

# Fractionl dynamics

Editorial

Richard L. Magin<sup>1\*</sup>, Raoul R. Nigmatullin<sup>2†</sup>

*1 Department of Bioengineering, Science and Engineering Offices, University of Illinois at Chicago, 851 S. Morgan Street, Chicago, Illinois 60607-7052, USA*

*2 Deparment of Theoretical Physics, Institute of Physics, Kazan Federal University 18 Kremlevskaya Street, Kazan 420008, Russia*

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This topical issue of the Central European Journal of Physics (CEJP) highlights the emerging field of fractional dynamics; a domain in the realm of mathematical physics that applies the methods of fractional calculus to problems in physics, chemistry, biology and engineering. This issue is another in a recent series focusing on advances in statistical physics, nanoscience and nanotechnology, lasers, and solid-state interfaces.

The proliferation of 'fractional calculus' and its synonyms is evident in the lexicon of titles selected for recent workshops, international symposia, and the publication of special issues of technical journals. This fecundity speaks to the vitality and growth of the field, and in particular to its natural extension into modeling complex processes and materials whose dynamics are naturally expressed via fractional order integrals and derivatives.

For example, the International Symposium on Fractional Partial Differential Equations (Newport, RI, USA, June 5-8, 2013) brought together many advocates for fractional calculus and the conference web site available

at (<http://www.dam.brown.edu>) provides complete files (pdf) for all of the talks. However, while conferences and presentations are a quick way to catch up or to survey a field, special issues are also needed to provide depth and breath to emerging ideas.

In this topical issue on 'fractional dynamics' we are happy to bring together a collection of new material suitable for both the beginner and the expert. In analyzing the submitted manuscripts, the reviewers and the guest editors have screened the content of all papers to ensure that only those with new and original results are presented.

The utility of fractional calculus is well established in physics and engineering where the behavior of dielectrics and viscoelastic materials over wide ranges of time and frequency can only be described by fractional order dynamic models that encode fractal structures, e.g., self-similarity in the order of the differential operators. However, while the utility may be clear, for a novice, many aspects of fractional order modeling need further elaboration. It is a goal of this special issue to provide new examples illustrating modern techniques in fractional order modeling.

The papers in this topical issue on fractional dynamics can be classified under three general headings: theoretical techniques, numerical methods, and practical appli-

\*E-mail: [rmagin@uic.edu](mailto:rmagin@uic.edu)

†E-mail: [renigmat@gmail.com](mailto:renigmat@gmail.com)

cations. The theoretical papers extend from fundamental studies of the properties of fractional derivatives and integrals, to the existence and uniqueness of solutions to specific fractional order differential equations (FDE), and finally to the identification of the characteristics of systems of linear and non-linear FDE. The numerical papers address the critical issues surrounding the proper assumptions, approximations and boundary conditions needed for the discrete, computer friendly formulation of FDE. The application papers bring together theory and numerical methods to establish new approaches for solving practical problems in fractional dynamics.

Thus, in this special issue, the authors have the rare opportunity of making new contributions to an established field; contributions that highlight new structures, different relationships, rare functions, novel dynamics, and interesting intuitive behaviors that extend the conventional integer order models. We hope that the reader will find useful information here that will inspire and inform.

An overriding idea expressed in this special issue is addressed to both beginners and experts alike. This idea concerns the need – when applying fractional dynamics – to think about first expressing conservation laws specifically in terms of fractional integrals and derivatives. A related problem that remains largely unsolved is associated with the physical and geometrical meaning of the spatial fractional integral (as a continuous operation) that literally ‘emerges’ from the discrete self-similar structure embedded in the model. In such cases, the simple replacement of the integer, integro-differential operator by its fractional replica is not sufficient, without further justification. Here, it is necessary to find the strict mathe-

matical arguments that allow transforming the averaging procedure of a smooth function over a fractal structure into the desired fractional integral. The inverse problem is also important; finding a proper fractal structure that corresponds to the given fractional integral or derivative. The resolution of these important problems is needed to increase the position of fractional dynamics among allied fields where researchers are sometimes skeptical toward fractional order development of this area. Another important problem is associated with fractal (self-similar) analysis of different random sequences. It is necessary in such cases to justify arguments that allow reducing the information contained in a long-time series (e.g., containing  $10^6$  or more data points) to a finite set of stable parameters. The well-known detrended fluctuation analysis used for this purpose can be considered as the first attempt in this direction. These reduced parameter sets can be considered as a dynamic fractal generator that creates the fully developed fractal dynamical structure inside a long-time series. The solution of these key problems will be extremely helpful in advancing the future development of this new field. Timely solutions can fertilize the conventional natural sciences, especially in addressing dynamical problems associated with understanding of the dynamic behavior of complex systems both in time and space.

Finally, the editors wish to thank the authors who have submitted papers to this topical issue, the reviewers who have taken time to analyze the submissions and the Editors and Publishers of CJEP for bringing together all this work into a complete and comprehensive assessment of the emerging field of ‘fractional dynamics’.