega$, the line element (1) is not well-defined anymore. For these values of the radial coordinate, the particle is placed outside of the light-cone because its velocity is greater than the velocity of the light [40]. In this way, we have that the range $0 < \rho < \frac{1}{\omega}$ imposes a constraint where the wave function must be defined. Hence, without loss of generality, we can consider the geometry of the manifold as playing the role of a hard-wall confining potential due to noninertial effects. From now on, our focus is to discuss how this constraint of the radial coordinate can be used to confine a neutral particle in analogous way to a quantum dot with a hard-wall confining potential [33–37].

Taking into account the cylindrical symmetry of the system, we can work the Dirac spinors by using the mathematical formulation of the spinor theory in curved spacetime [38]. In a general coordinate system, spinors are defined locally by introducing a local reference frame where a spinor transforms under infinitesimal Lorentz transformations: $\psi'(x) = D(\Lambda(x))\,\psi(x)$, where $D(\Lambda(x))$ is the spinor representation of the infinitesimal Lorentz group, and $\Lambda(x)$ corresponds to the local Lorentz transformations [38]. A local reference frame can be built through a noncoordinate basis $\hat{\theta}^a = e^a{}_\mu(x)\, dx^\mu$, whose components $e^a{}_\mu(x)$ are called *Vierbein* or *tetrads* and satisfy the relation [39, 41, 42]: $g_{\mu\nu}(x) = e^a{}_\mu(x)\, e^b{}_\nu(x)\, \eta_{ab}$, where $\eta_{ab} = \mathrm{diag}(-+++)$ is the Minkowski tensor. In this notation, the indices μ, ν denote either the spacetime indices or the curvilinear coordinates, while the indices $a, b = 0, 1, 2, 3$ denote the local reference frame of the observers. The inverse of the tetrads is defined as $dx^\mu = e^\mu{}_a(x)\, \hat{\theta}^a$, where the relations $e^a{}_\mu(x)\, e^\mu{}_b(x) = \delta^a{}_b$, $e^\mu{}_a(x)\, e^a{}_\nu(x) = \delta^\mu{}_\nu$ are satisfied. A Fermi-Walker reference frame is defined by taking the components of the noncoordinate basis in the rest frame of the observers at each instant, $\hat{\theta}^0 = e^0{}_t(x)\, dt$, where the spatial components of the noncoordinate basis $\hat{\theta}^i$ do not rotate [39]. In the Fermi-Walker reference frame we can observe non-inertial effects from the action of external forces without any effects from arbitrary rotations of the local spatial axis. Following the above definition of the Fermi-Walker reference frame, we can write the tetrads and its inverse in the form [17, 24–26]:

$$
\begin{aligned}
e^{a}{}_{\mu}(x) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \omega\rho & 0 & \rho & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \\
e^{\mu}{}_{a}(x) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\omega & 0 & \frac{1}{\rho} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (2)
\end{aligned}
$$

By using the local reference frame (2), we can solve the Maurer-Cartan structure equations [42] and obtain the connections 1-form $\omega^{a}{}_{b} = \omega_{\mu}{}^{a}{}_{b}(x)\,dx^{\mu}$. In the absence of the torsion field, the Maurer-Cartan structure equations are written as: $d\hat{\theta}^{a} + \omega^{a}{}_{b} \wedge \hat{\theta}^{b} = 0$, where the operator d corresponds to the exterior derivative and the symbol $\wedge$ means the wedge product. By solving the Maurer-Cartan structure equations, the non-null components of the connecetions 1-form are $\omega_{\varphi}{}^{1}{}_{2}(x) = -\omega_{\varphi}{}^{2}{}_{1}(x) = -1$ and $\omega_{t}{}^{1}{}_{2}(x) = -\omega_{t}{}^{2}{}_{1}(x) = -\omega$.

Now, let us discuss how the noninertial effects of the Fermi-Walker reference frame can induce a field configuration which allow us to obtain either a Landau system for a neutral particle with a permanent electric dipole moment [24–26] or confine the neutral particle to a quantum dot. First of all, we consider a uniform electric field along the z axis in the rest frame of the observer, thus, we write this electric field in the form: $E^{3} = e^{3}{}_{Z}\,E^{Z} = \mathbf{E}_{0}$. In the local nonrotating reference frame of the observer or in the Fermi-Walker reference frame (2), the fields are given by [43–45]: $F^{\mu\nu}(x) = e^{\mu}{}_{a}(x)\,e^{\nu}{}_{b}(x)\,F^{ab}(x)$, where $F^{\mu\nu}$ is the electromagnetic tensor with $F^{0i} = -F^{0i} = -E^{i}$, $F^{ij} = -F^{ji} = -\epsilon^{ijk}B_{k}$. Thus, the non-null components of the electric and magnetic fields, when the local reference frames of the observers are Fermi-Walker transported, are

$$E^{z} = E^{3} = \mathbf{E}_{0}; \quad B^{\rho} = -\omega\rho\,E^{3} = -\omega\,\mathbf{E}_{0}\,\rho. \qquad (3)$$

Let us introduce the relativistic description of a quantum dynamics of a neutral particle with permanent electric dipole moment. The relativistic quantum dynamics of such a particle interacting with external magnetic and electric fields can be described by introducing a nonminimal coupling into the Dirac equation given by [46–49]

$$i\gamma^{\mu}\nabla_{\mu} \rightarrow i\gamma^{\mu}\nabla_{\mu} + i\frac{d}{2}\Sigma^{\mu\nu}\gamma^{5}F_{\mu\nu}(x), \qquad (4)$$

where d is the permanent electric dipole moment of the neutral particle, $F_{\mu\nu}$ is the electromagnetic field tensor, and ∇_{μ} represents the components of the covariant derivative of a spinor in curved spacetime which is given by $\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}(x)$. The term $\Gamma_{\mu}(x) = \frac{i}{4}\,\omega_{\mu ab}(x)\,\Sigma^{ab}$ corresponds to the spinorial connection [38, 41, 42], and $\Sigma^{ab} = \frac{i}{2}\left[\gamma^{a}, \gamma^{b}\right]$. The γ^{a} matrices are defined in the local reference frame, and correspond to the Dirac matrices given in the Minkowski spacetime [41, 49], *i.e.*,

$$
\begin{aligned}
\gamma^{5} &= \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}; \quad \gamma^{0} = \hat{\beta} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \\
\gamma^{i} &= \hat{\beta}\,\hat{\alpha}^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ -\sigma^{i} & 0 \end{pmatrix}; \quad \Sigma^{i} = \begin{pmatrix} \sigma^{i} & 0 \\ 0 & \sigma^{i} \end{pmatrix}, \qquad (5)
\end{aligned}
$$

with I being the 2×2 identity matrix, and $\vec{\Sigma}$ being the spin vector. The matrices σ^{i} correspond to the Pauli matrices and satisfy the relation $\left(\sigma^{i}\,\sigma^{j} + \sigma^{j}\,\sigma^{i}\right) = 2\,\eta^{ij}$ $(i, j, k = 1, 2, 3)$. The γ^{μ} matrices are related to the γ^{a} matrices via $\gamma^{\mu} = e^{\mu}{}_{a}(x)\,\gamma^{a}$ [41]. Taking the expressions for the connections 1-form, $\omega_{\varphi}{}^{1}{}_{2}(x) = -\omega_{\varphi}{}^{2}{}_{1}(x) = -1$ and $\omega_{t}{}^{1}{}_{2}(x) = -\omega_{t}{}^{2}{}_{1}(x) = -\omega$, we can calculate the spinorial connection $\Gamma_{\mu}(x)$, and obtain $\gamma^{\mu}\Gamma_{\mu} = \frac{\gamma^{1}}{2\rho}$ [24–26, 30, 31]. Hence, the Dirac equation in curvilinear coordinates with the interaction of the permanent electric dipole moment of the neutral particle with external fields (3) is given by

$$
\begin{aligned}
i\frac{\partial\psi}{\partial t} &= m\hat{\beta}\psi + i\omega\frac{\partial\psi}{\partial\varphi} - i\hat{\alpha}^{1}\left(\frac{\partial}{\partial\rho} + \frac{1}{2\rho}\right)\psi \\
&- i\frac{\hat{\alpha}^{2}}{\rho}\frac{\partial\psi}{\partial\varphi} - i\hat{\alpha}^{3}\frac{\partial\psi}{\partial z} + id\hat{\beta}\vec{\alpha}\cdot\vec{B}\psi + d\hat{\beta}\vec{\Sigma}\cdot\vec{E}\psi, \qquad (6)
\end{aligned}
$$

where the electric and magnetic fields in the Dirac equation (6) are given in (3). At this moment, our aim is to obtain the nonrelativistic equation of motion for a neutral particle under the influence of the noninertial effects of Fermi-Walker reference frame. The procedure in obtaining the nonrelativistic limit of the Dirac equation (6) can be given by writing first the Dirac spinors in the form $\psi = e^{-imt}\left(\phi\ \chi\right)^{T}$, with ϕ and χ being two-spinors, and after considering ϕ as being the “large” component and χ as being the “small” component [49]. Thus, substituting this solution into the Dirac equation (6), we obtain two coupled equation of ϕ and χ. After some calculations to decouple the equation for ϕ and χ, the Schrödinger-Pauli equation for a neutral particle with a permanent electric dipole moment interacting with external electric and magnetic fields in the Fermi-Walker reference frame is [24–26]

$$
\begin{aligned}
i\frac{\partial\phi}{\partial t} \approx & -\frac{1}{2m}\left[\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}\right]\phi \\
& + \frac{i}{2m}\frac{\sigma^3}{\rho^2}\frac{\partial\phi}{\partial\varphi} - i\frac{d\omega \mathrm{E}_0}{m}\sigma^3\frac{\partial\phi}{\partial\varphi} + \frac{1}{8m\rho^2}\phi \\
& + \frac{d\omega\mathrm{E}_0}{2m}\phi + \frac{d^2\omega^2\mathrm{E}_0^2}{2m}\rho^2\phi + i\omega\frac{\partial\phi}{\partial\varphi} + d\mathrm{E}_0\,\sigma^3\phi.
\end{aligned} \tag{7}
$$

We can see in the equation (7) that ϕ is an eigenfunction of σ^3, whose eigenvalues are $s = \pm 1$. Thus, we can write $\sigma^3\phi_s = \pm\phi_s = s\phi_s$. Moreover, we can observe that the Hamiltonian given in the right-hand-side of Eq. (7) commutes with the operators $\hat{p}_z = -i\frac{\partial}{\partial z}$, and $\hat{J}_z = -i\frac{\partial}{\partial\varphi}$ [50]. Hence, the solutions of the equation (7) are given in the form: $\phi_s = e^{-i\mathcal{E}t}\,e^{i\left(l+\frac{1}{2}\right)\varphi}\,e^{ikz}\,R_s(\rho)$[1], where $l = 0, \pm 1, \pm 2, \ldots$ and k is a constant. Substituting this solution into the Schrödinger-Pauli equation (7), we obtain the radial equation:

$$
\left[\frac{d^2}{d\rho^2}+\frac{1}{\rho}\frac{d}{d\rho}-\frac{\zeta_s^2}{\rho^2}-\delta^2\rho^2-\beta_s\right]R_s(\rho)=0, \tag{8}
$$

where we have defined in (8) the parameters: $\zeta_s = l + \frac{1}{2}(1-s)$, $\delta = d\omega\mathrm{E}_0$, and $\beta_s = 2m\left[\mathcal{E}+\omega(l+1/2) - sd\mathrm{E}_0 - s\frac{\delta}{m}\zeta_s - \frac{\delta}{m}\right]$. We wish a solution of the radial equation (8) that is regular at the origin, thus, the solution can be given in the form:

$$
R_s(\xi) = e^{-\frac{\xi}{2}}\,\xi^{\frac{|\zeta_s|}{2}}\;{}_1F_1\left(\frac{|\zeta_s|}{2}+\frac{1}{2}-\frac{\beta_s}{4\delta}, |\zeta_s|+1, \xi\right), \tag{9}
$$

where $\xi = \delta\rho^2$, and ${}_1F_1\left(\frac{|\zeta_s|}{2}+\frac{1}{2}-\frac{\beta_s}{4\delta}, |\zeta_s|+1, \xi\right)$ is the confluent hypergeometric function or the Kummer equation [51]. It is well-known in the literature [51, 52] that the radial part of the wave function becomes finite everywhere when the parameter $\frac{|\zeta_s|}{2}+\frac{1}{2}-\frac{\beta_s}{4\delta}$ of the confluent hypergeometric function is equal to a non-positive integer number, making the confluent hypergeometric series to be a polynomial of degree n. By observing that the line element (1) is valid only for values of the radial coordinate inside the range $0 < \rho < \frac{1}{\omega}$, we have that the condition of making the confluent hypergeometric series to be a polynomial of degree n means that the wave function is defined both in the physical region of the spacetime $0 < \rho < \frac{1}{\omega}$ and in the non-physical region $\rho \geq 1/\omega$. Hence, we cannot just impose the condition where the confluent hypergeometric series becomes a polynomial in order to normalize the wave function of the neutral particle.

In previous work [24–26], we discussed a way of achieving the Landau-He-McKellar-Wilkens quantization via noninertial effects by imposing a condition of the induced fields given by $dE_0 \ll \omega$, and making the confluent hypergeometric series to be a polynomial of degree n. With this condition on the induced fields ($dE_0 \ll \omega$), we have that the amplitude of probability becomes very small for values $\rho \geq 1/\omega$ because the parameter $\xi = \delta\rho^2 \ll 1$ when $\rho \to \frac{1}{\omega}$. Thus, without loss of generality, we can consider the wave function being normalized in the range $0 < \rho < \frac{1}{\omega}$ (since $R_s(\xi) \approx 0$ when $\rho \to \frac{1}{\omega}$). A similar analysis has been made in [30–32] to achieve discrete energy levels of a neutral particle with permanent magnetic dipole moment confined to a parabolic potential analogous to the Tan-Inkson potential for a quantum dot [53–55], and in [21] to achieve the Landau-Aharonov-Casher quantization in a rotating frame. Here, we bring the discussion of obtaining bounds states for a neutral particle in the present noninertial system without imposing the above condition on the induced fields.

In this way, in order to obtain a normalized wave function inside the physical region of the spacetime $0 < \rho < \frac{1}{\omega}$, we first consider the radial wave function as vanishing at $\rho \to 1/\omega$. Note that this boundary condition corresponds to having the geometry of the manifold playing the role of a hard-wall confining potential due to the influence of noninertial effects. In this case, we have for a fixed radius $\rho_0 = 1/\omega$ that

$$
R_s\left(\xi_0 = \delta\,\rho_0^2\right) = 0. \tag{10}
$$

Our next step is to assume that the intensity of the electric field E_0 is given in such a way that the parameter $\delta = dE_0\omega$ can be considered small. In this way, by taking a fixed value for the parameter $b = |\zeta_s|+1$ of the confluent hypergeometric function and a fixed radius $\rho_0 = 1/\omega$, we can consider the parameter $a = \frac{|\zeta_s|}{2}+\frac{1}{2}-\frac{\beta_s}{4\delta}$ of the confluent hypergeometric function being large. In this way, we can write the Kummer function of first kind in the form [51]:

$$
\begin{aligned}
{}_1F_1\left(a, b, \xi_0 = \omega dE_0\,\rho_0^2\right) \approx & \frac{\Gamma(b)}{\sqrt{\pi}}\,e^{\frac{\xi_0}{2}}\left(\frac{b\xi_0}{2}-a\xi_0\right)^{\frac{1-b}{2}} \\
& \times \cos\left(\sqrt{2b\xi_0 - 4a\xi_0} - \frac{b\pi}{2}+\frac{\pi}{4}\right),
\end{aligned} \tag{11}
$$

where $\Gamma(b)$ is the gamma function. Substituting (11) into (9), and by applying the boundary condition (10) (with

[1] *It has been shown in Ref. [50] that the z-component of the total angular momentum in cylindrical coordinates is given by $\hat{J}_z = -i\partial_\varphi$, where the eigenvalues are $j = l \pm \frac{1}{2} = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$.*

$\beta_s = 2m\left[\mathcal{E} + \omega\left(l + 1/2\right) - sd\mathrm{E}_0 - s\frac{\delta}{m}\zeta_s - \frac{\delta}{m}\right]$), we obtain a discrete energy spectrum given by

$$\mathcal{E}_{n,\,l} \approx \frac{1}{2m\rho_0^2}\left[n\pi + \zeta_s\frac{\pi}{2} + \frac{3\pi}{4}\right]^2 + \frac{d\omega E_0}{m}\left[s\,\zeta_s + 1\right] + s\,dE_0 - \omega\left[l + \frac{1}{2}\right]. \tag{12}$$

The energy levels (12) corresponds to the bound states of a nonrelativistic neutral particle with a permanent electric dipole moment interacting with external fields confined to the region of the spacetime $0 < \rho < 1/\omega\eta$. Comparing the result (12) with the studies of the confinement of particles to a quantum dot made in [33–37], we have in this case that the geometry of the spacetime plays the role of a hard-wall confining potential due to the presence of noninertial effects that restrict the physical region of the spacetime where the wave function can be defined. Moreover, we have that the energy levels (12) are proportional to n^2 in contrast to recent studies of the analogous confinement of a neutral particle to a quantum dot [30, 31] (given by imposing a condition on the induced fields $\mu\lambda \ll \omega$), where the energy levels are proportional to n in an analogous way to the Tan-Inkson model for a quantum dot [53–55]. Hence, the spectrum of energy (12) is analogous to having a neutral particle with a permanent electric dipole moment confined to a quantum dot described by a hard-wall confining potential [33–37].

Finally, we can also see in the expression (12) the coupling between the angular velocity ω and the quantum number l induced by noninertial effects, which is called in the literature as the Page-Werner *et al.* term [11–13].

3. Conclusions

In this work, we have studied the nonrelativistic quantum dynamics of a neutral particle with a permanent electric dipole moment interacting with a field configuration induced by noninertial effects. We have seen that the geometry of the spacetime can play the role of a confining potential due to noninertial effects yielding bound states analogous to having a neutral particle confined to a quantum dot with a hard-wall confining potential as in Refs. [33–37]. Further, by assuming that the parameter $\delta = dE_0\omega$ is small, we have shown that the spectrum of energy is proportional to n^2 in contrast to previous studies of the analogous confinement of a neutral particle with a permanent magnetic dipole moment to a quantum dot induced by noninertial effects [30, 31], where the energy levels are proportional to n in an analogous way to the Tan-Inkson model for a quantum dot [53–55]. Moreover, we have also obtained the Page-Werner *et al.* term [11–13], which corresponds to the coupling between the quantum number l and the angular velocity ω.

We would like to add a comment on the confinement of a neutral particle to a quantum dot. By considering the presence of topological defects [56–58], we have that the presence of a topological defect related to a torsion, for instance a screw dislocation, can modify the electromagnetic field in the rest frame of the observers [58]. Thus, it should be interesting to study the influence of torsion, for instance a screw dislocation or a edge dislocation [56, 57], on the field configuration induced by the noninertial effects of the Fermi-Walker reference frame, and on the bound states. Since the presence of a torsion background can modify the electromagnetic field, one can expect new contributions to the energy levels of bound states.

Another interesting point of discussion should be the influence of noninertial effects on persistent currents. Persistent currents arise from the dependence of the energy levels of bound states on geometric quantum phases. In recent decades, persistent currents have been studied for spinless quantum particles confined to a quantum ring [59], two-dimensional quantum rings and quantum dots [60] due to the presence of the Aharonov-Bohm quantum flux. Other studies of persistent currents have been made based on the Berry phase [61, 62], the Aharonov-Anandan quantum phase [63, 64], and the Aharonov-Casher geometric phase [65–68]. Therefore, based on the Sagnac effect [1, 2], the Mashhoon effect [3] and the analogue of the Aharonov-Casher effect obtained in noninertial frames [16, 17], geometric phases induced by noninertial effects can yield new contributions to persistent currents, and it should also be interesting to study the arising of persistent currents in noninertial systems.

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