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Functional variable method to study nonlinear evolution equations

Research Article

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Abstract: The functional variable method is a powerful solution method for obtaining exact solutions of nonlinear evolution equations. This method presents a wider applicability for handling nonlinear wave equations. In this paper, the functional variable method is used to construct exact solutions of Davey-Stewartson equation, generalized Zakharov equation, $K(m,n)$ equation with generalized evolution term, (2 + 1)-dimensional long-wave-short-wave resonance interaction equation and nonlinear Schrödinger equation with power law nonlinearity. The obtained solutions include solitary wave solutions, periodic wave solutions.

Keywords: Functional variable method • Davey-Stewartson equation • Generalized Zakharov equation • $K(m,n)$ equation • Nonlinear Schrödinger equation

1. Introduction

Nonlinear evolution equations are widely used to describe complex phenomena in various sciences such as fluid physics, condensed matter, biophysics, plasma physics, nonlinear optics, quantum field theory and particle physics, etc. In recent years, various powerful methods have been presented for finding exact solutions of the nonlinear evolution equations in mathematical physics, such as ansatz method and topological solitons [1–4], [28, 29], tanh method [5, 6], multiple exp-function method [7], Hirotas direct method [8–13], transformed rational function method [14] and so on.

The powerful and effective method for finding exact solutions of nonlinear evolution equations was proposed in [15, 16] by Zerarka et al., which is called the functional variable method.

Using this method in work [15–17] exact solutions of the one-dimensional Boussinesq equation, the nonlinear

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PHI-four equation, the generalized form of the Boussinesq system, the regularized long-wave (RLW) equation, the Zakharov-Kuznetsov-modified equal-width (ZK-MEW) equation, the modified Benjamin-Bona- Mahony (mBBM) equation and the modified KdV-Kadomtsev-Petviashvili (KdV-KP) equation were obtained.

The aim of this paper is to find exact solutions of Davey-Stewartson equation, generalized Zakharov equation, $K(m,n)$ equation with generalized evolution term, (2 + 1)-dimensional long-wave-short-wave resonance interaction equation and nonlinear Schrödinger equation by using the functional variable method [15, 16].

The paper is arranged as follows. In Section 2, we describe briefly the functional variable method. In Sections 3, we apply this method to Davey-Stewartson equation, generalized Zakharov equation, $K(m,n)$ equation with generalized evolution term, $(2+1)$-dimensional long-wave-short-wave resonance interaction equation and nonlinear Schrödinger equation with power law nonlinearity.

2. The functional variable method

Consider a nonlinear evolution equation:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, ...) = 0, \qquad (1)$$

where P is a polynomial in u and its partial derivatives. To find the traveling wave solution of Eq. (1) we introduce the wave variable $\xi = x - ct$ so that

$$u(x,t) = U(\xi). \qquad (2)$$

The nonlinear partial differential equation can be converted to an ordinary differential equation (ODE) as

$$Q(U, U', U'', ...) = 0, \qquad (3)$$

where Q is a polynomial in U and its total derivatives and $' = \frac{d}{d\xi}$.

Let us make a transformation in which the unknown function $U(\xi)$ is considered as a functional variable in the form

$$U_\xi = F(U) \qquad (4)$$

and some successively derivatives of U are

$$U_{\xi\xi} = \frac{1}{2}(F^2)', \qquad (5)$$

$$U_{\xi\xi\xi} = \frac{1}{2}(F^2)''\sqrt{F^2},$$

$$U_{\xi\xi\xi\xi} = \frac{1}{2}[(F^2)'''F^2 + (F^2)''(F^2)'],$$

where $' = \frac{d}{dU}$.

The ODE (3) can be reduced in terms of U, F and its derivatives upon using the expressions of Eq. (5) into Eq. (3) gives

$$R(U, F, F', F'', F''', ...) = 0. \qquad (6)$$

The key idea of this particular form Eq. (6) is of special interest because it admits analytical solutions for a large class of nonlinear wave type equations. After integration, the Eq. (6) provides the expression of F, and this in turn together with Eq. (4) give the relevant solutions to the original problem. In order to illustrate how the method works we examine some examples treated by other approaches. This matter is exposed in the following section.

3. Applications

In this section, we present four examples to illustrate the applicability of the functional variable method to solve nonlinear partial differential equations.

Example 1. Let us first consider a class of nonlinear partial differential equations [18]

$$iu_t + n(u_{xx} + \alpha_1 u_{yy}) + \beta_1 |u|^2 u + \gamma_1 uv = 0, \qquad (a) \quad (7)$$

$$\alpha_2 v_{tt} + (v_{xx} - \beta_2 v_{yy}) + \gamma_2 (|u|^2)_{xx} = 0, \qquad (b)$$

where n, α_i, β_i, γ_i $(i = 1, 2)$ are real constants and $n \neq 0$, $\beta_1 \neq 0$, $\gamma_1 \neq 0$, $\gamma_2 \neq 0$. The important cases of Eq. (7) are as follows. In fact, if one takes

$$n = \frac{1}{2}\sigma^2, \quad \alpha_1 = 2n, \quad \beta_1 = K, \quad \gamma_1 = -1,$$

$$\alpha_2 = 0, \quad \beta_2 = \alpha_1, \quad \gamma_2 = -2K \quad \sigma^2 = \pm 1,$$

then Eq. (7) represent the Davey-Stewartson (DS) equations [25-27]

$$iu_t+\frac{1}{2}\sigma^2(u_{xx}+\sigma^2u_{yy})+K|u|^2u-uv=0, \qquad (a) \quad (8)$$

$$v_{xx}-\sigma^2v_{yy}-2K(|u|^2)_{xx}=0. \qquad (b)$$

Also, if one takes

$$n=1, \quad \alpha_1=0, \quad \beta_1=-2K, \quad \gamma_1=2,$$

$$\beta_2=0, \quad \alpha_2=-1, \quad \gamma_2=-1,$$

then Eq. (7) become the generalized Zakharov (GZ) equations

$$iu_t+u_{xx}-2K|u|^2u+2uv=0, \qquad (a) \quad (9)$$

$$v_{tt}-v_{xx}+(|u|^2)_{xx}=0. \qquad (b)$$

We use the transformations

$$u(x,y,t)=e^{i(px+qy+kt)}U(\xi),$$
$$v(x,y,t)=V(\xi),$$
$$\xi=x+cy-2n(p+\alpha_1qc)t, \qquad (10)$$

where p,q,k and c are constants to be determined later. Substituting (10) into (9), we obtain ordinary differential equation:

$$-(K+p^2n+n\alpha_1q^2)U+(n+n\alpha_1c^2)U''+\beta_1U^3+\gamma_1UV=0, \qquad (a) \quad (11)$$

$$(4n^2\alpha_2(p+\alpha_1qc)^2-\beta_2c^2+1)V''+\gamma_2(U^2)''=0. \qquad (b)$$

Integrating Eq. (11b) twice with respect to ξ and taking the integration constant as zero yields

$$V(\xi)=-CU^2(\xi), \qquad (12)$$

where

$$C=\frac{\gamma_2}{4n^2\alpha_2(p+\alpha_1qc)^2-\beta_2c^2+1}.$$

Substituting Eq. (12) into Eq. (11a) yields

$$U''-AU-BU^3=0, \qquad (13)$$

where

$$A=\frac{K+p^2n+n\alpha_1q^2}{n+n\alpha_1c^2},$$

$$B=\frac{-\beta_1(4n^2\alpha_2(p+\alpha_1qc)^2-\beta_2c^2+1)+\gamma_1\gamma_2}{(n+n\alpha_1c^2)(4n^2\alpha_2(p+\alpha_1qc)^2-\beta_2c^2+1)}.$$

According to Eq. (5), we get from Eq. (13) the expression of the expression of the function $F(U)$ reads

$$F(U)=\sqrt{A}U\sqrt{1+\frac{B}{2A}U^2}. \qquad (14)$$

Using transformation (4), and then setting the constants of integration to zero, we can obtain the following result:

$$U(\xi) = \pm\sqrt{\frac{2A}{B}}csch(\sqrt{A}\xi). \qquad (15)$$

When $A>0$, We have the following hyperbolic solutions:

$$u_1(x,y,t)= \pm\sqrt{\frac{2A}{B}}csch[\sqrt{A}(x+cy-2n(p+\alpha_1qc)t)]e^{i(px+qy+kt)}, \qquad (16)$$

$$v_1(x,y,t)=-\frac{2AC}{B}csch^2[\sqrt{A}(x+cy-2n(p+\alpha_1qc)t)].$$

$$u_2(x,y,t)= \pm\sqrt{\frac{-2A}{B}}sech[\sqrt{A}(x+cy-2n(p+\alpha_1qc)t)]e^{i(px+qy+kt)}, \qquad (17)$$

$$v_2(x,y,t)=\frac{2AC}{B}sech^2[\sqrt{A}(x+cy-2n(p+\alpha_1qc)t)].$$

When $A<0$, we can obtain the following periodic solutions

$$u_3(x,y,t)= \pm\sqrt{\frac{-2A}{B}}\csc[\sqrt{-A}(x+cy-2n(p+\alpha_1qc)t)]e^{i(px+qy+kt)}, \qquad (18)$$

$$v_3(x,y,t)=\frac{2AC}{B}\csc^2[\sqrt{-A}(x+cy-2n(p+\alpha_1qc)t)].$$

$$u_4(x,y,t)=\pm\sqrt{\frac{-2A}{B}}\sec[\sqrt{-A}(x+cy-2n(p+\alpha_1 qc)t)]e^{i(px+qy+kt)}, \tag{19}$$

$$v_4(x,y,t)=\frac{2AC}{B}\sec^2[\sqrt{-A}(x+cy-2n(p+\alpha_1 qc)t)].$$

Comparing our results with Ebadi and Biswas's results [25] then it can be seen that the results are same.

Example 2. Next, we consider the (2 + 1)-dimensional long-wave-short-wave resonance interaction equation [19],[28]

$$i(u_t+u_y)-u_{xx}+uv=0, \quad (a) \tag{20}$$

$$v_t-2(|u|^2)_x=0. \quad (b)$$

where u and v denote the the short surface wave packets and long interfacial wave respectively. Eq. (20) describe the long and short waves propagating at an angle of each other in a two-layer fluid. This system has been demonstrated to have both bright and dark two-soliton solutions. Using the wave transformations

$$u(x,y,t)=e^{i(px+qy+kt)}U(\xi),$$
$$v(x,y,t)=V(\xi),$$
$$\xi=x+(d-2p)y+dt, \tag{21}$$

where p, q, k and d are real constant.
Substituting (21) into (20), we have

$$U''(\xi)+(q+k-p^2)U(\xi)-U(\xi)V(\xi)=0, \quad (a) \tag{22}$$

$$dV'(\xi)-2(U^2(\xi))'=0, \quad (b)$$

Integrating Eq. (22b) with respect to ξ and taking the integration constant as zero yields

$$V=\frac{2}{d}U^2. \tag{23}$$

Substituting Eq. (23) into Eq. (22a) yields

$$\frac{2}{d}U^3-(q+k-p^2)U=U''. \tag{24}$$

Following Eq. (5), it is easy to deduce from (24) the expression of the function $F(U)$ which reads as

$$F(U)=\sqrt{p^2-q-k}U\sqrt{1-\frac{1}{d(q+k-p^2)}U^2}. \tag{25}$$

The solution of Eq. (24) is obtained as

$$U(\xi)=\pm\sqrt{d(p^2-q-k)}csch(\sqrt{p^2-q-k}\xi). \tag{26}$$

We can easily obtain the following hyperbolic solutions:

$$u_1(x,y,t)=\pm\sqrt{d(p^2-q-k)}\, csch(\sqrt{p^2-q-k}(x+(d-2p)y+dt))e^{i(px+qy+kt)}, \tag{27}$$

$$v_1(x,y,t)=2(p^2-q-k)$$

$$csch^2(\sqrt{p^2-q-k}(x+(d-2p)y+dt)).$$

$$u_2(x,y,t)=\pm\sqrt{d(q+k-p^2)}\, sech(\sqrt{p^2-q-k}(x+(d-2p)y+dt))e^{i(px+qy+kt)}, \tag{28}$$

$$v_2(x,y,t)=2(q+k-p^2)$$

$$sech^2(\sqrt{p^2-q-k}(x+(d-2p)y+dt)).$$

For $p^2>q+k$, it is easy to see that solutions (27) and (28) can reduce to periodic solutions as follows:

$$u_3(x,y,t)=\pm\sqrt{d(q+k-p^2)}\, \sec(\sqrt{q+k-p^2}(x+(d-2p)y+dt))e^{i(px+qy+kt)}, \tag{29}$$

$$v_3(x,y,t)=2(q+k-p^2)$$

$$\sec^2(\sqrt{q+k-p^2}(x+(d-2p)y+dt)).$$

$$u_4(x,y,t)=\pm\sqrt{d(q+k-p^2)}$$
$$\csc(\sqrt{q+k-p^2}(x+(d-2p)y+dt))e^{i(px+qy+kt)}, \quad (30)$$

$$v_4(x,y,t)=2(q+k-p^2)$$

$$\csc^2(\sqrt{q+k-p^2}(x+(d-2p)y+dt)).$$

Example 3. In this section we study the nonlinear Schrödinger equation with power law nonlinearity [20] in the following form:

$$i\omega_t+\omega_{xx}+A|\omega|^{2n}\omega=0, \quad (31)$$

where A is a real parameter and $\omega=\omega(x,t)$ is a complex -valued function of two real variables x,t. Eq. (31) has important application in various fields, such as nonlinear optics, plasma physics, superconductivity and quantum mechanics. More details are presented [21–24].
We use the transformation

$$\omega(x,t)=e^{i(\alpha x+\beta t)}U(\xi), \quad \xi=k(x-2\alpha t), \quad (32)$$

where k,α and β are constants to be determined later.
Substituting (32) into (31), we obtain ordinary differential equation:

$$-(\beta+\alpha^2)U(\xi)+k^2U''(\xi)+AU^{2n+1}(\xi)=0, \quad (33)$$

According to Eq. (5), we get from Eq. (37) the expression of the function $F(U)$ reads

$$F(U)=\frac{\sqrt{\beta+\alpha^2}}{k}U\sqrt{1-\frac{A}{(\beta+\alpha^2)(n+1)}U^{2n}}. \quad (34)$$

The solution of Eq. (33) is obtained as

$$U(\xi)=\{-\frac{(\beta+\alpha^2)(n+1)}{A}csch^2(\frac{n\sqrt{\beta+\alpha^2}}{k}\xi)\}^{\frac{1}{2n}}. \quad (35)$$

We can easily obtain the following hyperbolic solutions:

$$\omega_1(x,t)=\{\frac{(\beta+\alpha^2)(n+1)}{A}$$
$$sech^2(\frac{n\sqrt{\beta+\alpha^2}}{k}(k(x-2\alpha t)+\xi_0))\}^{\frac{1}{2n}}e^{i(\alpha x+\beta t)}, \quad (36)$$

and

$$\omega_2(x,t)=\{-\frac{(\beta+\alpha^2)(n+1)}{A}$$
$$csch^2(\frac{n\sqrt{\beta+\alpha^2}}{k}(k(x-2\alpha t)+\xi_0))\}^{\frac{1}{2n}}e^{i(\alpha x+\beta t)}, \quad (37)$$

for $\beta>-\alpha^2$, it is easy to see that solutions (19) and (20) can reduce to periodic solutions as follows:

$$\omega_3(x,t)=\{\frac{(\beta+\alpha^2)(n+1)}{A}$$
$$\sec^2(\frac{n\sqrt{-(\beta+\alpha^2)}}{k}(k(x-2\alpha t)+\xi_0))\}^{\frac{1}{2n}}e^{i(\alpha x+\beta t)}, \quad (38)$$

and

$$\omega_4(x,t)=\{\frac{(\beta+\alpha^2)(n+1)}{A}$$
$$\csc^2(\frac{n\sqrt{-(\beta+\alpha^2)}}{k}(k(x-2\alpha t)+\xi_0))\}^{\frac{1}{2n}}e^{i(\alpha x+\beta t)}, \quad (39)$$

for $\beta<-\alpha^2$.
On comparison, we observe that our solutions (36)-(39) include the solutions of Wazwaz [20]
Example 4. Consider the $K(m,n)$ equation with generalized evolution term [2] with following form:

$$(q^l)_t+aq^mq+b(q^n)_{xxx}=0, \quad (40)$$

where, the first term is the generalized evolution term, while the second term represents the nonlinear term and the third term is the dispersion term. Also, a and b are arbitrary constants, while l,m and $n\in Z^+$.
Eq. (40) reduces to the $K(m,n)$ equation for $l=1$. Therefore, for $l=1$, $K(1,1)$ is the KdV equation while $K(2,1)$

is the mKdV equation. More details are presented [2].

For solving Eq. (40) with the functional variable method, the wave variable $\xi = x - ct$ where c as the wave speed, it can be converted to the ODE

$$-c(U^l)' + aU^m U' + b(U^n)''' = 0, \tag{41}$$

where prime denotes the derivative with respect to ξ. The Eq. (41) is then integrated term by term one time where integration constant are considered zero. This convert Eq. (41) into

$$-cU^l + \frac{a}{m+1}U^{m+1} + b(U^n)'' = 0. \tag{42}$$

Let $l = n$, we use the transformation

$$U(\xi) = V^{\frac{1}{n}}(\xi), \tag{43}$$

that will reduce (42) into the ODE

$$-cV + \frac{a}{m+1}V^{\frac{m+1}{n}} + bV'' = 0. \tag{44}$$

Following Eq. (5), it is easy to deduce from Eq. (44) the expression of the function $F(V)$ reads

$$F(V) = \sqrt{\frac{c}{b}}V\sqrt{1 - \frac{2an}{c(m+1)(m+n+1)}V^{\frac{m+1}{n}-1}}. \tag{45}$$

After making the change of variables

$$Z = \frac{2an}{c(m+1)(m+n+1)}V^{\frac{m+1}{n}-1}, \tag{46}$$

and using the relation (4), the solution of the Eq. (44) is in the following form

$$V(\xi) = \{-\frac{c(m+1)(m+n+1)}{2an} csch^2(\frac{m-n+1}{2n}\sqrt{\frac{c}{b}}\xi)\}^{\frac{n}{m-n+1}}. \tag{47}$$

Using the transformation (43) , we can easily obtain the following hyperbolic solutions of Eq. (40):

$$q_1(x,t) = \{\frac{c(m+1)(m+n+1)}{2an} sech^2(\frac{m-n+1}{2n}\sqrt{\frac{c}{b}}(x-ct))\}^{\frac{1}{m-n+1}}, \tag{48}$$

and

$$q_2(x,t) = \{-\frac{c(m+1)(m+n+1)}{2an} csch^2(\frac{m-n+1}{2n}\sqrt{\frac{c}{b}}(x-ct))\}^{\frac{1}{m-n+1}}, \tag{49}$$

for $\frac{c}{b} > 0$, it is easy to see that solutions (48) and (49) can reduce to periodic solutions as follows:

$$q_3(x,t) = \{\frac{c(m+1)(m+n+1)}{2an} \sec^2(\frac{m-n+1}{2n}\sqrt{-\frac{c}{b}}(x-ct))\}^{\frac{1}{m-n+1}}, \tag{50}$$

and

$$q_4(x,t) = \{\frac{c(m+1)(m+n+1)}{2an} \csc^2(\frac{m-n+1}{2n}\sqrt{-\frac{c}{b}}(x-ct))\}^{\frac{1}{m-n+1}}, \tag{51}$$

for $\frac{c}{b} < 0$.

Comparing our results with Ebadi and Biswas's results [2] then it can be seen that the results are same.

4. Conclusion

We observed that the functional variable method could be applied to nonlinear evolution equations which could be converted to a second order ODE through the traveling wave transformation. From our results, we can see that the technique used in this paper is very effective and can be steadily applied to nonlinear problems. On the other hand, it can be applied some nonintegrable equations, arising in applied mathematics. As a result, many exact solutions are obtained including soliton solutions presented by hyperbolic functions sech and cosech, periodic solutions presented by sec and cosec solutions. The functional variable method was successfully used to establish exact solutions of Davey-Stewartson equation, generalized Zakharov equation, $K(m,n)$ equation with generalized evolution term, $(2+1)$-dimensional long-wave-short-wave resonance interaction equation and nonlinear Schrödinger equation with power law nonlinearity. Thus, we conclude that the proposed method can be extended to solve the nonlinear problems which arise in the theory of solitons and other areas.

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