Catalogue of the J - Q trajectories for CC(T) and SEN(T) specimens in tension

Research Article

Marcin Graba*

Kielce University of Technology,
Faculty of Mechatronics and Machine Design,
Chair of Fundamentals of Machine Design,
Al. 1000-lecia PP 7, 25-314 Kielce, Poland

Received 14 February 2011; accepted 6 May 2011

Abstract: In this paper a short theoretical background about elastic-plastic fracture mechanics is presented and the O'Dowd-Shih theory is discussed. Using FEM, the values of the Q -stress determined for various elastic-plastic materials for two specimens in tension - SEN(T) specimen and CC(T) specimen are presented. The influence of geometry of the specimen, crack length and material properties (work-hardening exponent and yield stress) on the Q -parameter are tested. The numerical results were approximated by closed form formulas. The results are summarized in a catalogue of the Q -stress value, which may be used in engineering analysis for calculation of the real fracture toughness and the stress distribution near crack tip.

Keywords: Fracture mechanics • Q -stress • Stress fields • HRR solution • J -integral • O'Dowd theory • Fracture toughness • Fracture criteria.

© Versita sp. z o.o.

1. Theoretical background on elastic-plastic fracture mechanics

In 1968 Hutchinson [1] published fundamental work that characterized stress fields in front of a crack for non-linear Ramberg-Osgood (R-O) material in the form:

$$\sigma_{ij} = \sigma_0 \left(\frac{J}{\alpha \sigma_0 \epsilon_0 l_n r} \right)^{\frac{1}{1+n}} \tilde{\sigma}_{ij}(\theta, n) \quad (1)$$

where r and θ are polar coordinates of the coordinate system located at the crack tip, σ_{ij} are the components of

the stress tensor, J is the J -integral, n is R-O exponent, α is R-O constant, σ_0 is yield stress, ϵ_0 is strain related to σ_0 through $\epsilon_0 = \sigma_0/E$. Functions $\tilde{\sigma}_{ij}(n, \theta)$, $l_n(n)$ must be found by solving the fourth order non-linear homogenous differential equation independently for plane stress and plane strain [1]. These functions may be solved using algorithm and computer program presented by Gałkiewicz and Graba [2]. Equation (1) is commonly called the "HRR solution" (Fig. 1).

The HRR solution includes the first term of the infinite series only. Numerical analysis shows that results obtained using the HRR solution differ from the results obtained using the finite element method - FEM (Fig. 2). To eliminate this difference, it is necessary to use additional terms in the HRR solution.

*E-mail: mgraba@tu.kielce.pl

Figure 1. The crack opening stress distribution for elastic-plastic materials, obtained using the HRR solution.

$E = 206000\text{MPa}$ $n = 5$ $\nu = 0.30$
 $\sigma_0 = 315\text{MPa}$ $\varepsilon_0 = \sigma_0/E = 0.00153$
 $a/W = 0.50$ $W = 40\text{mm}$ (results for angle $\theta = 0$)

Figure 2. Comparison the FEM results and HRR solution for plane stress and plane strain for single edge notched specimen in bending (SEN(B)) and centre cracked plate in tension (CC(T)).

Li and Wang [3] used two terms in the Airy function, obtained the second term of the asymptotic expansion for the two materials described by $n=3$ and $n=10$. Next, they compared their results with the HRR fields and FEM results. Their analysis showed that using the two term solution to describe the stress field near the crack tip, brings

closer analytical results to FEM results. The two term solution much better describes the stress field near the crack tip; the value of the second term, which may not be negligible, depends on the material properties and the geometry of the specimen.

Yang et al. [4], using the Airy function with the separate variables in the infinite series form, proposed that stress field near the crack tip may be described by the Eq. (2) in the infinite series form:

$$\frac{\sigma_{ij}}{\sigma_0} = \sum_{k=1}^{+\infty} A_k \bar{r}^{s_k} \tilde{\sigma}_{ij}^{(k)}(\theta) \quad (2)$$

where k is the number of the series terms, A_k is the amplitude for the k series term, $\bar{r}$ is the normalized distance from the crack tip, s_k is power exponent for the k series term, and $\tilde{\sigma}_{ij}^{(k)}$ is "stress" function.

When limited to only three terms, Eq. (2) may be written in the following form:

$$\frac{\sigma_{ij}}{\sigma_0} = A_1 \bar{r}^s \tilde{\sigma}_{ij}^{(1)}(\theta) + A_2 \bar{r}^t \tilde{\sigma}_{ij}^{(2)}(\theta) + \frac{A_2^2}{A_1} \bar{r}^{2t-s} \tilde{\sigma}_{ij}^{(3)}(\theta) \quad (3)$$

where the $\tilde{\sigma}_{ij}^{(k)}$ functions must be found by solving the fourth order non-linear homogenous differential equation independently for plane stress and plane strain [4], s is the power exponent, which is identical to the one in the HRR solution (s may be calculated as $s=-1/(n+1)$), t is the power exponent for the second term of the asymptotic expansion, which must be found numerically by solving the fourth order non-linear homogenous differential equation independently for plane stress and plane strain [4], $\bar{r}$ is the normalized term of the asymptotic expansion, which must be found numerically by solving the fourth order non-linear homogenous differential equation independently for plane stress and plane strain [4], $\bar{r}$ is the normalized distance from the crack tip calculated as $\bar{r} = r/(J/\sigma_0)$, A_1 is the amplitude of the first term of the infinite series evaluated as $A_1 = (\alpha \varepsilon_0 I_n)^{-1/(n+1)}$, and A_2 is the amplitude of the second term, which is calculated by fitting the Eq. (3) to the numerical results of the stress fields close to crack tip.

Based on Li and Wang [3], O'Dowd et al. [5–7] proposed a simpler solution than that of Yang et al.. They assumed that the FEM results are exact and computed the difference between the numerical and HRR results. They proposed that the stress field near the crack tip may be described by following equation:

$$\frac{\sigma_{ij}}{\sigma_0} = \left(\frac{J}{\alpha \varepsilon_0 I_n r} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta; n) + Q \left(\frac{r}{J/\sigma_0} \right)^q \hat{\sigma}_{ij}(\theta; n) \quad (4)$$

where $\hat{\sigma}_{ij}(\theta; n)$ are functions evaluated numerically, q is the power exponent, which value changes in the range (0; 0,071), and Q is the parameter, which is the amplitude of the second term asymptotic solution.

O'Dowd and Shih [5, 6] tested the Q -parameter in the range $J/\sigma_0 < r < 5J/\sigma_0$ near the crack tip. They showed that the Q -parameter weakly depends on crack tip distance in the range of the $\pm\pi/2$ angle. O'Dowd and Shih proposed only two terms to describe the stress field near the crack tip:

$$\sigma_{ij} = (\sigma_{ij})_{HRR} + Q\sigma_0\hat{\sigma}_{ij}(\theta) \quad (5)$$

where $(\sigma_{ij})_{HRR}$ is the HRR solution.

To avoid the ambiguity during the calculation of the Q -stress, O'Dowd and Shih [5, 6] have suggested where the Q -stress may be evaluated. It was assumed that the Q -stress should be computed at $r=2J/\sigma_0$ for $\theta=0$ direction. O'Dowd and Shih postulated that for $\theta=0$ the function $\hat{\sigma}_{\theta\theta}(\theta=0)$ is equal to 1. Therefore the Q -stress may be calculated from following relationship:

$$Q = \frac{(\sigma_{\theta\theta})_{FEM} - (\sigma_{\theta\theta})_{HRR}}{\sigma_0} \quad \text{for } \theta=0 \text{ and } \frac{r\sigma_0}{J} = 2 \quad (6)$$

where $(\sigma_{\theta\theta})_{FEM}$ is the stress value calculated using FEM and $(\sigma_{\theta\theta})_{HRR}$ is stress value evaluated from HRR solution.

During analysis, O'Dowd and Shih show that in the range of $\theta=\pm\pi/4$, the following relationships take place: $Q\hat{\sigma}_{\theta\theta} \approx Q\hat{\sigma}_{rr}$, $\hat{\sigma}_{\theta\theta}/\hat{\sigma}_{rr} \approx 1$ and $Q\hat{\sigma}_{r\theta} \approx 0$ (because $Q\hat{\sigma}_{r\theta} \ll Q\hat{\sigma}_{\theta\theta}$). Thus, the Q -stress value determines the level of the hydrostatic stress. For plane stress, the Q -parameter is equal to zero (Fig. 3a), but for plane strain, the Q -parameter is in most cases smaller then zero (Fig. 3b and Fig. 4). The Q -stress value for plane strain depends on external loading and distance from crack tip – especially for large external loads (Fig. 3b), and also on the geometry of the specimen (Fig. 4)

2. Discussion on engineering aspects of the J -A2 and J -Q theory fracture criteria based on the O'Dowd's approach

The HRR solution can be extended with more terms, to overcome the deficiencies aforementioned, using, for example, the J -A2 theory suggested by Yang et al. [4], or the O'Dowd and Shih approach – the J -Q theory [5–7].

A lot of the analysis carried out in the 1990's showed that the multi-terms description, using the three terms

of asymptotic solution [4] is better than O'Dowd's approach [5–7]. The A2 amplitude, which is used in J -A2 theory suggested by Yang et al. [4], is nearly independent from the distance of its determination, in contrast to Q -stress, which depends on the position where it is calculated. But J -A2 theory can be very burdensome, since firstly one must solve fourth order nonlinear differential equation to determine the $\hat{\sigma}_{ij}^{(k)}$ function and the t power exponent. Secondly, using FEM results, the A2 amplitude is calculated by fitting Eq. 3 to numerical results.

In the O'Dowd approach, one needs only the Q -stress (which is calculated numerically). That is why the O'Dowd approach is more convenient to use than the J -A2 theory. The J -Q theory found application in European Engineering Programs, like SINTAP [8] or FITNET [9]. The Q -stress is applied to construct the fracture criterion and for the assessment of fracture toughness of the structural component. Thus O'Dowd's theory has practical application in engineering issues. Several fracture criteria based on the O'Dowd theory that have engineering aspects are presented below.

Based on the J -Q theory, O'Dowd [10] proposed the following fracture criterion:

$$J_C = J_{IC} \left(1 - \frac{Q}{(\sigma_c/\sigma_0)} \right)^{n+1} \quad (7)$$

where J_C is the real fracture toughness for structural element characterized by geometrical constraint defined by Q -stress (which value usually is smaller then zero), J_{IC} is the fracture toughness for plane strain condition for $Q=0$ and σ_c is the critical stress according to the Ritchie-Knott-Rice hypothesis [11].

The fracture criterion proposed by O'Dowd was discussed by Neimitz et al. [12], where the authors proposed another criterion form. They modified Eq. (7) by replacing the critical stress σ_c by maximum opening stress σ_{max} , which must be evaluated numerically using large strain formulation. Proposed by Neimitz et al., the fracture criterion has the following form:

$$J_C = J_{IC} \left(1 - \frac{Q}{(\sigma_{max}/\sigma_0)} \right)^{n+1} \quad (8)$$

For single edge notch in bending (SEN(B)), Neimitz et al. [12], using finite element method and the large strain formulation, estimated the maximum opening stress σ_{max} for several materials (different R-O exponent, different yield stress) and for several crack lengths.

Real fracture toughness K_{mat}^C , may be evaluated using the formula proposed by Ainsworth and O'Dowd [13]. Ainsworth and O'Dowd have shown that the increase in

Figure 3. The J - Q trajectories measured at six distance near the crack tip for center cracked plate in tension specimen (CC(T)): a) plane stress; b) plane strain.

Figure 4. The comparison of the J - Q trajectories (a) and $Q=f(\log(J/(a \cdot \sigma_0)))$ trajectories (b) for CC(T) and SEN(B) specimen measured at the distance $r=2.0 \cdot J/\sigma_0$ near the crack tip.

fracture in both the brittle and ductile regimes may be represented by an expression of the form:

$$K_{mat}^C = \begin{cases} K_{mat} & \beta L_r > 0 \\ K_{mat} [1 + \alpha (-\beta L_r)^k] & \beta L_r < 0 \end{cases} \quad (9)$$

where K_{mat} is the fracture toughness for plane strain condition obtained using FITNET procedures, and β is the parameter calculated using following formula:

$$\beta = \begin{cases} T/(L_r \sigma_0) & \text{for elastic materials} \\ Q/L_r & \text{for elastic-plastic materials} \end{cases} \quad (10)$$

where L_r is the ratio of the actual external load P and the limit load P_0 (or the reference stress), which may be calculated using FITNET procedures [9].

The parameters α and k in Eq. (9) are material and temperature dependent constants (see Table 1). Sherry et

Table 1. Some values of the α and k parameters in Eq. (9) [8, 9].

Material	Temperature	Fracture mode	α	k
A533B (steel)	-75°C	cleavage	1.0	1.0
A533B (steel)	-90°C	cleavage	1.1	1.0
A533B (steel)	-45°C	cleavage	1.3	1.0
Low Carbon Steel	-50°C	cleavage	1.3	2.0
A515 (steel)	+20°C	cleavage	1.5	1.0
ASTM 710 Grade A	+20°C	ductile	0.0	1.0
			0.6	1.0
			1.0	2.0

al. [14, 15] proposed designation procedures to calculate the constants α and k . Using any fracture criterion (such as Eq. (7), Eq. (8), Eq. (9), or others presented in [16]), one can estimate fracture toughness quite fast, if the Q -stress is known.

Sometimes the application of J - Q theory may be limited, because there of lack of value of the Q -stress for a given material or specimen. Literature does not provide the Q -stress catalogue and Q -stress value as a function of external load, material properties or the geometry of the specimen. Only in some articles one may find the J - Q graphs for a certain group of material. An ideal solution is the production of a catalogue of the J - Q graphs for materials characterized by various yield strength and different work-hardening exponent. Such catalogue should take into consideration the influence of the external load, the type of specimen (SEN(B) specimen - bending, CC(T) - tension or SEN(T) - tension) and the geometry of the specimen. For SEN(B) specimens, such catalogue is presented in [17, 18], which presents Q -stress values for specimens with a predominance of bending for different materials and crack length. The numerical analysis presented in [17, 18] shows that the Q parameter depends on material properties, specimen geometry and external load. In these works authors presented the full catalogue of the numerical solutions (the Q -stress values, J - Q trajectories) for single edge notch in bending specimen (SEN(B)) for different materials (different yield stress and work-hardening exponents) and different crack lengths.

In the literature there is no similar catalogue for specimens with predominance of tension. Therefore in the next sections of this paper, the values of the Q -stress will be determined for various elastic-plastic materials for center cracked plate in tension (CC(T)) and for single edge notch in tension specimen (SEN(T)). Both specimens - CC(T) and SEN(T), are the basic structural element used in the FITNET procedures [9] for modeling real structures. All results will be presented in the graphic form - $Q = f(J)$ graphs. Finally, the numerical results will be approximated by closed form formulas.

3. Details of numerical models and numerical analysis

In the numerical analysis, the centre cracked plate in tension (CC(T)) (Figure 5a) and the single edge notch in tension specimen (SEN(T)) (Figure 5b), were used. Dimensions of the specimens satisfy the standard requirement of finite element method (FEM) calculation, $L \geq 2W$, where W is the width of the specimen and L is the measuring length of the specimen. Computations were performed for plane strain using small strain option. The relative crack length was a $a/W = \{0.05 ; 0.20 ; 0.50 ; 0.70\}$ where a is the crack length and the width of specimens W was equal to 40 mm. (for this case, the measuring length $L \geq 80$ mm). All geometrical dimension of the CC(T) and SEN(T) specimens are presented in Table 2.

The choice of the CC(T) and SEN(T) specimens was intentional, as both specimens are used in the FITNET procedures [9] for modeling real structural elements. Also in FITNET procedures, the limit load and stress intensity factors solutions for CC(T) and SEN(T) specimens are presented. However in the EPRI procedures [19], the hybrid method for calculating the J -integral, crack opening displacement (COD) or crack tip opening displacement (CTOD) are given. Also experimental tests may be performed in order to determine the critical values of the J -integral, using the CC(T) specimen, as is presented in [20].

Computations were performed using ADINA SYSTEM 8.4 [21, 22]. Due to the symmetry, only a quarter of the specimen was modeled for CC(T) specimens and a half of the specimen was modeled for SEN(T) specimens. The finite element mesh was filled with 9-node plane strain elements. The size of the finite elements in the radial direction was decreasing towards the crack tip, while in the angular direction the size of each element was kept constant. The crack tip region was modeled using 36 semi-circles. The first of which was 20 times smaller than the last one. This also means that the first finite element behind to crack tip is 2000 times smaller than the width of the specimen. The crack tip was modeled as quarter of the arc with a radius equal to $rw = (1 / 2.5) \cdot 10^{-6}$ m $((0.000025 / 0.0000625) \times W)$. The whole CC(T) specimen was modeled using about 450 finite elements and 1700 nodes. The whole SEN(T) specimen was modeled using 323 finite elements and 1353 nodes. External load was applied to the top and bottom edge of the specimen (Figure 6). A sample of the finite element model for SEN(T) specimen used in the numerical analysis is presented in Figure 6, where the crack tip model for CC(T) specimen is shown.

In the FEM simulation, the deformation theory of plasticity and the von Mises yield criterion were adopted. In the model the stress-strain curve was approximated by the relation:

$$\frac{\varepsilon}{\varepsilon_0} = \begin{cases} \sigma/\sigma_0 & \text{for } \sigma \leq \sigma_0 \\ \alpha (\sigma/\sigma_0)^n & \text{for } \sigma > \sigma_0 \end{cases} \quad (11)$$

where $\alpha = 1$. The tensile properties for the materials which were used in the numerical analysis are presented in Table 3. In the FEM analysis, calculations were performed for sixteen materials, with different yield stresses and work hardening exponents.

The J -integral was calculated using two methods. The first method, called the "virtual shift method" [21, 22], uses the concept of the virtual crack growth to compute the virtual energy change. One of the twelve integral contours for calculation of the J -integral using virtual shift method is presented in Figure 6c. The second method is based on the J -integral definition:

$$J = \int_C [w dx_2 - t (\partial u / \partial x_1) ds] \quad (12)$$

where w is the strain energy density, t is the stress vector acting on the contour C drawn around the crack tip, u denotes displacement vector and ds is the infinitesimal segment of contour C . Both methods gave the same J -integral value.

In the numerical analysis 64 CC(T) specimens and 64 SEN(T) specimens were modeled, which differed by crack length (different a/W) and material properties (different ratio σ_0/E and values of the power exponent n).

4. Numerical results - analysis of the J - Q trajectories for CC(T) and SEN(T) specimen

The analysis of the results obtained for CC(T) and SEN(T) specimens was made in the range $J/\sigma_0 < r < 6 J/\sigma_0$ near the crack tip, and it is shown that the Q -stress decreases if the distance from the crack tip increases (Figure 7). If the external load increases, the Q -stress decreases and the difference between Q -stress calculated in the subsequent measurement points increases.

For the sake of the fact that the Q -parameter, which is used in fracture criterion, is calculated at distance equal to $r = 2.0 \cdot J/\sigma_0$, it is necessary to note that:

- if the crack length decreases, then Q -stress reaches more negative value for the same J -integral level (Figure 8); this conclusion is true for CC(T) and for SEN(T) specimens;

Figure 5. The comparison of the J - Q trajectories (a) and a) Centre cracked plate in tension (CC(T)); b) Single-edge notched plate in tension (SEN(T)).

Figure 6. a) Finite element model for SEN(T) specimen used in numerical analysis; b) Crack tip model used to modeling the CC(T) specimen; c) One of the twelve integral contours for calculation of the J -integral using virtual shift method.

Table 2. Geometrical dimension of the CC(T) and SEN(T) specimens used in numerical analysis.

width, W [mm]	measuring length, $4W$ [mm]	total length $2L$ [mm]	relative crack length, a/W	crack length, a [mm]	length of the non-cracked section of the specimen $b = (W - a)$ [mm]
40	160	176	0.05	2	38
			0.20	8	32
			0.50	20	20
			0.70	28	12

Table 3. The mechanical properties of the materials used in numerical analysis.

yield stress, σ_0 [MPa]	Young's modulus, E [MPa]	Poisson's ratio, ν	strain corresponding to yield stress, $\epsilon_0 = \sigma_0/E$	constant used in the constitutive relationship (11), α	work hardening exponent for relationship (11), n	Functions characterized for HRR solution	
						$\bar{\sigma}_{\theta\theta}(\theta = 0)$	ln
315	206000	0.3	0.00153	1	3	1.94	5.51
500			0.00243		5	2.22	5.02
1000			0.00485		10	2.50	4.54
1500			0.00728		20	2.68	4.21

Figure 7. "The J-Q family curves" for CC(T) and SEN(T) specimens calculated at six distances r for plane strain - $a/W=0.50$, $n=10$, $\sigma_0=1000$ MPa.**Figure 8.** The influence of the relative crack length for CC(T) and SEN(T) specimens for $n=10$, $\sigma_0=1000$ MPa (plane strain for distance from crack tip $r=2.0 \cdot J/\sigma_0$).

- for short cracks in CC(T) specimens, it is observed that Q -parameter changes faster if the external load is increased (Figure 8a);
- for CC(T) specimens characterized by short crack, the J - Q curves reach the saturation level faster, compared to CC(T) specimens characterized by normative ($a/W = 0.50$) and long ($a/W = 0.70$) cracks;
- for short cracks in SEN(T) specimens, the Q -stress value drops more rapidly than for long ones in the range of small external loads (Figure 8b);
- for SEN(T) specimen with long cracks ($a/W = 0.70$), another nature of the J - Q trajectories was observed relative to specimen with relative cracks length $a/W \leq 0.50$ (Figure 8b); this may be a consequence of the absence of the stress field due to the bending stress near the crack tip in the analysis, discussed by Chao et al. [23];
- if the yield stress increases, the Q -parameter, for all CC(T) and SEN(T) specimens with different crack length a/W increases too (Figure 9);

Figure 9. The influence of the yield stress on J - Q trajectories for CC(T) and SEN(T) specimens for $a/W=0.50$, $n=10$ (plane strain for distance from crack tip $r=2.0 J/\sigma_0$).

- for smaller yield stress for CC(T) and SEN(T) specimens, the J - Q trajectories run lower and it is observed that the Q -parameter changes faster if the external load increases (Figure 9);
- for smaller yield stress, the J - Q curves for CC(T) specimens reach the saturation level for bigger external loads than J - Q curves for CC(T) specimens characterized by large yield stress;
- for CC(T) specimen, if the yield stress decreases, the differences between the J - Q trajectories characterized for materials described by different work-hardening exponents are larger;
- comparing the J - Q trajectories for different values of σ_0/E , it is observed that the largest differences are characterized for materials with small work-hardening exponent ($n = 3$ – strongly work-hardening materials) and the smallest differences are found for materials characterized by large work-hardening exponent ($n = 20$ – weakly work-hardening materials). If the crack length increases, this differences somewhat increase too; this conclusion is the same for both types of the specimens used in the numerical analysis;
- for CC(T) specimens, the ambiguous behavior of the J - Q trajectories dependency on the work-hardening exponent is observed more than for SEN(B) specimen, which was presented in [17, 18]; for most cases (different relative crack length a/W , different yield stress ($\sigma_0/E \geq 0.00364$)), if the work-hardening exponent is smaller (strongly work-hardening materials) then Q -stress value increases (Figure 11b); for small yield stress ($0.00153 \leq \sigma_0/E \leq 0.00200$)

Figure 10. The influence of the work-hardening exponent on J - Q (a) and $Q=f(\log(J/(a\sigma_0)))$ (b) trajectories for CC(T): $a/W=0.20$, $\sigma_0=315$ MPa (plane strain for distance from crack tip $r=2.0 J/\sigma_0$).

if the external load increases, then the Q -stress value decreases if the work-hardening exponent decreases (Figure 10a); for materials characterized by yield stress $\sigma_0/E = 0.00243$, the difference between the J - Q trajectories are small and the mutual intersecting and overlapping of the trajectories is observed (Figure 11a);

- for SEN(T) specimens, also the ambiguous behavior of the J - Q trajectories dependency on the work-hardening exponent is observed; for specimens with short cracks ($a/W \leq 0.20$) and the same yield stress, for smaller values of the work-hardening exponent n (e.g. $n = 5$), the Q -stress becomes less negative (Figure 12a); for specimens with the normative crack length ($a/W = 0.50$) or with the long cracks ($a/W = 0.70$), the interruption of the J - Q trajectories was observed (Figure 12b and Figure 13) – at first higher values of Q -stress were observed for specimens characterized by strongly hardening material, but for increasing external loads a reversal of the trend took place and higher Q -stresses were observed for specimens characterized by weakly hardening materials.

5. Approximation of the numerical results for CC(T) and SEN(T) specimens

In literature, the mathematic formulas for calculating the Q -stress taking into consideration the level of external load, material properties and the geometry of the specimen

Figure 11. The influence of the work-hardening exponent on J - Q trajectories for CC(T): a) $a/W=0.50$, $\sigma_0=500\text{MPa}$; b) $a/W=0.70$, $\sigma_0=1000\text{MPa}$; (plane strain at distance from crack tip $r=2.0 \cdot J/\sigma_0$).

Figure 12. The influence of the work-hardening exponent on J - Q trajectories for SEN(T) specimens: a) SEN(T), $a/W=0.20$, $\sigma_0=315\text{MPa}$; b) SEN(T), $a/W=0.50$, $\sigma_0=500\text{MPa}$.

are not known for most cases. Presented in this paper are numerical computations provided with the J - Q catalogue and the universal formula (13), which allow the calculation of the Q -stress and take into consideration all the parameters influencing the value of the Q -stress. All results for CC(T) and SEN(T) specimens, are presented in the $Q = f(\log(J/(a \cdot \sigma_0)))$ graphical forms (e.g. Figure 10b and Figure 14). All graphs have been approximated by simple mathematical formulas, taking the material properties, external load and specimen geometry into consideration. All the approximations were made for results obtained at the distance $r = 2.0 \cdot J/\sigma_0$.

Each of the obtained trajectories $Q = f(\log(J/(a \cdot \sigma_0)))$ for CC(T) and for SEN(T) specimens, were approximated by

Figure 13. The influence of the work-hardening exponent on J - Q trajectories for SEN(T) specimens with relative crack length $a/W=0.70$.

Figure 14. The influence of the work-hardening exponent on $Q=f(\log(J/(a \cdot \sigma_0)))$ trajectories for CC(T) specimen: a) $a/W=0.50$, $\sigma_0=500\text{MPa}$; b) $a/W=0.70$, $\sigma_0=1000\text{MPa}$; which were used in the procedure of approximation.

the third order polynomial in the form:

$$Q(J, a, \sigma_0) = A + B \cdot \left(\log \left(\frac{J}{(a \cdot \sigma_0)} \right) \right) + C \cdot \left(\log \left(\frac{J}{(a \cdot \sigma_0)} \right) \right)^2 + D \cdot \left(\log \left(\frac{J}{(a \cdot \sigma_0)} \right) \right)^3 \quad (13)$$

where the A , B , C , D coefficients depend on the work-hardening exponent n , yield stress σ_0 and crack length a/W .

The squared residual of fitting the formula (13) to numerical results for CC(T) specimens for the worst case was equal $R^2 = 0.94$ for crack length $a/W = 0.05$. For other crack lengths $a/W = \{0.20, 0.50, 0.70\}$, the squared residual satisfy the condition $R^2 \geq 0.99$. For SEN(T) specimens, the squared residual for the worst case was equal $R^2 = 0.95$. For different work hardening exponents n , yield stresses σ_0 and ratios a/W , which were not included in the numerical analysis, the coefficients A , B , C and D may be evaluated using the linear or quadratic approximation.

Results of the numerical approximation using formula (13) for CC(T) specimens (all coefficients and regression residuals) are presented in Tables 4-7. Tables 8-11 present results of the approximation for SEN(T) specimens.

Figures 15 and 16 present the comparison of the numerical results and their approximation for J - Q trajectories for several cases of the CC(T) and SEN(T) specimens, respectively. The annexes attached to this article (Annexes A-H) present in the graphical form all numerical results obtained for CC(T) and SEN(T) specimens in plain strain. All results are presented using the J - Q trajectories for each analyzed case.

Figure 15. Comparison of the numerical results and their approximation for J - Q trajectories for CC(T) specimens: a) $a/W=0.50$; $\sigma_0=\{315, 500\}$ MPa; $n=\{5, 10\}$; b) $a/W=0.50$; $\sigma_0=\{1000, 1500\}$ MPa; $n=\{10, 20\}$.

Figure 16. Comparison of the numerical results and their approximation for J - Q trajectories for SEN(T) specimens with relative crack length $a/W=0.50$: a) $\sigma_0=\{315, 500\}$ MPa, $n=\{5, 10\}$; b) $\sigma_0=\{1000, 1500\}$ MPa, $n=\{10, 20\}$.

6. Conclusions

In this paper the values of the Q -stress were determined for various elastic-plastic materials for two types of the specimens under tensile loading – for centre cracked plate in tension (CC(T)) and for single edge notched specimens in tension (SEN(T)). The influence of the crack length and the material properties, such the yield stress and the work-hardening exponent on the Q -parameter (often called by Q -stress) was tested. The results of each case were presented in graphical form using the J - Q trajectories. All the numerical results were approximated by closed form formulas. The coefficients of the approximations were presented in eight tables.

The most important conclusions of the analyzed numerical results are summarized as follows:

- the Q -stress depends on specimens geometry and the external load; different values of the Q -stress are obtained for center cracked plane in tension (CC(T)) and for the SEN(T) specimen, which are characterized by the same material properties;
- the Q -parameter is a function of the material properties; its value depends on the work-hardening exponent n and the yield stress σ_0 ;
- if the crack length decreases, then Q -stress reaches more negative value under the same applied load.

In this paper a catalogue of the Q -stress value and J - Q trajectories for specimens with predominance of tension (CC(T) and SEN(T) specimens) are presented. These are complementary of the numerical solutions presented in [17, 18], which gave J - Q trajectories for specimens under predominantly bending load (SEN(B) specimens)). Both papers may be quite useful for solving engineering problems where the fracture toughness or the stress distribution near a crack tip must be estimated readily.

Acknowledgements

The support of the Kielce University of Technology – Faculty of Mechatronics and Machine Design through grant No 1.22/7.14 is acknowledged by the author of the paper.

Table 4. The coefficients of equation (13) for CC(T) specimen with the crack length $a/W=0.05$.

$a/W = 0.05$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-1.79540	-0.16046	0.00270	-0.05173	0.979	3	-1.54832	-0.56730	-0.33063	-0.11413	0.982
5	-1.84658	-0.29915	-0.12998	-0.07354	0.982	5	-1.72656	-0.63071	-0.33882	-0.10721	0.986
10	-1.84196	-0.61308	-0.41668	-0.13219	0.961	10	-1.49156	-0.10931	-0.03536	-0.05032	0.989
20	-1.74217	-0.60418	-0.44835	-0.13965	0.939	20	-1.60795	-0.40632	-0.28062	-0.10566	0.996
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-1.61802	-0.35121	-0.26183	-0.12290	0.991	3	-1.33418	-0.24308	-0.05234	-0.04542	0.951
5	-1.74621	-0.47823	-0.33828	-0.12560	0.980	5	-1.51558	-0.28024	-0.05331	-0.03969	0.970
10	-1.79245	-0.70894	-0.52808	-0.16144	0.969	10	-1.52391	-0.19560	-0.02477	-0.03644	0.981
20	-1.74847	-0.77333	-0.63058	-0.18304	0.934	20	-1.59474	-0.30780	-0.10917	-0.05471	0.984

Table 5. The coefficients of equation (13) for CC(T) specimen with the crack length $a/W=0.20$.

$a/W = 0.20$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-3.40016	-2.97172	-1.63678	-0.33292	0.995	3	-3.65130	-3.39214	-1.55538	-0.26722	0.973
5	-2.81279	-2.11444	-1.22345	-0.26738	0.990	5	-1.67933	-0.35103	-0.11710	-0.05261	0.995
10	-2.23934	-1.40529	-0.89907	-0.21962	0.999	10	-1.49619	-0.12541	-0.07218	-0.05963	0.997
20	-2.13638	-1.32808	-0.84394	-0.20595	1.000	20	-1.53751	-0.18282	-0.11915	-0.07285	0.991
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-2.27413	-1.18967	-0.58590	-0.13421	0.992	3	-1.27982	0.01996	0.14503	0.01481	0.989
5	-2.29981	-1.38659	-0.81232	-0.19438	0.997	5	-1.41550	-0.01120	0.11191	-0.00153	0.994
10	-2.42665	-1.88288	-1.19659	-0.27781	0.998	10	-1.54844	-0.25319	-0.11256	-0.06390	0.993
20	-2.57462	-2.29845	-1.48745	-0.33839	0.997	20	-1.67907	-0.45441	-0.25880	-0.10071	0.994

Table 6. The coefficients of equation (13) for CC(T) specimen with the crack length $a/W=0.50$.

$a/W = 0.50$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-3.85021	-2.64950	-1.05024	-0.17336	0.990	3	-3.42176	-2.52110	-0.90895	-0.13071	0.977
5	-2.54684	-1.13625	-0.51015	-0.11358	0.997	5	-1.63674	0.01781	0.19411	0.01940	0.994
10	-2.24656	-0.98456	-0.50146	-0.11605	0.997	10	-1.68070	-0.08345	0.05407	-0.02520	0.997
20	-3.18413	-2.55066	-1.29798	-0.24468	0.996	20	-1.88835	-0.39917	-0.13029	-0.06196	0.996
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-3.55938	-2.47891	-0.95900	-0.15488	0.983	3	-1.27394	-0.01699	0.04887	-0.01944	0.997
5	-0.96124	1.06111	0.56236	0.05471	0.998	5	-1.57516	-0.02788	0.20193	0.03045	0.994
10	-1.61943	-0.06732	-0.04079	-0.04818	0.999	10	-1.94130	-0.43244	0.01182	-0.00341	0.994
20	-2.39669	-1.42289	-0.78360	-0.17829	0.999	20	-2.07357	-0.56398	-0.06157	-0.02087	0.995

Table 7. The coefficients of equation (13) for CC(T) specimen with the crack length $a/W=0.70$.

$a/W = 0.70$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-3.39313	-1.86453	-0.70116	-0.12257	0.991	3	-2.93370	-1.54934	-0.40487	-0.04995	0.986
5	-1.86720	-0.11810	-0.07379	-0.05049	0.998	5	-2.16067	-0.26326	0.17639	0.02853	0.997
10	-3.70437	-2.86445	-1.35357	-0.24114	0.997	10	-2.42945	-0.26930	0.25072	0.04251	0.998
20	-5.11211	-4.87495	-2.27736	-0.37934	0.997	20	-2.39733	-0.11232	0.32626	0.05005	0.998
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-6.77352	-5.84374	-2.12683	-0.28448	0.981	3	-2.02517	-0.78494	-0.17842	-0.02840	0.992
5	-2.14513	-0.20561	0.07296	-0.00700	0.997	5	-2.05092	-0.46715	0.02955	0.00524	0.989
10	-2.09694	-0.47913	-0.18855	-0.06352	0.998	10	-2.10385	-0.19800	0.21559	0.03379	0.992
20	-2.21086	-0.88306	-0.48750	-0.12529	0.999	20	-2.31937	-0.32264	0.19251	0.03093	0.996

Table 8. The coefficients of equation (13) for SEN(T) specimen with the crack length $a/W=0.05$.

$a/W = 0.05$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-3.28530	-4.36483	-2.94099	-0.68053	0.992	3	-1.99711	-2.14485	-1.41276	-0.33698	0.992
5	-3.24056	-4.31160	-2.96935	-0.69539	0.995	5	-1.96974	-1.85874	-1.23020	-0.31170	0.994
10	-2.47635	-2.73475	-1.95688	-0.48987	0.997	10	-1.58899	-0.74181	-0.41549	-0.13920	0.996
20	-2.00489	-1.73313	-1.33384	-0.37069	0.998	20	-1.46327	-0.24652	-0.01392	-0.04870	0.996
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-2.99628	-4.03835	-2.75829	-0.64898	0.992	3	-1.56950	-1.32827	-0.82071	-0.19907	0.991
5	-2.79399	-3.50018	-2.40158	-0.56961	0.998	5	-1.59085	-1.04309	-0.59431	-0.15738	0.994
10	-2.19391	-2.17163	-1.55331	-0.40339	0.999	10	-1.41621	-0.30137	-0.00645	-0.02979	0.996
20	-1.79826	-1.20852	-0.89575	-0.26807	0.999	20	-1.39250	-0.00158	0.27440	0.03657	0.997

Table 9. The coefficients of equation (13) for SEN(T) specimen with the crack length $a/W=0.20$.

$a/W = 0.20$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-2.47638	-2.22144	-1.16457	-0.22834	0.993	3	-1.87486	-1.43826	-0.65090	-0.11928	0.958
5	-2.12775	-1.72171	-0.99857	-0.22328	0.997	5	-1.19787	0.00713	0.21704	0.03662	0.990
10	-1.75244	-0.99122	-0.60396	-0.16295	0.998	10	-1.06536	0.55158	0.62207	0.11569	0.995
20	-1.67749	-0.68307	-0.37942	-0.12146	0.997	20	-1.16311	0.53317	0.65408	0.12193	0.996
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-1.61800	-0.87556	-0.39728	-0.08676	0.986	3	-1.60078	-1.09898	-0.47734	-0.08861	0.982
5	-1.10512	0.10449	0.11856	-0.00844	0.996	5	-1.46909	-0.53684	-0.05638	-0.00207	0.990
10	-1.36486	-0.13911	0.02871	-0.02643	0.996	10	-1.40095	-0.07778	0.32773	0.08000	0.996
20	-1.46487	-0.14521	0.07469	-0.01694	0.996	20	-1.48602	-0.08540	0.36420	0.08777	0.996

Table 10. The coefficients of equation (13) for SEN(T) specimen with the crack length $a/W=0.50$.

$a/W = 0.50$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-2.74334	-1.60675	-0.45633	-0.05877	0.990	3	-4.00852	-4.03116	-1.62916	-0.22925	0.977
5	-2.90920	-1.51579	-0.33435	-0.03755	0.990	5	-2.66219	-1.86911	-0.54490	-0.05925	0.997
10	-0.62143	1.91271	1.29119	0.20480	0.996	10	-2.77342	-1.76034	-0.40331	-0.03218	0.996
20	0.23829	3.36391	2.03142	0.31983	0.996	20	-2.97077	-1.78947	-0.31156	-0.00628	0.997
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-3.92704	-3.61508	-1.43453	-0.20919	0.982	3	-2.61243	-2.33500	-0.94277	-0.13764	0.994
5	-3.38336	-2.41420	-0.72782	-0.08761	0.995	5	-2.50468	-1.89491	-0.62886	-0.07803	0.999
10	-2.00914	-0.13182	0.43492	0.09418	0.997	10	-2.55895	-1.68753	-0.41997	-0.03523	0.996
20	-1.81015	0.45038	0.81088	0.16036	0.997	20	-2.35690	-1.04098	0.04823	0.05900	0.997

Table 11. The coefficients of equation (13) for SEN(T) specimen with the crack length $a/W=0.70$.

$a/W = 0.70$											
$\sigma_0 = 315\text{MPa}$			$\sigma_0/E = 0.00153$			$\sigma_0 = 1000\text{MPa}$			$\sigma_0/E = 0.00485$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-6.05065	-4.76210	-1.51230	-0.17853	0.989	3	-6.70261	-6.47126	-2.32311	-0.28584	0.985
5	-3.28652	-0.87161	0.17082	0.04914	0.991	5	-7.23736	-6.93731	-2.45603	-0.30050	0.996
10	0.29014	3.71005	2.04549	0.29372	0.993	10	-7.64216	-7.19820	-2.48068	-0.29742	0.998
20	4.42431	8.93058	4.17516	0.57390	0.993	20	-8.52732	-8.05839	-2.74713	-0.32501	0.997
$\sigma_0 = 500\text{MPa}$			$\sigma_0/E = 0.00243$			$\sigma_0 = 1500\text{MPa}$			$\sigma_0/E = 0.00728$		
n	A	B	C	D	$R2$	n	A	B	C	D	$R2$
3	-8.57507	-8.07151	-2.81751	-0.34119	0.989	3	-5.57993	-5.46233	-2.02119	-0.25563	0.976
5	-10.4696	-9.90818	-3.41653	-0.40967	0.997	5	-5.81914	-5.57557	-2.01119	-0.24998	0.995
10	-11.0363	-9.95801	-3.22667	-0.36490	0.998	10	-5.99049	-5.60764	-1.96142	-0.23824	0.998
20	-0.75327	2.84628	1.97859	0.32498	0.993	20	-7.45282	-7.31464	-2.61738	-0.32245	0.999

References

- [1] Hutchinson J.W., 1968, Singular Behaviour at the End of a Tensile Crack in a Hardening Material, *Journal of the Mechanics and Physics of Solids*, 16, 13-31
- [2] Gatkiewicz J., Graba M., 2006, Algorithm for Determination of $\tilde{\sigma}_{ij}(n, \theta)$, $\tilde{\varepsilon}_{ij}(n, \theta)$, $\tilde{u}_i(n, \theta)$, $d_n(n)$ and $I_n(n)$ Functions in Hutchinson-Rice-Rosengren Solution and its 3d Generalization, *Journal of Theoretical and Applied Mechanics*, Vol. 44, No. 1, 19-30
- [3] Li Y., Wang Z., 1985, High-Order Asymptotic Field of Tensile Plane-Strain Nonlinear Crack Problems, *Scientia Sinica (Series A)*, Vol. XXIX, No. 9, 941-955
- [4] Yang S., Chao Y.J., Sutton M.A., 1993, Higher Order Asymptotic Crack Tip in a Power-Law Hardening Material, *Engineering Fracture Mechanics*, Vol. 45, No. 1, 99. 1-20
- [5] O'Dowd N.P., Shih C.F., 1991, Family of Crack-Tip Fields Characterized by a Triaxiality Parameter – I. Structure of Fields, *J. Mech. Phys. Solids*, vol. 39, No. 8, 989-1015
- [6] O'Dowd N.P., Shih C.F., 1992, Family of Crack-Tip Fields Characterized by a Triaxiality Parameter – II. Fracture Applications, *J. Mech. Phys. Solids*, vol. 40, No. 5, 939-963
- [7] Shih C.F., O'Dowd N.P., Kirk M. T., 1993, A1993, A Framework for Quantifying Crack Tip Constraint, "Constraint Effects in Fracture", ASTM STP 1171, E.M. Hackett, K.-H. Schwalbe, R.H. Dodds, Eds., American Society for Testing and Materials, Philadelphia, 2-20
- [8] SINTAP, 1999, SINTAP: Structural Integrity Assessment Procedures for European Industry. Final Procedure, Brite-Euram Project No BE95-1426. – Rotherham: British Steel
- [9] FITNET, 2006, FITNET Report, (European Fitness-for-service Network), Edited by M.Kocak, S.Webster,

- J.J.Janosch, R.A.Ainsworth, R.Koers, Contract No. G1RT-CT-2001-05071
- [10] O'Dowd N.P., 1995, Applications of two parameter approaches in elastic-plastic fracture mechanics, *Engineering Fracture Mechanics*, Vol. 52, No. 3, 445-465
- [11] Ritchie R.O., Knott J.F., Rice J.R., 1973, On The Relationship Between Critical Tensile Stress and Fracture Toughness in Mild Steel, *Journal of the Mechanics and Physics of Solids*, Vol. 21, 395-410
- [12] Neimitz A., Graba M., Gałkiewicz J., 2007, An Alternative Formulation of the Ritchie-Knott-Rice Local Fracture Criterion, *Engineering Fracture Mechanics*, Vol. 74, 1308-1322
- [13] Ainsworth R.A., O'Dowd N.P., 1994, A1994, A Framework of Including Constraint Effects in the Failure Assessment Diagram Approach for Fracture Assessment, *ASME Pressure Vessels and Piping Conference*, PVP-Vol 287/MD-Vol47, ASME
- [14] Sherry, A.H., Wilkes M.A., Beardsmore D.W., Lidbury D.P.G., 2005, Material constraint parameters for the assessment of shallow defects in structural components – Part I: Parameter solutions, *Engineering Fracture Mechanics*, 72, 2373-2395
- [15] Sherry A.H., Hooton D.G., Beardsmore D.W., Lidbury D.P.G., 2005, Material constraint parameters for the assessment of shallow defects in structural components – Part II: constraint based assessment of shallow cracks, *Engineering Fracture Mechanics*, 72, 2396-2415
- [16] Neimitz A., Dzioba I., Molasy R., Graba M., 2004, Wpływ więzów na odporność na pękanie materiałów kruchych, *Materiały XX Sympozjum Zmęczenia i Mechaniki Pęknięcia*, Bydgoszcz-Pieczyska, 27-30 kwietnia 2004r., 265-272 (in Polish)
- [17] Graba M., 2007b, The Influence of Material Properties on the Q -stress Value near the Crack Tip for Elastic-Plastic Materials. Theoretical Backgrounds, *Transcom 2007, proceedings of the section 7 - Machines and Equipment. Applied Mechanics*; 53-57
- [18] Graba M., 2007c, The Influence of Material Properties on the Q -stress Value near the Crack Tip for Elastic-Plastic Materials. Numerical Results and their Approximation, *Transcom 2007, proceedings of the section 7 - Machines and Equipment. Applied Mechanics*; 59-63
- [19] Kumar V., German M.D., Shih C.F., 1981, An Engineering Approach for Elastic-Plastic Fracture Analysis, EPRI Report NP-1931, Electric Power Research Institute, CityPalo Alto, CA
- [20] Sumpter J.D.G., Forbes A.T., 1992, Constraint Based Analysis of Shallow Cracks in Mild Steel, TWI/EWI/IS International Conference on Shallow Crack Fracture Mechanics Test and Application, M.G. Dawes, Ed., Cambridge, UK, paper 7
- [21] ADINA 2006a, ADINA 8.4.1: User Interface Command Reference Manual – Volume I: ADINA Solids & Structures Model Definition, Report ARD 06-2, ADINA R&D, Inc., 2006
- [22] ADINA 2006b, ADINA 8.4.1: Theory and Modeling Guide – Volume I: ADINA, Report ARD 06-7, ADINA R&D, Inc., 2006
- [23] Chao Y.J., Zhu X.K., Kim Y., Lar P.S., Pechersky M.J., Morgan M.J., 2004, Characterization of Crack-Tip Field and Constraint for Bending Specimens under Large-Scale Yielding, *International Journal of Fracture*, 127, 283-302

Annex A

Numerical results for CC(T) specimen in plane strain with crack length $a/W = 0.05$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 17. The influence of the yield stress on J - Q trajectories for CC(T) specimens with crack length $a/W=0.05$ for different power exponents in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex B

Numerical results for CC(T) specimen in plane strain with the crack length $a/W = 0.20$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 18. The influence of the yield stress on J - Q trajectories for CC(T) specimens with crack length $a/W=0.20$ for different power exponents in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex C

Numerical results for CC(T) specimen in plane strain with the crack length $a/W = 0.50$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 19. The influence of the yield stress on J - Q trajectories for CC(T) specimens with crack length $a/W=0.50$ for different power exponents in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex D

Numerical results for CC(T) specimen in plane strain with the crack length $a/W = 0.70$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 20. The influence of the yield stress on J - Q trajectories for CC(T) specimens with crack length $a/W=0.70$ for different power exponents in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex E

Numerical results for SEN(T) specimen in plane strain with the crack length $a/W = 0.05$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 21. The influence of the yield stress on J - Q trajectories for SEN(T) specimens with crack length $a/W=0.05$ for different power exponents n in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex F

Numerical results for SEN(T) specimen in plane strain with the crack length $a/W = 0.20$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 22. The influence of the yield stress on J - Q trajectories for SEN(T) specimens with crack length $a/W=0.20$ for different power exponents n in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex G

Numerical results for SEN(T) specimen in plane strain with the crack length $a/W = 0.50$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 23. The influence of the yield stress on J - Q trajectories for SEN(T) specimens with crack length $a/W=0.50$ for different power exponents n in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.

Annex H

Numerical results for SEN(T) specimen in plane strain with the crack length $a/W = 0.70$ (distance from crack tip $r = 2.0 \cdot J/\sigma_0$)

Figure 24. The influence of the yield stress on J - Q trajectories for SEN(T) specimens with crack length $a/W = 0.70$ for different power exponents n in R-O relationship: a) $n=3$; b) $n=5$; c) $n=10$; d) $n=20$.