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OSCILLATION OF SECOND-ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH DAMPING

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ABSTRACT. We study oscillatory properties of solutions to a class of nonlinear second-order differential equations with a nonlinear damping. New oscillation criteria extend those reported in [ROGOVCHENKO, Yu. V.—TUNCAY, F.: *Oscillation criteria for second-order nonlinear differential equations with damping*, Nonlinear Anal. **69** (2008), 208–221] and improve a number of related results.

1. Introduction

In this paper, we are concerned with the oscillation of a second-order nonlinear differential equation with damping

$$\left(r(t)\left(x'(t)\right)^{\gamma}\right)' + p(t)\left(x'(t)\right)^{\gamma} + q(t)f(x(t)) = 0, \tag{1}$$

where $t \geq t_0 > 0$, $\gamma \geq 1$ is a ratio of odd positive integers, the functions r, p, q, f are such that $r \in \mathrm{C}^1([t_0, +\infty), (0, +\infty))$, $p, q \in \mathrm{C}([t_0, +\infty), \mathbb{R})$, $f \in \mathrm{C}(\mathbb{R}, \mathbb{R})$, $xf(x) > 0$ for $x \neq 0$, and $f(x)/x^{\gamma} \geq \mu$, for some $\mu > 0$ and for all $x \neq 0$. We also suppose that, for all $t \geq t_0$, $q(t) \geq 0$, and $q(t)$ does not vanish eventually.

In what follows, we assume that solutions to equation (1) exist for all $t \geq t_0$. As usual, a solution of equation (1) is called oscillatory if it has arbitrarily large zeros; otherwise we call it nonoscillatory. Equation (1) is said to be oscillatory if all its solutions are oscillatory.

Over the past decade, oscillation of differential equations with damping attracted significant attention of researchers, see, for instance, the papers [3]–[13] and the references cited therein. The purpose of this note is to extend recent

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oscillation results due to Rogovchenko and Tuncay [9] obtained for a differential equation

$$(r(t)\,x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0$$

to a more general equation (1) that may be also viewed as a special case of second-order damped differential equations with a p-Laplacian.

2. Main results

In the sequel, all functional inequalities are assumed to be satisfied for all t sufficiently large. Let

$$\mathbb{D} = \big\{(t,s):\ t_0 \le s \le t < +\infty\big\} \qquad \text{and} \qquad \mathbb{D}_0 = \big\{(t,s):\ t_0 \le s < t < +\infty\big\}.$$

We say that a function $H \in \mathrm{C}(\mathbb{D},[0,+\infty))$ belongs to a class $\mathcal{W}_\gamma$ if:

(i) $H(t,t) = 0$, and $H(t,s) > 0$ for all $(t,s) \in \mathbb{D}_0$;

(ii) H has a non-positive continuous partial derivative with respect to the second variable satisfying condition

$$\frac{\partial}{\partial s}H(t,s) = -h(t,s)(H(t,s))^{\gamma/(\gamma+1)},$$

for some function $h \in \mathcal{L}_{\mathrm{loc}}(\mathbb{D},\mathbb{R})$.

Note that for $\gamma = 1$, $\mathcal{W}_\gamma$ reduces to the class of functions $\mathcal{W}$ used in [9].

The following auxiliary result collects two useful inequalities that can be found, respectively, in Jiang and Li [2] and Hardy et al. [1].

Lemma 1. *Let $\gamma \ge 1$ be a ratio of two odd numbers. Then,*

$$A^{1+1/\gamma} - (A-B)^{1+1/\gamma} \le \frac{B^{1/\gamma}}{\gamma}\left[(\gamma+1)\,A - B\right], \tag{2}$$

and

$$C^{(\gamma+1)/\gamma} - \frac{\gamma+1}{\gamma}CD^{1/\gamma} \ge -\frac{1}{\gamma}D^{(\gamma+1)/\gamma}. \tag{3}$$

Theorem 2. *Suppose that there exists a function $\rho \in \mathrm{C}^1([t_0,+\infty),\mathbb{R})$ such that, for some $\beta \ge 1$ and for some $H \in \mathcal{W}_\gamma$,*

$$\limsup_{t\to+\infty} \frac{1}{H(t,t_0)}\int_{t_0}^{t}\Big[H(t,s)\psi(s) - \frac{\beta^\gamma}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\Big]\,\mathrm{d}s = +\infty, \tag{4}$$

where

$$\psi(t) = v(t)\big[\mu q(t) + r(t)\rho^{(\gamma+1)/\gamma}(t) - p(t)\rho(t) - (r(t)\rho(t))'\big]$$

and

$$v(t) = \exp\left[-(\gamma+1)\int_{t_0}^{t}\left(\rho^{1/\gamma}(s) - \frac{p(s)}{(\gamma+1)r(s)}\right)\mathrm{d}s\right]. \tag{5}$$

Then, equation (1) *is oscillatory.*

P r o o f. Let $x(t)$ be a non-oscillatory solution of (1). Observe that assumptions on γ and $f(x)$ imply that if $x(t)$ is a solution of (1), $-x(t)$ is also a solution to this equation. Thus, without loss of generality, we can assume that there exists a $T_0 \ge t_0$ such that $x(t) > 0$, for all $t \ge T_0$. Define a generalized Riccati substitution by

$$u(t) = v(t)r(t)\left[\left(\frac{x'(t)}{x(t)}\right)^{\gamma} + \rho(t)\right], \qquad t \ge T_0. \tag{6}$$

Differentiating (6), we obtain

$$u'(t) = \frac{v'(t)}{v(t)}u(t) + v(t)\frac{(r(t)(x'(t))^{\gamma})'}{x^{\gamma}(t)} - \gamma v(t)r(t)\left[\frac{u(t)}{v(t)r(t)} - \rho(t)\right]^{(\gamma+1)/\gamma} + v(t)(r(t)\rho(t))'. \tag{7}$$

Let $A = u(t)/(v(t)r(t))$ and $B = \rho(t)$. Using the inequality (2), we conclude that

$$\left[\frac{u(t)}{v(t)r(t)} - \rho(t)\right]^{(\gamma+1)/\gamma} \ge \left(\frac{u(t)}{v(t)r(t)}\right)^{(\gamma+1)/\gamma} - \frac{\rho^{1/\gamma}(t)}{\gamma}\left[(\gamma+1)\frac{u(t)}{v(t)r(t)} - \rho(t)\right].$$

Thus, (1), (5), and (7) yield

$$u'(t) \le -\psi(t) - \gamma\left(\frac{u^{\gamma+1}(t)}{v(t)r(t)}\right)^{1/\gamma}. \tag{8}$$

Multiplying both sides of (8) by $H(t,s)$ and integrating the resulting inequality from T_1 to t, we obtain, for all $\beta \ge 1$ and all $t \ge T_1 \ge T_0$,

$$\int_{T_1}^{t} H(t,s)\psi(s)\,\mathrm{d}s + \int_{T_1}^{t} h(t,s)(H(t,s))^{\gamma/(\gamma+1)}u(s)\,\mathrm{d}s + \frac{\gamma}{\beta}\int_{T_1}^{t} H(t,s)\left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma}\mathrm{d}s \tag{9}$$

$$\le H(t,T_1)u(T_1) - \frac{\gamma(\beta-1)}{\beta}\int_{T_1}^{t} H(t,s)\left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma}\mathrm{d}s.$$

Define now

$$C = \left(\frac{\gamma H(t,s)u^{(\gamma+1)/\gamma}(s)}{\beta v^{1/\gamma}(s)r^{1/\gamma}(s)}\right)^{\gamma/(\gamma+1)}$$

and

$$D = -\left(\frac{\gamma\beta^{\gamma}v(s)r(s)h^{\gamma+1}(t,s)}{(\gamma+1)^{\gamma+1}}\right)^{\gamma/(\gamma+1)}.$$

Applying the inequality (3), we conclude that

$$h(t,s)(H(t,s))^{\gamma/(\gamma+1)}u(s) + \frac{\gamma}{\beta}H(t,s)\left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma} \le -\frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s). \tag{10}$$

By (9) and (10),

$$\int_{T_1}^{t}\left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]\,\mathrm{d}s \le H(t,T_1)u(T_1) - \frac{\gamma(\beta-1)}{\beta}\int_{T_1}^{t}H(t,s)\left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma}\,\mathrm{d}s. \tag{11}$$

Using monotonicity of H, we conclude that, for all $t \ge T_1$,

$$\int_{T_1}^{t}\left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]\,\mathrm{d}s \le H(t,T_1)|u(T_1)| \le H(t,t_0)|u(T_1)|.$$

Thus,

$$\int_{t_0}^{t}\left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]\,\mathrm{d}s \le H(t,t_0)\left[|u(T_1)| + \int_{t_0}^{T_1}|\psi(s)|\,\mathrm{d}s\right].$$

Hence,

$$\limsup_{t\to+\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)\psi(s)-\frac{\beta^\gamma}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]\mathrm{d}s$$

$$\leq |u(T_1)|+\int_{t_0}^{T_1}|\psi(s)|\,\mathrm{d}s<+\infty,$$

which contradicts (4). The proof is complete. □

If condition (4) in Theorem 2 fails to hold, the following result can be used instead.

Theorem 3. *Suppose that there exist functions $H\in\mathcal{W}_\gamma$, $\rho\in\mathrm{C}^1([t_0,+\infty),\mathbb{R})$, and $\phi\in\mathrm{C}([t_0,+\infty),\mathbb{R})$ such that, for all $T\geq t_0$ and for some $\beta>1$,*

$$0<\inf_{s\geq t_0}\left[\liminf_{t\to+\infty}\frac{H(t,s)}{H(t,t_0)}\right]\leq+\infty \tag{12}$$

and

$$\limsup_{t\to+\infty}\frac{1}{H(t,T)}\int_{T}^{t}\left[H(t,s)\psi(s)-\frac{\beta^\gamma}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]\mathrm{d}s\geq\phi(T),$$

where ψ and v are as in Theorem 2. *If*

$$\int_{t_0}^{+\infty}\left(\frac{\phi_+^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma}\mathrm{d}s=+\infty, \tag{13}$$

where $\phi_+(t)=\max\{\phi(t),0\}$, then equation (1) *is oscillatory.*

P r o o f. Without loss of generality, assume again that (1) possesses a solution $x(t)$ such that $x(t)>0$ on $[T_0,+\infty)$ for some $T_0\geq t_0$. Proceeding as in the proof of Theorem 2, we arrive at the inequality (11), which yields, for all $t>T_1$ and for any $\beta\geq1$,

$$\phi(T_1)\leq\limsup_{t\to+\infty}\frac{1}{H(t,T_1)}\int_{T_1}^{t}\left[H(t,s)\psi(s)-\frac{\beta^\gamma}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]\mathrm{d}s$$

$$\leq u(T_1)-\frac{\gamma(\beta-1)}{\beta}\liminf_{t\to+\infty}\frac{1}{H(t,T_1)}\int_{T_1}^{t}H(t,s)\left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma}\mathrm{d}s.$$

The latter inequality implies that, for all $t > T_1$ and for all $\beta \geq 1$,

$$\phi(T_1) + \frac{\gamma(\beta-1)}{\beta} \liminf_{t\to+\infty} \frac{1}{H(t,T_1)} \int_{T_1}^{t} H(t,s) \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma} \mathrm{d}s \leq u(T_1).$$

Consequently,

$$\phi(T_1) \leq u(T_1), \tag{14}$$

and

$$\begin{aligned} \liminf_{t\to+\infty} \frac{1}{H(t,T_1)} \int_{T_1}^{t} H(t,s) \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma} \mathrm{d}s \\ \leq \frac{\beta}{\gamma(\beta-1)}(u(T_1) - \phi(T_1)) < +\infty. \end{aligned} \tag{15}$$

Assume now that

$$\int_{T_1}^{+\infty} \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma} \mathrm{d}s = +\infty. \tag{16}$$

Condition (12) implies existence of a $\vartheta > 0$ such that

$$\liminf_{t\to+\infty} \frac{H(t,s)}{H(t,t_0)} > \vartheta. \tag{17}$$

It follows from (16) that, for any positive constant η, there exists a $T_2 > T_1$ such that, for all $t \geq T_2$,

$$\int_{T_1}^{t} \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma} \mathrm{d}s \geq \frac{\eta}{\vartheta}. \tag{18}$$

Using integration by parts and (18), we have, for all $t \geq T_1$,

$$\begin{aligned} &\frac{1}{H(t,T_1)} \int_{T_1}^{t} H(t,s) \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)}\right)^{1/\gamma} \mathrm{d}s \\ &= \frac{1}{H(t,T_1)} \int_{T_1}^{t} H(t,s)\, \mathrm{d}\left[\int_{T_1}^{s} \left(\frac{u^{\gamma+1}(\xi)}{v(\xi)r(\xi)}\right)^{1/\gamma} \mathrm{d}\xi\right] \\ &= \frac{1}{H(t,T_1)} \int_{T_1}^{t} \left[\int_{T_1}^{s} \left(\frac{u^{\gamma+1}(\xi)}{v(\xi)r(\xi)}\right)^{1/\gamma} \mathrm{d}\xi\right] \left[-\frac{\partial H(t,s)}{\partial s}\right] \mathrm{d}s \\ &\geq \frac{\eta}{\vartheta} \frac{1}{H(t,T_1)} \int_{T_2}^{t} \left[-\frac{\partial H(t,s)}{\partial s}\right] \mathrm{d}s = \frac{\eta}{\vartheta} \frac{H(t,T_2)}{H(t,T_1)} \geq \frac{\eta}{\vartheta} \frac{H(t,T_2)}{H(t,t_0)}. \end{aligned}$$

By virtue of (17), there exists a $T_3 \geq T_2$ such that, for all $t \geq T_3$,

$$\frac{H(t,T_2)}{H(t,t_0)} \geq \vartheta,$$

which implies that

$$\frac{1}{H(t,T_1)} \int_{T_1}^{t} H(t,s) \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)} \right)^{1/\gamma} \mathrm{d}s \geq \eta, \qquad t \geq T_2.$$

Since η is an arbitrary positive constant,

$$\liminf_{t \to +\infty} \frac{1}{H(t,T_1)} \int_{T_1}^{t} H(t,s) \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)} \right)^{1/\gamma} \mathrm{d}s = +\infty,$$

and the latter contradicts (15). Consequently,

$$\int_{T_1}^{+\infty} \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)} \right)^{1/\gamma} \mathrm{d}s < +\infty,$$

and, by virtue of (14),

$$\int_{T_1}^{+\infty} \left(\frac{\phi_+^{\gamma+1}(s)}{v(s)r(s)} \right)^{1/\gamma} \mathrm{d}s \leq \int_{T_1}^{+\infty} \left(\frac{u^{\gamma+1}(s)}{v(s)r(s)} \right)^{1/\gamma} \mathrm{d}s < +\infty,$$

which contradicts (13). This completes the proof. □

Remark 4. Choosing different combinations of functions H and ρ, one can derive from general Theorems 2 and 3 a variety of efficient tests for oscillation of equation (1) and its particular cases.

If $f(x) = \mu x^\gamma$ for some $\mu > 0$, equation (1) reduces to a second-order half-linear damped differential equation

$$\left(r\,(t)\,(x'\,(t))^\gamma\right)' + p(t)\,(x'\,(t))^\gamma + \mu q(t) x^\gamma(t) = 0. \tag{19}$$

Equation (19) has been recently studied by Liu et al. [6] and Zhang et al. [13] under the assumptions

$$r(t) > 0, \qquad p(t) \geq 0, \qquad q(t) > 0.$$

Theorems 2 and 3 yield the following oscillation result.

Corollary 5. *Let $f(x) = \mu x^\gamma$, for some $\mu > 0$. Suppose further that either*

(i) (4) *holds or*

(ii) *assumptions of* Theorem 3 *are satisfied.*

Then, equation (19) *is oscillatory.*

***Example* 6.** For $t \geq 1$, consider a second-order half-linear differential equation

$$\left(\frac{1}{t}\left(x'(t)\right)^{\gamma}\right)' + \cos t\,(x'(t))^{\gamma} + \left[\frac{1}{t^3} - \frac{\alpha(t)}{t}\left(\frac{t\cos t - 2t^{-1}}{\gamma+1}\right) + \alpha(t)\cos t + \left(\frac{\alpha(t)}{t^{\gamma}}\right)'\right] x^{\gamma}(t) = 0, \tag{20}$$

where $\gamma \geq 1$ is a quotient of odd positive integers and

$$\alpha(t) = \left(\frac{t\cos t - 2t^{-1}}{\gamma+1}\right)^{\gamma}.$$

Let $H(t,s) = (t-s)^2$ and $\rho(t) = \alpha(t)$. Then, $h(t,s) = 2(t-s)^{(1-\gamma)/(1+\gamma)}$, $v(t) = t^2$, and $\psi(t) = t^{-1}$. Consequently, for any $\beta \geq 1$,

$$\limsup_{t\to+\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} \left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}} v(s)r(s)h^{\gamma+1}(t,s)\right] \mathrm{d}s$$

$$= \limsup_{t\to+\infty} \frac{1}{t^2} \int_{1}^{t} \left[\frac{(t-s)^2}{s} - \frac{2^{\gamma+1}\beta^{\gamma}}{(\gamma+1)^{\gamma+1}} s(t-s)^{1-\gamma}\right] \mathrm{d}s = +\infty,$$

and equation (20) is oscillatory by Corollary 5, part (i).

***Example* 7.** For $t \geq 1$, consider a second-order half-linear differential equation

$$\left(\alpha(t)\,(x'(t))^{\gamma}\right)' + \frac{3}{t}\alpha(t)\,(x'(t))^{\gamma} + t^{-3}\left(\left(1 - t^3 + 2t^2 - 6t\right)\sin t + 12t\right) x^{\gamma}(t) = 0, \tag{21}$$

where $\gamma \geq 1$ is a quotient of odd positive integers and

$$\alpha(t) = \left(1 + \frac{1}{2t^3}\right)(2 + \sin t).$$

Let $\beta = 2^{-1}(1+\gamma)^{(\gamma+1)/\gamma}$, $H(t,s) = (t-s)^2$, and $\rho(t) = 0$. Then, $v(t) = t^3$, $h(t,s) = 2(t-s)^{(1-\gamma)/(\gamma+1)}$, and $\psi(t) = \left(1 - t^3 + 2t^2 - 6t\right)\sin t + 12t$. One has

$$\limsup_{t\to+\infty} \frac{1}{H(t,T)} \int_{T}^{t} \left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}} v(s)r(s)h^{\gamma+1}(t,s)\right] \mathrm{d}s$$

$$= \limsup_{t\to+\infty} \frac{1}{t^2} \Big[(t-s)^2 \left(\left(1 - s^3 + 2s^2 - 6s\right)\sin s + 12s\right) - 2^{\gamma}\beta^{\gamma}(\gamma+1)^{-(\gamma+1)}(2s^3+1)(2+\sin s)(t-s)^{1-\gamma}\Big] \mathrm{d}s$$

$$\geq \limsup_{t\to+\infty} \frac{1}{t^2} \left[(t-s)^2 \left(\left(1-s^3+2s^2-6s\right)\sin s+12s\right)-(2s^3+1)(2+\sin s)\right] \mathrm{d}s$$
$$= 16 - T^3\cos T + T^2(2\cos T - 6 + 3\sin T) - 4T\sin T - 3\cos T = \phi(T).$$

It is not difficult to verify that condition (13) is satisfied. Therefore, equation (21) is oscillatory by Corollary 5, part (ii).

Remark 8. Note that Theorems 2 and 3 include, as a particular case ($\gamma = 1$), corresponding results derived by Rogovchenko and Tuncay [9: Theorem 17, Theorem 19] and improve those obtained in the papers [6,12,13] under more restrictive sign conditions on the coefficients p and q.

Remark 9. It would be interesting to find another method to investigate (1) when $\gamma < 1$, since the inequality (2) does not hold in this case.

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