Review Article · DOI: 10.2478/s13230-010-0007-z · JBR · 1(1) · 2010 · 66-72

Abstract intelligence and cognitive robots

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Received 11 February 2010

Accepted 19 March 2010

Abstract

Abstract intelligence is the human enquiry of both natural and artificial intelligence at the neural, cognitive, functional, and logical levels reductively from the bottom up. According to the abstract intelligence theory, a ***cognitive robot*** is an autonomous robot that is capable of thought, perception, and learning based on the three-level computational intelligence known as the imperative, autonomic, and cognitive intelligence. This paper presents the theoretical foundations of cognitive robots based on the latest advances in abstract intelligence, cognitive informatics, and denotational mathematics. A formal model of intelligence known as the *Generic Abstract Intelligence Mode* (GAIM) is developed, which provides a foundation to explain the mechanisms of advanced natural intelligence such as thinking, learning, and inference. A set of denotational mathematics is introduced for rigorously modeling and manipulating the behaviors of cognitive robots. A case study on applications of a denotational mathematics, visual semantic algebra (VSA), is presented in architectural and behavioral modeling of cognitive robots based on the theory of abstract intelligence.

Keywords

cognitive informatics · cognitive computing · abstract intelligence · cognitive robots · computational intelligence · denotational mathematics · cognitive models · cognitive behaviors · behavioral modeling

1. Introduction

It is recognized that traditional machines are invented to extend human physical capability, while modern information processing machines such as computers, communication networks, and robots are developed for extending human intelligence, memory, and the capacity of information processing. Therefore, any machine that may even partially implement human behaviors and actions in information processing possesses some extent of intelligence. Therefore, one of the key objectives in cognitive informatics [22, 24] and computational intelligence is to seek a coherent theory for explaining the nature and mechanisms of both natural and artificial intelligence [27].

The history of investigation into the brain and natural intelligence is as long as the history of mankind, which can be traced back to the Aristotle's era and earlier. Early studies on intelligence are represented by works of Vygotsky, Spearman, and Thurstone [2, 11, 12, 16, 17, 32]. Vygotsky (1896 - 1934) presents a communication view that perceives intelligence as inter- and intra-personal communication in a social context. Spearman (1863 - 1945) and Thurstone (1887 - 1955) proposed the *factor theory* [11], in which seven factors of intelligence are identified known *erbal comprehension, word fluency, number facility, spatial visualization, associative memory, perceptual speed*, and *reasoning*.

Gardner's *multiple intelligence theory* [5] identified eight forms of intelligence, i.e., *linguistic, logical-mathematical, musical, spatial, bodily-kinesthetic, naturalist, interpersonal,* and *intrapersonal* intelligence. He perceived that intelligence is an ability to solve a problem or create a product within a specific cultural setting. Sternberg's *triarchic theory* [18–20] modeled intelligence in three dimensions known as the *analytic, practical*, and *creative* intelligence. Lefton and his colleagues [11] defined intelligence as the overall capacity of the individual to act purposefully, to think rationally, and to deal effectively with the social and cultural environment. They perceived that intelligence is not a thing but a process that is affected by a person's experiences in the environment.

McCarthy, Minsky, Rochester, and Shannon proposed the term *Artificial Intelligence* (AI) in 1955 [13, 14]. Kleene analyzed the relations of automata and nerve nets [10], and Widrow initiated the technology of *Artificial Neural Networks* (ANNs) in the 1950s [31] based on multilevel, distributed, dynamic, interactive, and self-organizing nonlinear networks [1, 4]. The concepts of robotics [3] were developed in the 1970s on the basis of AI studies. Then, expert systems [6], intelligent systems [15], computational intelligence [9], and software agents [7, 8, 24] emerged in the 1980s and 1990s. A *Layered Reference Model of the Brain* (LRMB) has been developed by Wang and his colleagues [29], which reveals that natural intelligence encompasses 43 cognitive processes at seven layers known as the *sensation, memory, perception, action, meta-cognitive, meta-inference*, and *higher-cognitive layers* from the bottom up.

This holistic view has led to the theory of *abstract intelligence* [27] in order to unify all paradigms of intelligence such as natural, artificial, machinable, and computational intelligence.

Definition 1. *Abstract intelligence*, αI, is a form of driving force that transfers information into behaviors or actions.

αI is a human enquiry of both natural and artificial intelligence at the levels of neural, cognitive, functional, and logical embodiment from the bottom up. Therefore, a cognitive robot is an embodiment or derived entity of abstract intelligence.

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Definition 2. A *cognitive robot* is an autonomous robot that is capable of thought, perception, and learning based on the three-level computational intelligence at the imperative, autonomic, and cognitive levels.

This paper presents the theoretical foundations of cognitive robots on the basis of the latest advances in abstract intelligence, cognitive informatics, and denotational mathematics. In the remainder of this paper, the theory of abstract intelligence is introduced in Section 2, which describes the hierarchical levels of intelligence, explores the taxonomy and paradigm of abstract intelligence, and develops a generic abstract intelligence model. The architectural and behavioral models of cognitive robots that mimic the brain of the natural intelligence are formally elaborated in Section 3. A set of denotational mathematical structures for rigorously modeling and manipulating the intelligent behaviors of cognitive robots is explored in Section 4. Typical denotational mathematics applied in cognitive robots and computational intelligence are concept algebra, system algebra, real-time process algebra (RTPA), visual semantic algebra (VSA), and granular algebra, which are adopted in the behavioral and architectural modeling of cognitive robots based on the theory of abstract intelligence.

2. The theory of Abstract Intelligence

According to the principle of *functional reductionism*, a logical model of the general form of intelligence is needed, known as the abstract intelligence, in order to formally explain the high-level mechanisms of the brain on the basis of observations at the biological, physiological, functional, and logical levels. Based on the logical model of abstract intelligence, the studies on the natural, artificial, machinable, and computational intelligence may be unified into a common framework as developed in cognitive informatics [22, 24].

2.1. The hierarchical levels of intelligence

In the *narrow sense*, αI is a human or a system ability that transforms information into behaviors. While, in the *broad sense*, αI is any human or system ability that autonomously transfers the forms of abstract information between *data, information, knowledge*, and *behaviors* in the brain or systems.

Definition 3. The *behavioral model* of αI, $\S\alpha I\mathbf{ST}$, is an abstract logical model denoted by a set of parallel behaviors that encompasses those of the *imperative* intelligence I_I, *autonomic* intelligence I_A, and *cognitive* intelligence I_C from the bottom-up, i.e.:

$$\begin{aligned} &\S\alpha\,\mathrm{IST}\ \mathrm{A}(I_I, I_A, I_C) \\ &\quad = \{(B_e, B_t, B_{int}) && //I_I \\ &\quad ||(B_e, B_t, B_{int}, B_g, B_d) && //I_A \\ &\quad ||(B_e, B_t, B_{int}, B_g, B_d, B_p, B_{\mathrm{inf}}) && //I_C \\ &\quad \} \end{aligned} \tag{1}$$

According to Definition 3, the relationship among the three-form intelligence is as follows:

$$I_I \subseteq I_A \subseteq I_C. \tag{2}$$

Both Eqsuations 1 and 2 indicate that any lower layer intelligence and behavior is a subset of those of a higher layer. In other words, any higher layer intelligence and behavior is a natural extension of those of the lower layers.

2.2. The Generic Abstract Intelligence Model (GAIM)

On the basis of the conceptual models developed in previous subsection, the mechanisms of αI can be described by a Generic Abstract Intelligence Model (GAIM) as shown in Figure 1.

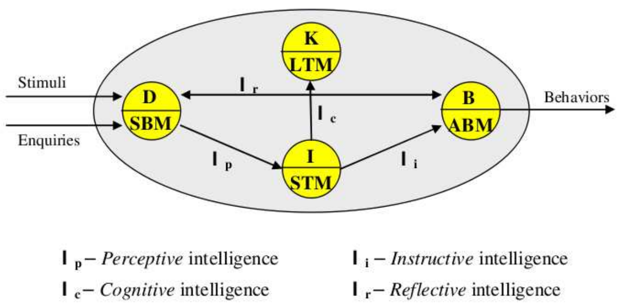

Figure 1. *The Generic Abstract Intelligence Model (GAIM).*

In the GAIM model as shown in Figure 1, different forms of intelligence are described as a driving force that transfers between a pair of abstract objects in the brain such as *data* (D), *information* (I), *knowledge* (K), and *behavior* (B). It is noteworthy that each abstract object is physically retained in a particular type of memory. This is the neural informatics foundation of natural intelligence, and the physiological evidences of why natural intelligence can be classified into four forms as given in the following theorem.

Theorem 1. The *nature of intelligence* states that *abstract intelligence* αI can be classified into four forms called the *perceptive intelligence* I_p, *cognitive intelligence* I_c, *instructive intelligence* I_i, and *reflective intelligence* I_r as modeled below:

$$\begin{aligned} \alpha I\quad \mathrm{A}\ & I_p : D \to I && \text{(Perceptive)} \\ ||& I_c : I \to K && \text{(Cognitive)} \\ ||& I_i : I \to B && \text{(Instructive)} \\ ||& I_r : D \to B && \text{(Reflective)} \end{aligned} \tag{3}$$

According to Definition 3 and Theorem 1 in the context of the GAIM model, the narrow sense of αI corresponds to the instructive and reflective intelligence; while the broad sense of αI includes all four forms of intelligence, that is, the perceptive, cognitive, instructive and reflective intelligence.

2.3. Paradigms of Abstract Intelligence

With the clarification of the intension and extension of the concept of αI, its paradigms and concrete forms in the real-world can be derived as summarized in Table 1. In the paradigms of αI, *Natural Intelligence* (NI) is an embodying form of αI that implements intelligent mechanisms and behaviors by naturally grown biological and physiological organisms such as human brains and those of other well developed species. *Artificial Intelligence* (AI) is an embodying form of αI that implements intelligent mechanisms and behaviors by cognitively-inspired artificial models and man-made systems such as intelligent systems, knowledge systems, decision-making systems, and distributed agent systems. *Machinable Intelligence* (MI) is an embodying form of αI that implements intelligent mechanisms and behaviors by complex machine and circuit systems such as computers, robots, circuits, neural networks, and autonomic mechanical machines. *Computational Intelligence* (CoI) is an embodying form of αI that implements intelligent mechanisms and behaviors by computational methodologies and software systems.

The GAIM architectural model reveals that NI and AI share the same cognitive informatics foundations on the basis of abstract intelligence. The *compatible intelligent capability* states that NI, AI, MI, and CoI are compatible by sharing the same mechanisms of intelligent capability and behaviors. In other words, at the logical level, NI of the brain shares the same mechanisms as those of AI and computational intelligence. The differences between NI and AI are only distinguishable by the means of implementation and the extent of intelligent ability.

3. The behavioral model of cognitive robots

As described in Definition 2, a *cognitive robot* is an autonomous robot that is capable of thought, perception, and learning based on the three-level computational intelligence known as the imperative, autonomic, and cognitive intelligence. This section develops a reference model of cognitive robots based on the theory of αI and the reference model of LRMB [29].

Definition 4. The *architectural model of a cognitive robot*, §CR-A**ST**, is a logical structure with a set of parallel intelligent engines, such as the *Sensory Engine* (SE), *Memory Engine* (ME), *Perception Engine* (PE), *Action Engine* (AE), *Meta-Cognition Engine* (CE), *Meta-Inference Engine* (IE), and *Higher Cognition Engine* (HCE), from the bottom up according to LRMB, i.e.:

$$\begin{array}{llll} \S CR-A\mathbf{ST} & A\ SE & // & \text{Sensory engine} \\ & ||ME & // & \text{Memory engine} \\ & ||PE & // & \text{Perception engine} \\ & ||AE & // & \text{Action engine} \\ & ||CE & // & \text{Cognitive engine} \\ & ||IE & // & \text{Inference engine} \\ & ||HCE & // & \text{Higher cognitive engine} \end{array} \quad (4)$$

where || denotes the parallel relation between given components of the system.

In Definition 4, each intelligent engine of §CR-A**ST** is further refined by detailed structures and functions as given in Figure 2. In addition, a relative system clock §t**TM** is provided in §CR-A**ST** for synchronizing activities and behaviors of the cognitive robot.

Definition 5. The *behavioral model of cognitive robot*, §CR-B**ST**, is a parallel structure of a set of behavioral processes as modeled in Figure 3.

The integration of architectural and behavioral models of cognitive robots, CR-A**ST** || CR-B**ST**, provides a reference model for the generic behaviors of cognitive robots and the layout of their conceptual platforms.

Therefore, the studies on NI and AI in general, and cognitive robots in particular, may be unified into a coherent framework based on cognitive informatics and αI, which can be formalized by contemporary denotational mathematics.

The Architectural Model of Cognitive Robots

```
§CR-AST A
{  <SEST:   R_{ptrP=0}^{n-1} SENSORS[ptrP]ST>     // Layer 1: Sensation engine
 || <MEST:  R_{addrP=0}^{5} MEM[addrP]ST>         // Layer 2: Memory engine
                = <SBMST || STMST || CSMST || LTMST || ABMST>
 || <PEST:  R_{iP=0}^{7} PROC[iN]ST>              // Layer 3: Perception engine
                = <AttentionST || MotivationST || EmotionST || AttitudeST
                   || SensOfSpatialityST || SensOfTimeST || SensOfMotionST>
 || <AEST:  R_{ptrP=0}^{n_SERVO N-1} SERVOS[ptrP]ST>  // Layer 4: Action engine
 || <CEST:  R_{iP=0}^{10} PROC[iN]ST >            // Layer 5: Meta-cognition engine
                = <ObjectIdentificationST || AbstractionST || ConceptEstablishmentST
                   || SearchST || CategorizationST || ComparisonST || MemorizationST
                   || QualificationST || QuantificationST || SelectionST>
 || <IEST:  R_{iP=0}^{6} PROC[iN]ST >             // Layer 6: Meta-inference engine
                = <DeductionST || InductionST || AbductionST || AnalogyST
                   || AnalysisST || SynthesisST>
 || <HCEST: R_{iP=0}^{7} PROC[iN]ST >             // Layer 7: Higher cognition engine
                = <ComprehensionST || LearningST || PlanningST || ProblemSolvingST
                   || DecisionMakingST || CreationST || PattenRecognitionST>
 || <§tTM>                                        // Relative clock
}
```

Figure 2. The architectural model of a cognitive robot.

4. Formally modeling cognitive robot behaviors by denotational mathematics

Applied mathematics can be classified into two categories known as *analytic* and *denotational* mathematics [25]. The former are mathematical structures that deal with functions of variables as well as their operations and behaviors; while the latter are mathematical structures that formalize rigorous expressions and inferences of system architectures and behaviors with abstract concepts, complex relations, and dynamic processes. The denotational and expressive needs in cognitive informatics, αI, cognitive robots, computational intelligence, software science, and knowledge engineering have led to new forms of mathematics collectively known as denotational mathematics.

4.1. Emergence of denotational mathematics

Denotational mathematics is a collection of higher order functions on complex mathematical entities. The term denotational mathematics is first introduced by Yingxu Wang in the emerging disciplines of *cognitive informatics* and *cognitive computing* [22, 24].

Definition 6. *Denotational mathematics* is a category of expressive mathematical structures that deals with high-level mathematical entities beyond numbers and simple sets, such as abstract objects, complex relations, behavioral information, concepts, knowledge, processes, intelligence, and systems.

Denotational mathematics is viewed as a new approach to formal inference on both complex architectures and intelligent behaviors to meet

Table 1. *Taxonomy of Abstract Intelligence and its Embodying Forms.*

No.	Form of intelligence	Embodying means	Paradigms
1	Natural intelligence (NI)	Naturally grown biological and physiological organisms	Human brains and brains of other well developed species
2	Artificial intelligence (AI)	Cognitively-inspired artificial models and man-made systems	Intelligent systems, knowledge systems, decision-making systems, and distributed agent systems
3	Machinable intelligence (MI)	Complex machine and wired systems	Computers, robots, autonomic circuits, neural networks, and autonomic mechanical machines
4	Computational intelligence (CoI)	Computational methodologies and software systems	Expert systems, fuzzy systems, autonomous computing, intelligent agent systems, genetic/evolutionary systems, and autonomous learning systems

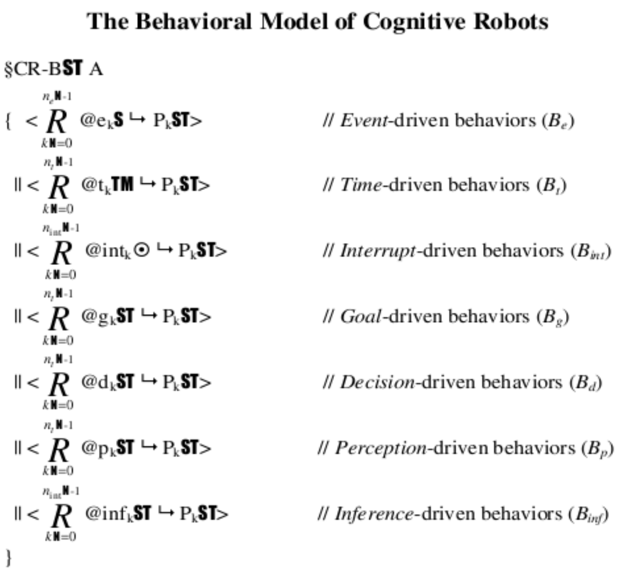

Figure 3. *The behavioral model of a cognitive robot.*

modern challenges in understanding, describing, and modeling natural and machine intelligence. The emergence of denotational mathematics is driven by practical needs in cognitive informatics, computational intelligence, cognitive robots, computing science, software science, and knowledge engineering, because all these modern disciplines study complex human and machine intelligence and their rigorous treatments.

4.2. Paradigms of denotational mathematics

A set of denotational mathematics, known as *Concept Algebra* (CA) [26], *System Algebra* (SA) [30], *Real-Time Process Algebra* (RTPA) [21], *Visual Semantic Algebra* (VSA) [28], and *Granular Algebra* (GA) [25], have been developed by Wang and his colleagues. Typical paradigms of denotational mathematics are comparatively presented in Table 2, where their structures, mathematical entities, algebraic operations, and usages are contrasted.

4.2.1. Concept Algebra

Concept algebra is an abstract mathematical structure for the formal treatment of concepts as the basic unit of human reasoning and their algebraic relations, operations, and associative rules for composing complex concepts.

Definition 7. A *concept algebra CA* on a given semantic environment Θ_C is a triple, i.e.:

$$CAA(C, OP, \Theta_C) = (\{O, A, R^c, R^i, R^o\}, \{\bullet_r, \bullet_c\}, \Theta_C) \qquad (5)$$

where $OP = \{\bullet_r, \bullet_c\}$ are the sets of *relational* and *compositional* operations on abstract concepts.

Concept algebra provides a denotational mathematical means for algebraic manipulations of abstract concepts. Concept algebra can be used to model, specify, and manipulate generic "to be" type problems, particularly system architectures, knowledge bases, and detail-level system designs, in cognitive informatics, computational intelligence, cognitive robots, computing science, software engineering, and knowledge engineering. Detailed relational and compositional operations of concept algebra may be found in [26].

4.2.2. System Algebra

System algebra is an abstract mathematical structure for the formal treatment of abstract and general systems as well as their algebraic relations, operations, and associative rules for composing and manipulating complex systems [23].

Definition 8. A *system algebra SA* on a given universal system environment U is a triple, i.e.:

$$SAA(S, OP, \Theta) = (\{C, R^c, R^i, R^o, B, \Omega\}, \{\bullet_r, \bullet_c\}, \Theta) \qquad (6)$$

where $OP = \{\bullet_r, \bullet_c\}$ are the sets of *relational* and *compositional* operations, respectively, on abstract systems.

System algebra provides a denotational mathematical means for algebraic manipulations of all forms of abstract systems. System algebra can be used to model, specify, and manipulate generic "to be" and "to have" type problems, particularly system architectures and high-level system designs, in cognitive informatics, computational intelligence,

Table 2. Paradigms of Denotational Mathematics.

No	Paradigm	Structure	Mathematical entities	Algebraic operations	Usage
1	Concept algebra (CA)	CA A (C, OP, Θ) = ({O, A, R^c, R^i, R^o}, {•_r, •_c}, Θ_C)	c A (O, A, R^c, R^i, R^o)	·_r A {« , Ŕ, p, f, =, @, : , @} ·_c A {Ţ, Ţ, Ţ, Ţ, â, ă, É, , a }	Algebraic manipulations on abstract concepts
2	System algebra (SA)	SA A (S, OP, Θ) = ({C, R^c, R^i, R^o, B, Ω} {•_r, •_c}, Θ)	S A (C, R^c, R^i, R^o, B, Ω, Θ)	·_r A {Ŕ, « , Ő, =, ô, ő } ·_c A {Ţ, Ţ, Ţ, Ţ, ', â, ă, É, }	Algebraic manipulations on abstract systems
3	Real-time process algebra (RTPA)	RTPA A (T, P, N)	P A {:=, ∨, ⇒, ⇐, —, , , |, |, @, ≅, ↑, ↓, !, ⊗,), §} T A {N, Z, R, S, BL, B, H, P, TI, D, DT, RT, ST, @eS, @tTM, @inte ,&sBL}	R A {→, ∠, |, |...|..., R^*, R^+, R^i, ©, ° , ‖, ∯, ‖‖, », ϟ, ↳_t, ↳_e, ↳_i}	Algebraic manipulations on abstract processes
4	Visual semantic algebra (VSA)	VSA A (O, ·_VSA)	O A {H ČS ČF ČL}	·_VSA A { -, Ż¬, ®, e, Ä, (, R, @(p), @(x,y,x), Đ , a , $\mathop{R}_{iN=0}^{n-1N}(A_i \text{ a } A_{i+1})$}	Algebraic manipulations on abstract visual objects/patterns
5	Granular algebra (GrA)	GA A (G, •_r, •_p, •_c) = ((C, R^c, R^i, R^o, B, Ω), •_r, •_p, •_c)	G A (C, R^c, R^i, R^o, B, Ω, Θ)	·_r A {Ŕ, « , Ö, =, ô, ő } ·_p A {Ţ · Ţ · Ţ · Ţ } ·_c A {â, ă, É }	Algebraic manipulations on abstract granules

cognitive robots, computing science, software engineering, and system engineering. Detailed relational and compositional operations on abstract systems may be found in [30].

4.2.3. Real-Time Process Algebra (RTPA)

A process algebra is a set of formal notations and rules for describing algebraic relations of software engineering processes. RTPA [21] is a real-time process algebra that can be used to formally and precisely describe and specify architectures and behaviors of human and software systems.

Definition 9. *Real-Time Process Algebra* (RTPA) is a denotational mathematical structure for algebraically denoting and manipulating system behavioural processes and their attributes by a triple, i.e.:

$$RTPA \triangleq (\mathrm{T}, \mathrm{P}, \mathrm{N}) \tag{7}$$

where T is a set of 17 primitive types for modeling system architectures and data objects, P a set of 17 meta-processes for modeling fundamental system behaviors, and R a set of 17 relational process operations for constructing complex system behaviors.

RTPA provides a coherent notation system and a formal engineering methodology for modeling both software and intelligent systems. RTPA can be used to describe both *logical* and *physical* models of systems, where logical views of the architecture of a software system and its operational platform can be described using the same set of notations. When the system architecture is formally modeled, the static and dynamic behaviors that perform on the system architectural model can be specified by a three-level refinement scheme at the system, class, and object levels in a top-down approach. Detailed syntaxes and formal semantics of RTPA meta-processes and process relations may be found in [21, 23].

4.2.4. Visual Semantic Algebra (VSA)

A new form of denotational mathematics known as Visual Semantic Algebra (VSA) is presented for abstract visual object and architecture manipulations.

Definition 10. *Visual Semantic Algebra* (VSA) is a denotational mathematical structure that formally manipulates visual objects by algebraic operations on symbolic or semantic objects in geometric analyses and compositions, i.e.:

$$VSA \triangleq (O, \bullet_{VSA}) \tag{8}$$

where O is a finite set of basic abstract visual objects and $\bullet_{VSA}$ is a finite set of algebraic operations on O.

VSA provides a new paradigm of denotational mathematical means for relational visual object manipulation [28]. VSA can be applied not only in machine visual and spatial reasoning, but also in computational intelligence system designs as a powerful man-machine language in representing and dealing with the high-level inferences in complex visual patterns and systems. On the basis of VSA, computational intelligence systems such as robots and cognitive computers can process and reason visual and image objects and their spatial relations rigorously and efficiently at conceptual level.

4.2.5. Granular Algebra

With the establishment of the formal model of abstract granules in [25], a set of relational, reproduction, and compositional operations on granules can be rigorously defined. This leads to the establishment of a mathematical structure, known as granular algebra, for the formal treatment of abstract and generic granules as well as their algebraic relations, operations, and associative rules for composing and manipulating complex granular systems.

Definition 11. A *granular algebra GA* is a 4-tuple on the given universal granular environment U, i.e.:

$$\begin{aligned} GA &\triangleq (G, \bullet_r, \bullet_p, \bullet_c) \\ &= ((C, R^c, R^i, R^o, B, \Omega), \bullet_r, \bullet_p, \bullet_c) \end{aligned} \tag{9}$$

where $\bullet_r$, $\bullet_p$ and $\bullet_c$ are sets of *relational*, *reproductive*, and *compositional* operations, respectively, on abstract granules.

Granular algebra provides a denotational mathematical means for algebraic manipulations of all forms of granular systems. Granular algebra can be used to model, specify, and manipulate system architectures and high-level system designs in computing, cognitive robots, software science, system engineering, and cognitive informatics.

The above five forms of denotational mathematics provide a powerful mathematical means for modeling and formalizing cognitive robots and systems. Not only the architectures of cognitive robots, but also their dynamic behaviors can be rigorously and systematically manipulated by denotational mathematics. Denotational mathematics has found a wide range of real-world applications in cognitive informatics and computational intelligence [23, 25], from the cognitive processes of the brain to the generic model of software systems, from rigorous system manipulation to knowledge network modeling, and from autonomous machine learning to cognitive computers [24].

Applications of denotational mathematics in cognitive robots and computational intelligence will be elaborated in the next subsection, which demonstrate that denotational mathematics is an ideal mathematical means for dealing with concepts, knowledge, behavioral processes, and natural/machine intelligence in cognitive robots.

Figure 4. *A robot walks down stairs (Honda ASIMO, courtesy of Wikipedia).*

Algorithm 1. A Robot Walks Down Stairs

```
WDS_Algorithm A WDS_Architecture§ ↔ WDS_Behavior †

WDS_Architecture§ (nN) A
  { // Layout
      StairsST : (S0ST ε ↑ (S1ST ε ↑ (S2ST ε ↑ (… (Sn-1ST ε ↑⊥) …))))
    // Robot
      Robot † : (LeftFootST || RightFootST)
    // Initial state
      Robot † @ (   StairsST.S0ST   )
    // Final state
      Robot † @ (   StairsST.⊥   )
  }

WDS_Behavior † A WDS(Robot † , WDS_Architecture§, nN) ::
 {
   n−1N
    R   (Robot † .LeftFootST ∠ ⊗Ż@(StairsST.SiN+1ST)
   iN=0
          α  Robot † .RightFootST ∠ ⊗Ż@(StairsST.SiN+1ST
         )
   α  Robot † .LeftFootST ∠ ⊗Ż@(StairsST.⊥)
   α  Robot † .RightFootST ∠ ⊗Ż@(StairsST.⊥)
 }
```

Figure 5. *The WDS algorithm for a cognitive robot in VSA.*

4.3. A case study on modeling the autonomous

Behaviors of Cognitive Robots

A case study on applications of one of the denotational mathematics, VAS, in robotic behavioral description is illustrated in Figure 4, where a robot with autonomously cognition about the special environment while walking done stairs. The visual walk planning mechanisms and processes can be described by the ***walking down stairs*** (WDS) algorithm in VSA as given in Figure 5.

Algorithm 1. The WDS algorithm can be described in VSA as shown in Figure 5. The WDSAlgorithm encompasses the architecture WDSArchitecture§, the robot behaviors WDSBehaviors, and their interactions. WDSArchitecture§ describes the layout, initial and final states of the system. WDSBehaviors describes the actions of the robot based on its visual interpretation about the stairs' visual structure.

The case study presented above demonstrates that VSA provides a new paradigm of denotational mathematical means for relational visual object manipulations. VSA can be applied not only in machine visual and spatial reasoning, but also in computational intelligence system designs as a powerful man-machine language in representing and dealing with the high-level inferences in complex visual patterns and systems. On the basis of VSA, computational intelligence systems such as robots and cognitive computers can process and reason with visual and image objects and their spatial relations rigorously and efficiently at a conceptual level.

5. Conclusions

This paper has introduced the theory of abstract intelligence for cognitive robots based on studies in cognitive informatics. Abstract intelligence has been described as a form of driving force that transfers information into behaviors or actions. Abstract intelligence has been classified into four forms known as the perceptive, cognitive, instructive, and reflective intelligence in the Generic Abstract Intelligence Mode (GAIM), which provides a foundation upon which to explain the mechanisms of advanced natural intelligence such as thinking, learning, and inference for cognitive robots.

It has been recognized that suitable mathematical means known as denotational mathematics beyond sets and logic are needed to rigorously model the architectures and behaviors of cognitive robots. The latest paradigms of denotational mathematics have been explored such as concept algebra, system algebra, RTPA, VSA, and granular algebra. A cognitive robot has been modeled as an autonomous robot that is capable of thought, perception, and learning based on the imperative, autonomic, and cognitive intelligence. Both the generic architectural and behavioral models of cognitive robots have been developed based on studies in abstract intelligence and denotational mathematics. A case study on modeling the autonomous behaviors of cognitive robots has been presented and discussed.

Acknowledgement

The author would like to acknowledge the Natural Science and Engineering Council of Canada (NSERC) for its partial support to this work. The author would like to thank Dr. Andrzej Ruta and anonymous reviewers for their valuable comments and suggestions to this work.

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