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Generalized model for the nonlinear dynamics of plasma waves driven unstable by energetic ions near the stability threshold

Research Article

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Abstract: In this paper, an attempt to develop the original single mode theory describing the nonlinear dynamics of a linearly unstable plasma wave, excited by the resonant interaction with energetic ions near the stability threshold to the case of n interacting plasma modes has been made. The effects of an energetic ion source and classical collisional processes represented by the Krook, diffusion and dynamical friction (drag) collision operators are included in the model. For numerical purposes, the problem has been reduced to ten nonlinearly coupled integro-differential equations. In comparison to the previous papers, the system revealed similar (the steady-state, oscillation, and blow-up solutions), as well as quite new types of the amplitudes behaviour, i.e. different levels of competition between the modes.

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1. Introduction

The nonlinear evolution of kinetic instabilities driven by high-energy ions is a phenomenon of fundamental importance in fusion plasma physics. The existence of energetic ions in fusion plasmas, such as 3.51 Mev α-particles, may induce wave micro-instabilities, which in turn may cause large losses of α-particles, that consequently has an important impact on plasma parameters, such as plasma stability, confinement and ignition condition. An investigation of these instabilities is necessary to determine the nonlinear dynamics of waves and final saturation levels [1, 2].

The theory describing the nonlinear dynamics of a single unstable plasma wave driven resonantly by high-energy ions just above the stability threshold has been proposed by H. Berk, B. Breizman et al. [3], and was developed in a number of papers [4–6]. The Berk-Breizman theory has been also identified to some extent in tokamak experiments. In particular, it was applied to study the nonlinear evolution of Toroidal Alfvén Eigenmodes (TAE's) appearing in the Ion Cyclotron Resonance Heating (ICRH) and Neutral Beam Injection (NBI) heating experiments [7–11].

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In the model, the fast ion distribution function $F(t, x, \upsilon)$ is given by a non-equilibrium bump-on-tail model. The instability mechanism is due to particles resonantly interacting with a weakly unstable plasma mode for which the linear growth rate $\gamma = \gamma_L - \gamma_d$ (where γ_L is the particle kinetic drive for an instability, and γ_d is the linear damping due to the background plasma) is much less than the real part of the mode frequency ω. The instability arises when $\gamma_L > \gamma_d$. The particular emphasis is given to the case $\gamma_L/\gamma_d \sim 1$ and $\gamma_L - \gamma_d \ll \gamma_L$, which means that the plasma mode is excited just above the linear stability threshold. Furthermore, it has been assumed that the relevant nonlinear time scale $\tau \sim 1/\gamma$ is shorter than a typical bounce time of trapped particles in a wave potential ω_B^{-1}, where ω_B denotes the bounce frequency. It has been shown that the evolution of the mode is determined by an interplay between wave electric field, that tends to flatten the distribution function of energetic ions, and the relaxation processes restoring the distribution function due to the collisions, modelled via the collision operator

$$\left(\frac{\partial F}{\partial t}\right)_{coll} = -\nu F + \alpha^2 \frac{\partial F}{\partial \upsilon} + \beta^3 \frac{\partial^2 F}{\partial \upsilon^2}. \qquad (1)$$

In papers [3–5], it was shown that the nonlinear evolution of the wave amplitude strongly depends on values of the collision operator parameters ν (Krook) and β (diffusion), and four main regimes of the amplitude behaviour have been observed: the steady-state solution, a periodic oscillation, the chaotic and explosive regimes. However, when the dominant collision processes were modelled via the drag collision operator (dynamical friction effects), characterized by the parameter α, the explosive behaviour of the wave amplitude was the only possible solution [6].

The theory proposed by Berk and Breizman describes only the case of a single mode amplitude evolution, and in fact, it does not cover a more realistic case, when the resonance excitation of many plasma waves takes place. As a first step to this case, we considered in paper [12] the nonlinear dynamics of two interacting plasma modes driven by energetic ions at the linear stability threshold. According to the assumptions of the Berk-Breizman model, we derived the system of two coupling nonlinear integro-differential equations. In contrast to a single mode case, we assumed the existence of two plasma eigenmodes with the phase velocities ω_i/k_i (where $i = 1, 2$) lying on the positive slope of the fast ion velocity distribution function. The numerical examination showed new types of the amplitudes behaviour: the mode competition for survival, amplitude oscillation in the same and in the opposite phase, and the saturation of the modes at the same level.

In this paper, the two-mode model has been developed to the case of n linearly unstable plasma modes driven resonantly by fast ions in the vicinity of the stability threshold. We treat the problem as one-dimensional and assume that the phase velocities of the modes ω_i/k_i are on the positive slope of the bump-on-tail velocity distribution function. Following the Berk-Breizman theory we apply a perturbation analysis and derive the system of nonlinear integro-differential equations describing the dynamics of n independent but lying close to each other plasma modes. One should emphasize, however, that the number of modes should be relatively small. Otherwise the quasi-linear approach has to be taken into consideration, which actually is not a goal of the present work, and cannot be simply done in the framework of Berk-Breizman theory. For this reason, we have limited our consideration to the case of ten interacting plasma modes, what in our opinion should be a sufficient number to obtain new interesting results. Particularly, we are interested in - how the amplitudes behaviour changes, due to the increase in the number of plasma modes.

The present paper is organized as follows: In Sec. 2, we derive the integro-differential equations for the mode amplitudes driven resonantly by energetic ions in the vicinity of the stability threshold. In Sec. 3, a numerical examination of different nonlinear scenarios described by the derived set of equations has been presented. Finally, in Sec. 4 we summarize the present work.

2. Model equations

In accordance with [3–6, 12], we use the perturbative analysis to solve the kinetic equation describing the evolution of the fast ion velocity distribution function $F(t, x, \upsilon)$ in the presence of an electric field $E(t, x)$,

$$\frac{\partial F}{\partial t} + \upsilon\frac{\partial F}{\partial x} + \frac{q_e}{m}E(t, x)\frac{\partial F}{\partial \upsilon} = S(\upsilon) - \nu F + \alpha^2\frac{\partial F}{\partial \upsilon} + \beta^3\frac{\partial^2 F}{\partial \upsilon^2}. \qquad (2)$$

where q_e and m are particle charge and mass, respectively, and $S(\upsilon)$ is a constant source of fast ions determined by

$$S(\upsilon) = \nu F_0 - \alpha^2\frac{\partial F_0}{\partial \upsilon} - \beta^3\frac{\partial^2 F_0}{\partial \upsilon^2}. \qquad (3)$$

In our consideration, we have taken into account the Krook, the drag and the diffusion collision operators, characterized by the parameters ν, α and β, respectively. We represent the total electric field $E(t, x)$ as a sum of n discrete but laying close to each other background plasma eigenmodes with phase velocities ω_i/k_i (ω_i and k_i are the mode frequencies and the wave numbers, respectively)

$$E(x, t) = \sum_{i=1}^{n}\left[\hat{A}_i(t)\, e^{i(k_i x - \omega_i t)} + c.c.\right], \qquad (4)$$

where $\hat{A}_i(t)$ is a slowly varying complex wave amplitude given by

$$\hat{A}_i(t) = \frac{1}{2}\hat{E}_i(t)\, e^{i\varphi_i(t)}, \tag{5}$$

satisfying the condition

$$\left|\frac{1}{\hat{A}_i}\frac{d\hat{A}_i}{dt}\right| \ll \omega_i \ . \tag{6}$$

Here $\varphi_i(t)$ and $\hat{E}_i(t)$ are slowly varying phases and real amplitudes of the particular plasma modes. In the spirit of the Berk-Breizman theory, we represent the fast ion distribution function $F(t,x,\upsilon)$ as a Fourier series

$$\begin{aligned} F(t,x,\upsilon) =& F_0(\upsilon) + f_0(t,\upsilon) + \\ &+ \sum_{i=1}^{n}\left[f_i(t,\upsilon)e^{i\psi_i} + g_i(t,\upsilon)e^{2i\psi_i} + ... + c.c.\right] + \\ &+ \sum_{\substack{i=1\\ i\neq k}}^{n}\sum_{k=1}^{n} h^-_{ik}(t,\upsilon)e^{i(\psi_i-\psi_k)} + \\ &+ \sum_{\substack{i=1\\ i\neq k}}^{n}\sum_{k=1}^{n} h^+_{ik}(t,\upsilon)e^{i(\psi_i+\psi_k)}, \end{aligned} \tag{7}$$

where $\psi_i = k_i x - \omega_i t$, and h^+ and h^- correspond to the coupling between modes. Note, that for $i > k$, $h^+_{ik} = h^{+*}_{ki}$ and $h^-_{ik} = h^{-*}_{ki}$. In order to close our set of equations we have to determine the evolution equation of the electric field amplitudes. For the electrostatic plasma waves, the Maxwell equations give

$$\frac{\partial \hat{A}_i}{\partial t} + \frac{q_e}{\varepsilon_0}\int_{-\infty}^{\infty}\upsilon\, f_i(t,\upsilon)\, d\upsilon + \gamma_d \hat{A}_i = 0, \tag{8}$$

which describe the time evolution of the wave amplitudes due to the interaction with fast ions and in the presence of the damping due to the background plasma. Combining now Eqs. (2) and (7) and separating with respect to different Fourier harmonics, we obtain

$$\begin{aligned} &\frac{\partial f_0}{\partial t} - \beta^3\frac{\partial^2 f_0}{\partial v^2} - \alpha^2\frac{\partial f_0}{\partial v} + \nu f_0 = \\ &-\frac{q_e}{m}\left[\sum_{i=1}^{n}\left(\hat{A}_i\frac{\partial f_i^*}{\partial v} + \hat{A}_i^*\frac{\partial f_i}{\partial v}\right)\right], \\ &\frac{\partial f_i}{\partial t} + i(k_i v - \omega_i)f_i - \beta^3\frac{\partial^2 f_i}{\partial v^2} - \alpha^2\frac{\partial f_i}{\partial v} + \nu f_i = \\ &-\frac{q_e}{m}\left[\hat{A}_i\left(\frac{\partial F_0}{\partial v} + \frac{\partial f_0}{\partial v}\right) + \sum_{\substack{k=1\\ k\neq i}}^{n}\hat{A}_k\frac{\partial h^-_{ik}}{\partial v}\right], \\ &\frac{\partial h^-_{ik}}{\partial t} + i\left[\upsilon(k_i - k_k) - (\omega_i - \omega_k)\right]h^-_{ik} - \beta^3\frac{\partial^2 h^-_{ik}}{\partial \upsilon^2} - \\ &\alpha^2\frac{\partial h^-_{ik}}{\partial \upsilon} + \nu h^-_{ik} = -\frac{q_e}{m}\left[\hat{A}_i\frac{\partial f_k^*}{\partial \upsilon} + \hat{A}_k^*\frac{\partial f_i}{\partial \upsilon}\right]. \end{aligned} \tag{9}$$

Due to the model assumptions terms h^+_{ik}, g_i, and higher Fourier amplitudes do not contribute to the final result, therefore the equations for them have been omitted. Solving iteratively Eqs. (9) (at the assumption $F_0 \gg f_i \gg f_0, h^-_{ik}$) for f_i we can calculate the integrals $\int \upsilon\, f_i(t,\upsilon)\, d\upsilon$ of Eq. (8), which reduces to the following form

$$\begin{aligned} \frac{\partial \hat{A}_i}{\partial t} =& \gamma\hat{A}_i - \frac{\gamma_L}{2}\int_{t/2}^{t}d\tau\int_{t-\tau}^{\tau}d\sigma\Bigg[(t-\tau)^2\cdot\Bigg((q_e k_i/m)^2\hat{A}_i(\tau)\hat{A}_i(\sigma)\hat{A}_i^*(\tau - t + \sigma) \\ &+ \hat{A}_i(\tau)\sum_{\substack{k=1\\ k\neq i}}^{n}(q_e k_k/m)^2\hat{A}_k(\sigma)\hat{A}_k^*(\tau - t + \sigma)\cdot e^{-\omega_i(t-\tau)\left(\frac{\Delta k_{ik}}{k_i} - \frac{\Delta\omega_{ik}}{\omega_i}\right)}\Bigg) \\ &+ \hat{A}_i(\sigma)\sum_{\substack{k=1\\ k\neq i}}^{n}(q_e k_k/m)^2\hat{A}_k(\tau)\hat{A}_k^*(\tau - t + \sigma)\cdot e^{-\omega_i(2t-\tau-\sigma)\left(\frac{\Delta k_{ik}}{k_i} - \frac{\Delta\omega_{ik}}{\omega_i}\right)} \\ &\cdot(t-\tau)\left((t-\tau) + \frac{\Delta k_{ik}}{k_i}(t-\sigma)\right)\Bigg]e^{-\nu(2t-\tau-\sigma)-\beta^3(t-\tau)^2\left(\frac{2}{3}(t-\tau)+\tau-\sigma\right)+i\alpha^2(t-\tau)(t-\sigma)}. \end{aligned} \tag{10}$$

In deriving Eq. (10), we have assumed the overlapping of the resonance areas of the neighboring modes, at the condition $\left|\frac{\Delta k_{ik}}{k_i}\right|, \left|\frac{\Delta \omega_{ik}}{\omega_i}\right| \ll 1$ (now indices i, k involve only the neighboring modes). For this reason, terms $\hat{A}_1\hat{A}_3\hat{A}_3, \hat{A}_1\hat{A}_4\hat{A}_4, \ldots, \hat{A}_2\hat{A}_4\hat{A}_4$, etc., have been neglected in Eq. (10). Such terms may only appear when the phase shifts between modes are assumed to be small or, in other words, when the phase velocities of the first mode ω_1/k_1 and the n-mode ω_n/k_n are very close. By changing our variables to $\eta = t - \tau$, $\chi = t - \eta - \sigma$ and rescaling them according to $t \to \gamma t$, $\hat{A}_i \to [(q_e k_i/m)\hat{A}_i/\gamma^2](\gamma_L/\gamma)^{1/2}$, $\nu \to \nu/\gamma$, $\beta^3 \to \beta^3/\gamma^3$, $\alpha^2 \to \alpha^2/\gamma^2$, we obtain the final form of the dimensionless equations for the time evolution of the mode amplitudes

$$
\begin{aligned}
\frac{\partial \hat{A}_i}{\partial t} = \hat{A}_i - \frac{1}{2}\int_0^{t/2} d\eta \int_0^{t-2\eta} d\chi \Bigg[\eta^2 \cdot \Bigg(& \hat{A}_i(t-\eta)\hat{A}_i(t-\eta-\chi)\hat{A}_i^*(t-2\eta-\chi) \\
& + \hat{A}_i(t-\eta) \sum_{\substack{k=i-1 \\ 0<k<n}}^{k=i+1} \hat{A}_k(t-\eta-\chi)\hat{A}_k^*(t-2\eta-\chi) \cdot e^{-\omega_i \eta \left(\frac{\Delta k_{ik}}{k_i} - \frac{\Delta \omega_{ik}}{\omega_i} \right)} \Bigg) \\
& + \hat{A}_i(t-\eta-\chi) \sum_{\substack{k=i-1 \\ 0<k<n}}^{k=i+1} \hat{A}_k(t-\eta)\hat{A}_k^*(t-2\eta-\chi) \cdot e^{-\omega_i (2\eta-\chi) \left(\frac{\Delta k_{ik}}{k_i} - \frac{\Delta \omega_{ik}}{\omega_i} \right)} \eta \left(\eta + \frac{\Delta k_{ik}}{k_i}(\eta+\chi) \right) \Bigg] \cdot e^{-\nu(2\eta+\chi) - \beta^3 \eta^2 \left(\frac{2}{3}\eta + \chi \right) + i\alpha^2 \eta(\eta+\chi)}.
\end{aligned}
\tag{11}
$$

Now the set of Eqs. (11) depends only on the collision parameters ν, α, β, the phase shifts between modes, and the initial values of the mode amplitudes. In fact, the system of Eqs. (11) describes the evolution of n complex amplitudes $\hat{A}_i$ and, therefore, it contains n equations for the real amplitudes $\hat{E}_i$ and n equations for the real phases φ_i, where $i = 1, 2..., n$.

3. Numerical results

The set of integro-differential Eqs. (11) describes the nonlinear evolution of n coupled plasma eigenmodes $\hat{A}_i$ driven resonantly by high-energy ions in a plasma. In order to numerically solve the given set of equations, the C++ Language has been used. The derivatives in Eqs. (11) have been approximated by the finite – difference methods. For integrals, however, the numerical integration technique that uses the Newton-Cotes formulae (also called quadrature formulae) has been applied. The time step in a numerical calculation is $h = 0.05$, what is a sufficiently small value, due to the fact that even for a smaller h the results remain the same. It is noteworthy, that equivalent solutions may be obtained using other computing software, where even more accurate numerical techniques are used for solving such types of nonlinear equations, e.g. the Runge – Kutta method. Nevertheless, a simulation time is much longer than in the case of our algorithm.

For numerical purposes, we have considered the case $n = 10$ modes. In addition, we have assumed that each mode has the same frequency ω, but different wave number k_i.

In the absence of the drag collision operator, $\alpha = 0$, and for a relatively large value of the Krook (ν) operator and small phase shifts between neighbouring modes, the competition for survival has been observed, see Fig. 1. It seems interesting, that when the phase shifts between modes are small enough, half of the number of the wave amplitudes reaches the same saturation level, while the others go to zero values. The same situation is observed in Fig. 2, where the dominant collision processes were modelled via the diffusion operator, described by the parameter β. This fact may suggest, that both the Krook and the diffusion operators give the very similar behaviour of the mode amplitudes. For decreasing values of ν (or β) the competition between modes vanishes and the amplitudes oscillate: first more or less periodically, see Fig. 3, and for even smaller values of ν chaotically (Fig. 4). In Fig. 5 we see that for a sufficiently small value of ν the mode amplitudes go to infinity in finite time. The "blow-up" solution occurs. According to paper [13], where the explosive nature of the Berk-Breizman model was considered, we claim that the "blow-up" behaviour, as well as the chaotic regime are nonphysical phenomena, and they result from the breakdown of the model in the case of small values of the collision operators, i.e. the Krook and diffusion operators.

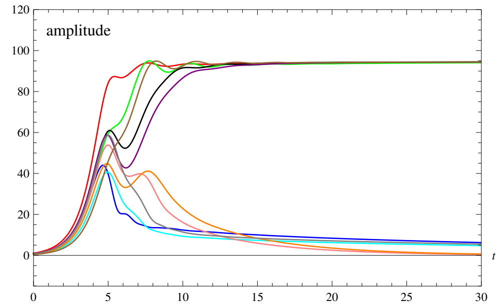

Figure 1. A mode competition for survival of Eqs. (11) for $\nu = 6$, $\beta = 0$, $\alpha = 0$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

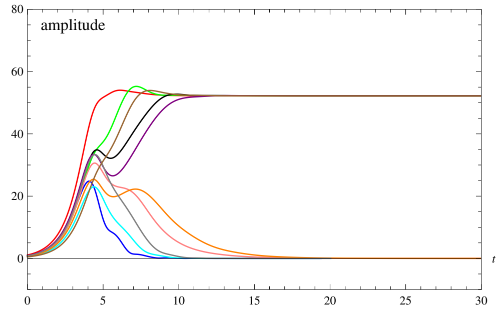

Figure 2. A mode competition for survival of Eqs. (11) for $\nu = 0$, $\beta = 6$, $\alpha = 0$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

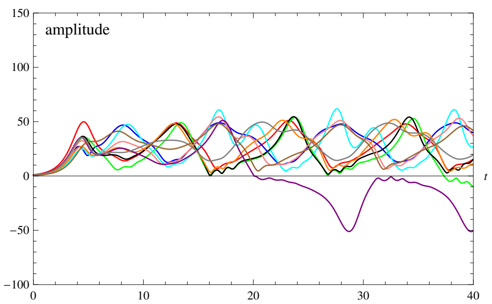

Figure 3. Oscillating regime of Eqs. (11) for $\nu = 4.2$, $\beta = 0$, $\alpha = 0$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

An inclusion of the dynamical friction effect, first considered in [6], introduces an oscillatory dependence to the integrals of Eqs. (11), and consequently the integrals may easily change sign. From a physical point of view, the presence of the dynamical friction effect leads to a continuous process of an exchange of fast ions in the resonance region. The fast ions with velocities slightly greater than the phase velocity come into the resonance region, while the ions with velocities smaller than the phase velocity leave this region. Consequently, in the bump-on-

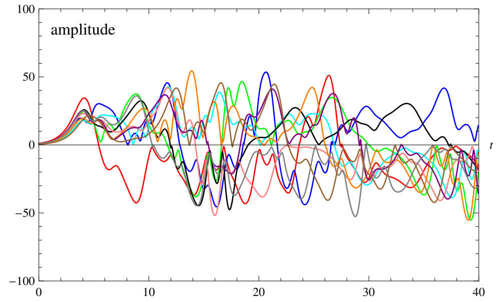

Figure 4. Chaotic regime of Eqs. (11) for $\nu = 3.2$, $\beta = 0$, $\alpha = 0$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

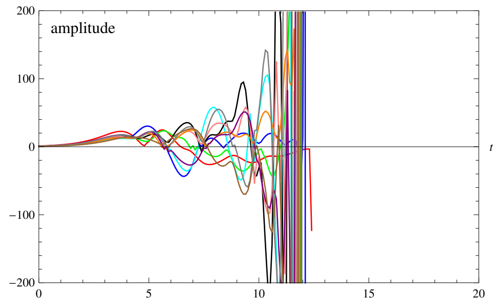

Figure 5. The blow-up behaviour of Eqs. (11) for $\nu = 2.3$, $\beta = 0$, $\alpha = 0$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

tail problem, the particle energy is constantly transferred to the waves. For this reason, in the case of a pure drag ($\nu = 0$ and $\beta = 0$) the steady state solution never occurs and the amplitudes behaviour is indeed explosive. The same situation is observed in the case of a non-zero and relatively small, in comparison to α, values of ν or β, see Fig. 6. In this case, the relaxation processes connected with the particle diffusion in the velocity space or with the Krook-type collisions are too weak to stabilize the system, which breaks into a chaotic oscillation and finally explodes. It should be emphasized, that the nature of the explosive evolution of the mode amplitudes is different in the case of the dynamical friction effect and in the case of relaxation processes, modelled via the Krook or the diffusion collision operator. In Fig. 7, it has been shown that if the Krook (or the diffusion operator) becomes the dominant component of the collision operator, the competition for survival between modes takes place again, but with a difference, that some mode amplitudes are assembling in different saturation levels, while the others go to the zero values. Such behaviour is a new feature of the model, which could not be observed in the two-mode case, because of an insufficient number of plasma modes.

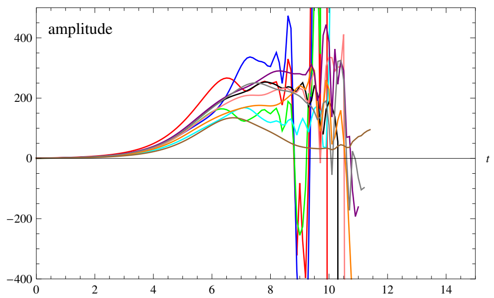

Figure 6. The blow-up behaviour of Eqs. (11) for $\nu = 6$, $\beta = 0$ and $\alpha = 7$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

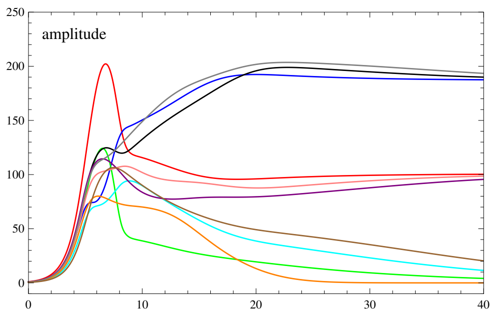

Figure 7. The amplitude behaviour of Eqs. (11) for $\nu = 6$, $\beta = 0$ and $\alpha = 5$ and the same value of the phase shifts between neighboring modes $\Delta k_{ik} = 0.001$.

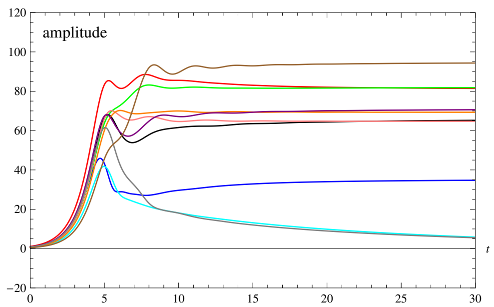

Figure 8. The amplitude behaviour of Eqs. (11) for $\nu = 6$, $\beta = 0$ and $\alpha = 0$ and the different value of the phase shifts between neighboring modes: $\Delta k_{12} = 0.001, \Delta k_{23} = 0.002, \Delta k_{34} = 0.0015, \Delta k_{45} = 0.0013, \Delta k_{56} = 0.0025, \Delta k_{67} = 0.003, \Delta k_{78} 0.0029, \Delta k_{89} = 0.0018, \Delta k_{910} = 0.0011$.

In contrast to the previous results, we consider now the case of different phase shifts between neighboring modes. For a fixed value of the Krook or the diffusion operator, and in the absence of the dynamical friction effects, we have observed in Fig. 8, that some amplitudes reach different steady states, while the others go to zero. For larger values of the phase shifts, the mode amplitudes start to oscillate in the same or in the opposite phase, see Fig. 9. An increase of the value of the phase shifts parameters weakens the interaction strength between the modes, and consequently each mode behaves practically independently, Fig. 10. An inclusion of the drag collision operator to the case of different values of the phase shifts between modes presents the effect as given in Fig. 11. One can observe that a nonzero value of α causes increase in the splits between particular saturation levels. In conclusion, such types of solution (presented in Fig. 11 and Fig. 10) are a result of different phase shifts between modes, and mainly depend on values of Δk_{ik}, as well as collision parameters, i.e. $A(t) = f(\nu, \alpha, \beta, \Delta k_{ik})$. For example, in the case of a constant value of Δk_{ik}, we see in Fig. 7, that Eqs. (11) have three main solutions. In the case $\Delta k_{12} \neq \Delta k_{23} \neq \Delta k_{34}$ ... (see Fig. 10 and Fig. 11), however, we observe ten different solutions, because of a larger number of free parameters Δk_{ik}.âĂIJ

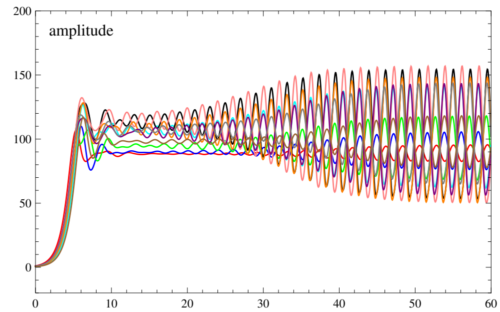

Figure 9. Oscillating regime of Eqs. (11) for $\nu = 6$, $\beta = 0$ and $\alpha = 0$ and the different value of the phase shifts between neighboring modes: $\Delta k_{12} = 0.005, \Delta k_{23} = 0.004, \Delta k_{34} = 0.0055, \Delta k_{45} = 0.0063, \Delta k_{56} = 0.0065, \Delta k_{67} = 0.0073, \Delta k_{78} 0.0049, \Delta k_{89} = 0.0058, \Delta k_{910} = 0.0061$.

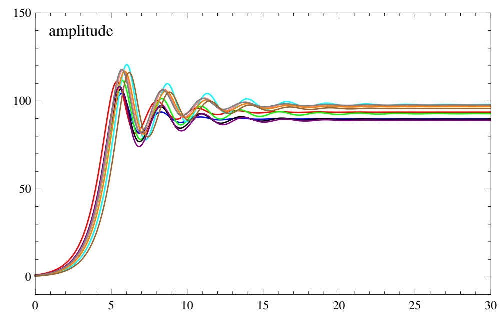

Figure 10. The amplitude behaviour of Eqs. (11) for $\nu = 6$, $\beta = 0$ and $\alpha = 0$ and the different value of the phase shifts between neighboring modes: $\Delta k_{12} = 0.06, \Delta k_{23} = 0.05, \Delta k_{34} = 0.065, \Delta k_{45} = 0.073, \Delta k_{56} = 0.064, \Delta k_{67} = 0.083, \Delta k_{78} 0.059, \Delta k_{89} = 0.068, \Delta k_{910} = 0.071$.

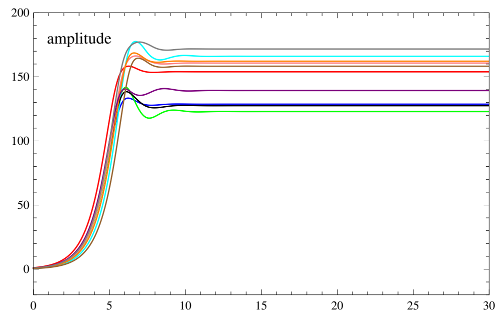

Figure 11. The amplitude behaviour of Eqs. (11) for $\nu = 6$, $\beta = 0$ and $\alpha = 7$ and the different value of the phase shifts between neighboring modes: $\Delta k_{12} = 0.06, \Delta k_{23} = 0.05, \Delta k_{34} = 0.065, \Delta k_{45} = 0.073, \Delta k_{56} = 0.064, \Delta k_{67} = 0.083, \Delta k_{78} 0.059, \Delta k_{89} = 0.068, \Delta k_{910} = 0.071$.

4. Summary

We have shown how the nonlinear theory of two unstable plasma modes excited by fast ions in the vicinity of the stability threshold may be generalized to the case of "many" interacting plasma waves. The main goal of the generalization was to examine how increasing the number of modes influences the modes dynamics. To simplify the problem, the examination has been limited to the case $n = 10$. It turned out, that a numerical solution of Eqs. (11) revealed very similar amplitudes behaviour, as well as new types, in comparison to the two-mode model [12]. When phase shifts between neighbouring modes were sufficiently small, and depending on the value of the collision parameters, we observed the competition for survival between modes, oscillation and chaotic regimes, and the "blow-up" solution. A new feature of the model has been noticed in the case of fixed values of the collision parameters and different phase shifts between modes. In contrast to the two-mode model, we have observed various types of competition between the modes, e.g. a different saturation level of each mode, or assembling of the mode amplitudes to a few steady-state levels. Further increasing the value of the phase shifts leads to the amplitudes oscillation in phase or in opposite phase. For even larger value of Δk_{ik} the interaction between modes becomes weaker, which in consequence leads to the independent behaviour of each plasma mode.

In summary, it should be emphasized that such generalization was not an attempt to convert the existing model to the multi-mode theory. In fact, such a task requires a more difficult approach, and cannot be done in the framework of the presented model in a simple manner. Nevertheless, the case $n = 10$ modes shows to a good approximation how the amplitudes behaviour changes due to an increase in the number of plasma modes, and in our opinion, similar solutions should occur in the case of even larger numbers of modes.

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References

[1] D. Anderson, H. Hamnen, M. Lisak, Thermonuclear Tokamak Plasmas in the Presence of Fusion Alpha Particles, (Chalmers University of Technology, Gothenburg, 1988)
[2] ITER Physics Basis Chapter 5, Nuclear Fusion, 39, 2471 (1999)
[3] H. L. Berk, B. N. Breizman, M. S. Pekker, Phys. Rev. Lett. 76, 1256 (1996)
[4] H. L. Berk, B. N. Breizman, M. S. Pekker, Plasma Phys. Rep. 23, 778 (1997)
[5] B. N. Breizman, H. L. Berk, M. S. Pekker, F. Porcelli, G. V. Stupakov, K. L. Wong, Phys. Plasmas 4, 1559 (1997)
[6] M. Lilley, B. N. Breizman, S. E. Sharapov, Phys. Rev. Lett. 102, 195003 (2009)
[7] A. Fasoli, B. N. Breizman, D. Borba, R. F. Heeter, M. S. Pekker, S. E. Sharapov, Phys. Rev. Lett. 81, 5564 (1998)
[8] R. F. Heeter, A. F. Fasoli, S. E. Sharapov, Phys. Rev. Lett. 85, 3177 (2000)
[9] S. D. Pinches et al., Plasma Phys. Controlled Fusion 46, B187 (2004)
[10] R. G. L. Vann, R. O. Dendy, M. P. Gryaznevich, Phys. Plasmas 12, 032501 (2005)
[11] M. P. Gryaznevich, S. E. Sharapov, Nucl. Fusion 46, S942 (2006)
[12] J. Zaleśny et al., Phys. Plasmas 18, 062109 (2011)
[13] J. Zaleśny, G. Galant, M. Lisak, P. Berczyński, S. Berczyński, Phys. Scr. 84, 065504 (2011)