Cent. Eur. J. Eng. • 3(3) • 2013 • 349-373
DOI: 10.2478/s13531-013-0108-7

VERSITA

Central European Journal of **Engineering**

Analytical engineering models of high speed normal impact by hard projectiles on metal shields

Review Article

Gabi Ben-Dor[1] Anatoly Dubinsky[1] Tov Elperin[1]*

1 *Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, P. O. Box 653, Beer-Sheva 84105, Israel*

Received 31 March 2013; accepted 18 June 2013

Abstract: In this review we consider mainly relatively simple engineering models which can be physically substantiated although usually their justification requires a large number of assumptions. These models are characterized by the following features in the case of normal impact: either they determine the relations between the "integral characteristics" of penetration (depth of penetration, ballistic limit velocity) in the explicit form (in algebraic form or including quadratures) or they describe local interactions between the shield and the penetrator in the points of the penetrator-shield contact surface that yield such integral characteristics. In this overview we present more or less comprehensively all widely used and also not well known analytical models which have been suggested for describing high-speed penetration into metal shields. This survey is characterized by the following distinguishing features: (i) includes an unprecedented large number of models; (ii) presents models suggested during recent years; (iii) analyzes models which have been originally published in Russian and are not well known in the West.

Keywords: Impact • Ballistic limit • High-speed penetration • Metal shield • Perforation

Nomenclature

a_i parameters determining models;

b thickness of shield;

BLV ballistic limit velocity;

CCE cylindrical cavity expansion;

d maximum diameter of impactor;

DOP depth of penetration;

E Young's modulus;

h instantaneous depth of penetration, Fig. 2 and Fig. 3;

H depth of penetration;

k_{plast} constant plastic modulus, Eq. (25);

K bulk modulus of shield material;

K_{CRH} caliber radius head of ogive nose;

LIM localized interaction model;

*E-mail: email@first.author.com

L length of nose of impactor, Fig. 2;

L_{imp} length of impactor;

m mass of impactor;

m_{piug} mass of plug;

n coefficient, Eq. (98);

R shank radius of impactor;

SCE spherical cavity expansion;

u $= \cos\phi$;

v instantaneous velocity of impactor;

v_{imp} impact velocity;

v_n local normal velocity at the surface of projectile;

v_{res} residual velocity of impactor;

v_{bl} ballistic limit velocity;

V velocity at hole boundary;

w $= v^2$;

W work done during perforation, minimum perforation energy;

x coordinate associated with impactor, Fig. 2 and Fig. 3;

y radius of hole;

Y yield stress of shield material;

α_c coefficient in CCE model;

α_s coefficient in SCE model;

β_c coefficient in CCE model;

β_s coefficient in SCE model;

γ_s coefficient in SCE model;

ε true strain;

$\dot{\varepsilon}$ true strain rate;

ξ coordinate, Fig. 3;

η_c parameters in CCE model, Eq. (66);

ϑ semi-vertex angle of cone-nosed impactor ;

θ,Θ functions determining impactor shield contact surface, Eqs. (5) and (6);

κ error control parameter, Eq. (35);

μ_{fr} friction coefficient;

ν Poisson's ratio of shield material;

ρ coordinate associated with impactor, Fig. 3;

ρ_{imp} material density of impactor;

ρ_{sh} material density of shield;

σ_u ultimate tensile strength of shield material;

σ true stress;

σ_c^{stat} static component of normal stress in CCE model;

σ_s^{stat} static component of normal stress in SCE model;

σ_r stress at the boundary of cavity;

σ_n normal stress (positive in compression);

σ_t tangential stress (positive in compression);

τ_s shear stress;

ϕ angle between direction of projectile motion and local external normal to the surface;

Φ function determining shape of impactor, Fig. 3;

χ_s parameters of SCE model, Eq. (52);

Ψ function determined by Eq. (45).

1. Introduction

Within the broad class of approximate engineering models we distinguish between two sub-classes: empirical (semi-empirical, phenomenological) models and analytical models. Relations between "integral characteristics" of penetration (for instance, between the impact velocity and the depth of penetration (DOP) for a semi-infinite shield and between the ballistic limit velocity (BLV) and the thickness of the plate for a shield of a finite thickness) that have been obtained by statistical analysis of the experimental results and are not based on the physical laws are known as "empirical model". Analytical engineering models that are the subject of this review, contrastingly, can be physically substantiated although usually their justification requires a large number of assumptions. We consider mainly the relatively simple engineering models which are characterized by the following features: either they determine the relations between the "integral characteristics" of penetration in the explicit form (in algebraic form or including quadratures) or they describe local interaction between the shield and the penetrator at the points of the penetrator-shield contact surface that yields such integral relations. A dedicated review by Børvik *et al.* [1] deals with penetration into metal shields. Surveys on this topic can be also found in reviews and original papers by Backman and Goldsmith [2]; Jonas and Zukas [3]; Neilson [4]; Brown [5]; Anderson and Bodner [6]; Aptukov [7]; Amde *et al.* [8]; Corbett *et al.* [9]; Ben-Dor *et al.* [1]; Aly and Li [11] and in the books by Zukas [12]; Recht [13]; Zukas and Walters [14]; Bulson [15] Bangash and Bangash [16]; Grigoryan [17]; Carlucci and Jacobson [18]; Bangash [19]; Szuladziński [20]. Altogether published overviews cover only a part of known analytical models and do not include the models suggested during the last decade and the models not published in English. The goal of this review is to fill this gap. The general criteria for including a particular study in this survey are the following: either the study contains explicit formulas for determining integral characteristics of penetration (BLV, DOP, residual velocity) or allows calculating these characteristics using standard procedures. Simplified models are not considered in this survey if they are characterized by one or several of the following features: (i) model does not yield explicit formulas for determining ballistic characteristics which can be determined by solving numerically algebraic or differential equations; (ii) model includes either some unspecified parameters which cannot be easily estimated in advance or empirical coefficients which are defined ambiguously; (iii) model is based on a certain set of experimental data and is in essence a best-fit approximation of the experimental results. It must be emphasized that the subject of this review are the models and not applied problems which can be solved using these models, e.g. shape optimization of penetrators, analysis of the effects of layering and spacing on the ballistic properties of shields, etc. If the original study contains expression for the minimum penetration energy which can be easily transformed into formula for the BLV, we usually present the expression from the original study. In order to unify notations all formulas with bulk modulus, K , are converted using the relation $K = E/[3(1-2\nu)]$, where E and ν are Young's modulus and Poisson's ratio, correspondingly. In the following we present a slightly simplified for engineering applications thickness-of-plate based classification of shields that was suggested by Backman and Goldsmith [2]. According to this classification we distinguish between the following types of shields: semi-infinite shields when the rear surface does not affect penetration (in the limiting case a shield can be viewed as a half-space); thick (non-thin) shields when the rear surface affects penetration either from the very beginning or after penetration at a certain depth; and thin shields when stresses and deformations in the shield during penetration are independent of the instantaneous depth of penetration. The main perforation mechanisms (Backman and Goldsmith [2]) that are approximately described by analytical models are as follows (see Fig. 1): hole enlargement (semi-infinite and thick shields), plugging (thick and thin shields), petalling (thin shields), and combination of hole enlargement and plugging (thick shields). Plug formation usually occurs during penetration by blunt-nosed projectiles.

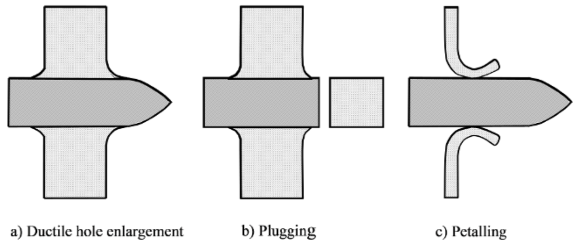

Figure 1. Main perforation mechanisms.

2. Localized interaction theory

The localized interaction theory (LIT) encompasses investigations base on the so called localized interaction models (LIMs) whereby the integral effect of the interaction between a shield and a penetrating projectile is described as a superposition of the independent local interactions of the projectile surface elements with the shield. Every local interaction is primarily determined by the local velocity of the surface element, v, and the angle ϕ between the vector $\overrightarrow{v}$ and the local outer normal vector to the projectile surface v_n as well as by some global parameters that take into account the integral characteristics of the shield (e.g., hardness, density, etc.). Consequently, the normal stress, σ_n, and the tangential stress, σ_t, respectively, which determine the the projectile shield interaction model, can be presented in the following form:

$$\sigma_n = \sigma_n(u, v), \;\; \sigma_t = \sigma_t(u, v), \;\; u = \cos\phi \qquad (1)$$

where global parameters are not indicated as arguments of functions σ_n and σ_t . The instantaneous resultant force acting on the projectile is determined by integrating instantaneous "local" force over the projectile-shield contact surface.

Many engineering models for penetration modeling belong to the category of LIMs; most of them are determined by the following relations:

$$\sigma_n = a_0 + a_2\omega(u)v^2, \quad (2)$$

$$\sigma_t = \mu_{fr}\sigma_n, \quad (3)$$

where μ_{fr} is the coefficient of friction between a projectile and a shield, a_0 and a_2 are "global" parameters. The choice of function $\omega(u)$ (in most cases $\omega(u) = u^2$) and "global" parameters is based on the used model of shield material, and in most cases it is based on cavity expansion approach (see below).

Application of analytical methods for solving penetration problems using LIMs is feasible only in the case of normal impact (impact velocity is normal to the impacted plate) of a rigid symmetric projectile, whereby projectile executes a translational motion under the effect of the resistance force, and relatively simple relationships for σ_n allow deriving analytical formulas for the DOP and BLV. In particular, this situation occurs for the model given by Eqs. (2) and (3)) while more involved situations (oblique impact, complicated model) require application of numerical methods. The formalism for the description of the impactor shield contact surface which was proposed by Ben-Dor *et al.* [21] for normal impact takes into account only partial immersion of the projectile in the shield. In accordance with this formalism, the moving contact surface of the impactor shield interaction can be described as follows (Fig. 2):

$$\theta(h) \le x \le \Theta(h) \quad (4)$$

where

$$\Theta(h) = \begin{cases} h & \text{if } 0 \le h \le L \\ L & \text{if } h \ge L \end{cases} \quad (5)$$

$\theta(h) = 0$ in the case of a semi infinite shield and

$$\theta(h) = \begin{cases} 0 & \text{if } 0 \le h \le b \\ h-b & \text{if } b \le h \le b+L \end{cases} \quad (6)$$

in the case of a shield having a finite thickness, the coordinate h , the instantaneous depth of penetration, is defined as the distance between the nose of impactor and the front surface of a shield, the coordinate x is associated with the impactor, b is thickness of a shield, and L is the length of the nose of impactor. In practice, a simplified version of LIM is often used when the stage where penetrator is only partially immersed in the shield is not taken into account.

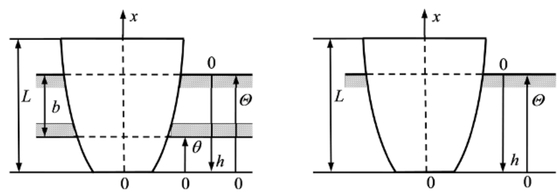

Figure 2. Description of impactor-shield interaction surface.

In the latter case $\theta(h) = 0$ and $\Theta(h) = L$. LIMs allows deriving formulas for the resistance force, the DOP, the BLV, etc. using standard mathematical manipulations. In order not to overload exposition we do not present these formulas here. Such formulas for different shapes of impactors can be found in the published studies and in the monograph by Ben-Dor *et al.* [22].

Generalization of the LIT to layered and spaced shields which takes into account incomplete immersion of a striker in a shield at the initial and final stages of penetration and simultaneous interaction of a striker with several different plates in a shield, was proposed by Ben-Dor *et al.* [23], [24]; a comprehensive description can be found in Ben-Dor *et al.* [22]. History of development of LIT for penetration modelling is surveyed in the monograph by Ben-Dor *et al.* [22]. Emergence and development of LIT in the field of gas dynamics is comprehensively described in the books by Alekseeva and Barantsev [25]; Bunimovich and Dubinsky [26] and Miroshin and Khalidov [27, 28]. Progress achieved by LIT in gas dynamics stimulated attempts to extend this approach also to penetration mechanics where LIMs are widely used for investigating various applied problems; extensive information on this topic can be found in Ben-Dor *et al.* [10, 22, 29, 30].

3. Cavity expansion approach

3.1. Introduction to the cavity expansion approach

Cavity expansion approach is a fairly universal approximate method that allows devising analytical models of penetration mechanics by describing local interaction between the shield and the penetrator at the points of the penetrator-shield contact surface and, consequently, allows determining the instantaneous local interaction force between the shield and the striker during its motion inside the shield. It is important that such models include explicitly parameters which determine mechanical properties of the material of a shield. This advantage is achieved by postulating a certain relation between penetration and expansion of a cavity in the material of a shield. Application of cavity expansion approach requires solving two fundamental problems. The first problem is to determine a law governing expansion of a cavity inside a material (generally from the zero initial radius). This problem is usually considered for the cases with axial or spherical symmetry. Multiplicity of formulations of this problem is related with numerous models of the material of the shield (compressible/incompressible, elastic-plastic with different strain-stress laws, etc.). It is a common practice to distinguish between three classes of models depending on the level of description of the cavity dynamics. Static (or occasionally called quasi-static) models describe stresses on the surface of a cavity in a static state. Quasi-dynamic models are the basis of cavity expansion approach in penetration mechanics. These models determine stresses on the cavity surface as a function of the constant rate of increase of the radius of the cavity. Dynamic models take into account acceleration of the cavity surface. The second problem is justifying a particular way of using a solution of cavity expansion problem in penetration mechanics. If the solution of the axially symmetric problem is used, then the method is called a cylindrical cavity expansion (CCE) approach/approximation while application of spherically symmetric solution is called a spherical cavity expansion (SCE). A survey of the state of the art up to the late 1950s regarding the problems of expansion of cavities in solids was written by Hopkins [31]. Useful information on this topic is summarized in the monograph of Yu [32]. Application of cavity expansion models in penetration mechanics have been described and analyzed by Teland 33 and Satapathy [34]. Only the studies directly associated with the application of cavity expansion methods in engineering modeling of ballistic impact for metal shields are considered below.

3.2. Spherical Cavity Expansion Approximation

Spherical cavity expansion approximation is widely used in impactor shield interaction models in a quasi dynamic version whereby expansion of a spherically symmetrical cavity from a zero initial radius at a constant velocity is described by the following formula:

$$\sigma_r = \sigma_r(V), \tag{7}$$

and the normal stress on the surface of the cavity, σ_r , is assumed to be known from the solution of the cavity expansion problem as a function of the velocity of the surface of the hole, V. Calculating the interaction force between the shield and the projectile is accomplished as follows. Consider some location on the surface of the impactor moving with the instantaneous velocity v inside the shield. Normal velocity at this location equals $v_n = v\cos\phi = uv$, where $u = \cos\phi$, ϕ is an angle between a direction of projectile motion and a local external normal to a surface. It is assumed that normal stress produced in the shield at this location is equal to the stress on the surface of a cavity that expands with a constant velocity, $V = v_n$. Therefore formula for normal stress on the surface of the impactor, σ_n , reads:

$$\sigma_n = \sigma_r(uv) \tag{8}$$

If the impactor has the shape of a body of revolution and is the equation of its surface (see Fig. 3) then

$$u = \cos\phi = \Phi'/\sqrt{\Phi'^2 + 1} \tag{9}$$

and Eq. (8) implies that

$$\sigma_n = \sigma_r\left(\frac{\Phi' v}{\sqrt{\Phi'^2 + 1}}\right) \tag{10}$$

Applications of the dynamic SCE models in penetration mechanics to projectiles having several typical shapes are described in Bernard and Hanagud [35]. Calculating the force acting on a striker at some location on the surface of a striker using the dynamic SCE model is ambiguous and does not allow a universal geometric interpretation. This is the reason why the dynamic SCE models are not widely used in penetration mechanics.

3.3. Cylindrical cavity expansion approximation

Another widely used in penetration mechanics approach is known as a cylindrical cavity expansion (CCE) approximation (model, method, etc.). Sometimes other names are used, e.g., method of plane sections (Sagomonyan [36]; Rahmatulin et al. [37]) and disks model (Yankelevsky and Adin [38]). CCE method can be justified more readily than a SCE method. In CCE approach, normal penetration of a slender body of revolution is usually considered, and it is assumed that particles of the material of a shield move in a radial direction during projectile penetration. The shield can be viewed as consisting of infinitely thin layers, and in each layer a cavity expansion caused by the moving impactor is modeled. This approach facilitates calculating the stress on the surface of a hole in each layer and, consequently, the force acting on the projectile at each location on the lateral surface of a projectile. The CCE approach can be described in a general case of a dynamic model of hole expansion for each layer as follows:

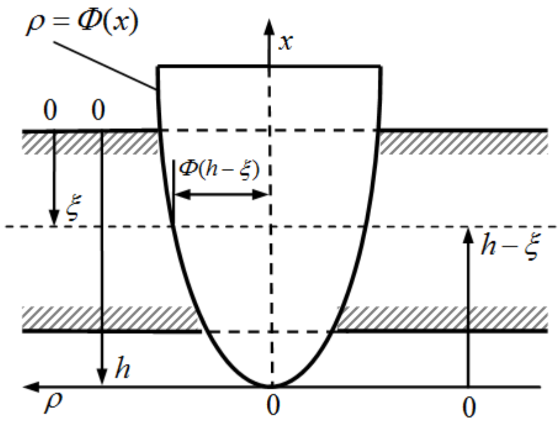

Figure 3. Cylindrical cavity expansion approximation.

$$\sigma_r = \sigma_r(y, \dot{y}, \ddot{y}), \dot{y} = V = dy/dt, \ddot{y} = d^2y/dt^2, \quad (11)$$

where y is the radius of the hole, t is time, the dot over a symbol denotes time derivative. Let $\rho = \Phi(x)$ be the equation of the surface of the projectile. Then, for the infinitesimal layer with the coordinate ξ (see Fig. 3), the conditions that the surface of the hole coincides with the surface of the impactor and that the velocity and acceleration of the hole surface are equal to the radial components of the same kinematical characteristics of the impactor yield (Rahmatulin et al. [37]):

$$y = \Phi(h - \xi) \quad (12)$$

$$\dot{y} = \Phi'(x)\dot{h}, \ddot{y} = \Phi''(x)\dot{h}^2 + \Phi'(x)\ddot{h}, x = h - \xi. \quad (13)$$

Expression for the normal stress on the surface of the impactor, σ_n , is obtained by substituting $y, \dot{y}$ and $\ddot{y}$ from Eqs. (12) and (13) into Eq. (11):

$$\sigma_n = \sigma_r(\Phi, \Phi'\dot{h}, \Phi''\dot{h}^2 + \Phi'\ddot{h}), \Phi = \Phi(x). \quad (14)$$

The case of a quasi dynamic model is similar to the case described by Eq. (7) for CCE model when

$$\sigma_r = \sigma_r(\dot{y}) = \sigma_r(V), \quad (15)$$

and Eq. (14) and (9) yields:

$$\sigma_n = \sigma_r(\Phi' v) = \sigma_r\left(\frac{uv}{\sqrt{1-u^2}}\right), \dot{h} = v \quad (16)$$

3.4. Typical quasi dynamic cavity expansion approximations

Two-term cavity expansion models are the most widely used models in penetration mechanics:

$$\sigma_r(V) = \alpha_s + \beta_s V^2, \quad (17)$$

$$\sigma_r(V) = \alpha_c + \beta_c V^2, \quad (18)$$

where subscripts "s" and "c" refer to the SCE model and the CCE model, correspondingly, α_s, β_s, α_c, β_c are coefficients of the models. Eqs. (17) and (18) imply the following relationships for the normal stress on the surface of the impactor, respectively:

$$\sigma_n = a_0 + a_2 u^2 v^2, a_0 = \alpha_s, a_2 = \beta_s \quad (19)$$

$$\sigma_n = a_0 + a_2 \frac{u^2 v^2}{1-u^2}, a_0 = \alpha_c, a_2 = \beta_c \quad (20)$$

Sometimes the following three-term CCE model with coefficients α_c, β_c, γ_c is used:

$$\sigma_r(V) = \alpha_s + \gamma_s V + \beta_s V^2, \quad (21)$$

which corresponds to the following formula:

$$\sigma_n = \alpha_0 + a_1 uv + a_2 u^2 v^2, a_0 = \alpha_s, a_1 = \gamma_s, a_2 = \beta_s. \quad (22)$$

Eqs. (19), (20) and (22) show that cavity expansion models given by Eqs. (17), (18) and (21) belong to the class of LIMs.

4. Cavity expansion models

4.1. Static Cavity Expansion Models

Static cavity models are widely used in penetration mechanics either directly or as elements of dynamic cavity expansion models. In the early works by Bishop et al. [39] and Hill [40], von Mises yield criterion was employed. Bishop et al. [39] proposed the following static SCE and CCE models for incompressible in the plastic region, elastic-plastic, linear strain-hardening materials:

$$\sigma_r = \sigma_s^{stat} = \frac{2Y}{3}\left[1 + ln\left(\frac{E}{(1+\nu)Y}\right)\right] + \frac{2\pi^2}{27}k_{plast}, \quad (23)$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{\sqrt{3}E}{2(1+\nu)Y}\right)\right] + \frac{\pi^2}{18}k_{plast}, \tag{24}$$

where k_{plast} is constant plastic modulus in the linear strain-hardening law:

$$\sigma = \begin{cases} E\varepsilon & \text{if } \varepsilon \le Y/E \\ Y + k_{plast}\varepsilon & \text{if } \varepsilon > Y/E \end{cases} \tag{25}$$

In the case of incompressible in the plastic region, elastic-perfectly plastic materials when

$$\sigma = \begin{cases} E\varepsilon & \text{if } \varepsilon \le Y/E \\ Y & \text{if } \varepsilon > Y/E \end{cases} \tag{26}$$

and Eqs. (23) and (24) can be rewritten for $k_{plast} = 0$ as follows:

$$\sigma_r = \sigma_s^{stat} = \frac{2Y}{3}\left[1 + ln\left(\frac{E}{(1+\nu)Y}\right)\right], \tag{27}$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{\sqrt{3}E}{2(1+\nu)Y}\right)\right], \tag{28}$$

In the case of incompressible over the whole stress range elastic-perfectly plastic materials ($\nu = 0.5$), Eqs. (27) and (28) read:

$$\sigma_r = \sigma_s^{stat} = \frac{2Y}{3}\left[1 + ln\left(\frac{2E}{3Y}\right)\right], \tag{29}$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{E}{\sqrt{3}Y}\right)\right], \tag{30}$$

Finally, in the case of incompressible over the whole stress range linear strain-hardening materials ($\nu = 0.5, k_{plast} > 0$), Eqs. (23) and (24) read:

$$\sigma_r = \sigma_s^{stat} = \frac{2Y}{3}\left[1 + ln\left(\frac{2E}{3Y}\right)\right] + \frac{2\pi^2}{27}k_{plast}, \tag{31}$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{E}{\sqrt{3}Y}\right)\right] + \frac{\pi^2}{18}k_{plast}, \tag{32}$$

Hill [40] proposed static SCE and CCE models for compressible over the whole stress range elastic-perfectly plastic materials:

$$\sigma_r = \sigma_s^{stat} = \frac{2Y}{3}\left[1 + ln\left(\frac{E}{3(1-\nu)Y}\right)\right], \tag{33}$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{\sqrt{3}E}{(5-4\nu)Y}\right)\right], \tag{34}$$

Regarding the expansion of a non-thin layer (shell), Hill [40] recommends using models which are valid for incompressible materials, i.e., the models determined by Eqs. (29)-(30) and (31)-(32). Masri and Durban [41] developed compressible cavity expansion models using a controlled error approach. Introducing an error control parameter, κ , they arrived at the expression for the cavitation pressure which depends only on one parameter. The optimal value of κ is such that allows predicting the cavitation pressure with a high accuracy. A number of CCE models were proposed in the absence of strain-hardening, namely, plane-strain models for von Mises and Tresca yield criteria, respectively (Masri and Durban [41]):

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left\{1 + ln\left[\frac{\sqrt{3}E}{[3+(1-2\kappa)(1-2\nu)]Y}\right]\right\}, \tag{35}$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{2}\left\{1 + ln\left[\frac{E}{2(1-\nu^2)Y}\right]\right\}, \tag{36}$$

Masri *et al.* [42] suggested the following plane-stress models for von Mises and Tresca yield criteria, correspondingly:

$$\sigma_r = \sigma_c^{stat} = 2,04Y\left[1 - (2e-1)\frac{Y}{E}\right], \tag{37}$$

$$\sigma_r = \sigma_c^{stat} = 2Y\left[1 - (2e-1)\frac{Y}{E}\right]. \tag{38}$$

Masri and Durban, 2007 used the value $\kappa = -0.4725$ in Eq. (35). Note that Eq. (35) coincides with Eq. (34) for $\kappa = -0.5$. Masri *et al.* [42] and Cohen et al. [43] examined validity of static CCE models for predicting the BLVs and the residual velocities on the basis of Eq. (94) with various relations for σ_c^{stat}. They used experimental data for shields manufactured from aluminum alloys and Weldox steels as well as numerical simulations for steel. Different versions of the models corresponding to Tresca or von Mises conditions, plane-stress or plane-strain patterns, were compared and the parameters of the models were determined. On the basis of numerical simulations Rosenberg and Dekel [?] proposed the following formula for the BLV of sharp-nosed projectiles penetrating into elastic-plastic shields:

$$v_{bl} = R\sqrt{2\pi b\sigma_*/m}, \quad (39)$$

where

$$\overline{b} = \frac{b}{d}, \frac{\sigma_*}{Y} = \begin{cases} 2/3 + 4\overline{b} & \text{if } 0 < \overline{b} \le 1/3 \\ 2.0 & \text{if } 1/3 < \overline{b} \le 1.0 \\ 2.0 + 0.8 ln\overline{b} & \text{if } \overline{b} > 1.0 \end{cases} \quad (40)$$

4.2. Quasi-Dynamic Cavity Expansion Models

Spherical and cylindrical quasi-dynamic models of penetration dynamics are generally based on cavity expansion laws described by Eqs. (7) and (15). Most of the cavity expansion laws are written in one of the following forms:

$$\sigma_r = \sigma_s^{stat} + \beta_s V^2 \quad (41)$$

$$\sigma_r = \sigma_c^{stat} + \beta_c V^2 \quad (42)$$

or in the form given by Eq. (21). Forrestal et al. [47] suggested quasi-dynamic SCE and CCE models for compressible elastic-perfectly plastic material similar to those given by Eq. (41) and Eq. (42), respectively, and used Eq. (33) and Eq. (34) for static component of normal stress while coefficients β_s and β_c were determined by curve-fitting of the corresponding results by Forrestal and Luk [48] and Forrestal [49]. Forrestal et al. [50] developed a CCE model for rate independent, power law strain hardening material that is described by Eq. (42) with

$$\sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + \left(\frac{E}{\sqrt{3}Y}\right)^n \Psi(1-\eta, n)\right], \quad (43)$$

$$\beta_c = \frac{\rho_{sh}}{2}\left(\frac{1 + ln2}{1 - \nu} + \eta - ln\eta - 1\right), \quad (44)$$

where

$$\eta = \frac{2(\nu+1)Y}{\sqrt{3}E}, \Psi(z,n) = \int_0^z \frac{(-ln\varsigma)^n}{l - \varsigma} d\varsigma, 0 < z < 1. \quad (45)$$

It was assumed that the plastic zone is incompressible while the elastic region is compressible. Parameter n (which is generally less than 0.5) is the coefficient in empirical stress-strain relation given by Eq. (98). For calculating the integral with singularity at $\varsigma = 0$ in Eq. (45), Forrestal and Romero [51] proposed the following formula:

$$\Psi(z,n) = 2.994 - 3.742n + 3.511n^2 + \frac{0.05^n - (1-z)^n}{n} + \frac{n[0.05^{n+1} - (1-z)^{n+1}]}{2(n+1)} \quad (46)$$

Luk and Amos [52] developed numerical CCE model for compressible, elastic-plastic, rate independent materials with power-law strain-hardening, compared numerical results obtained using their model with the results of analytical model by Forrestal *et al.* [50] for incompressible materials and concluded that application of the compressible model improved predictions for the thicker plates and $v_{imp} >> v_{bi}$. Luk et al. [53] proposed SCE model for elastic-plastic, rate independent material with power-law hardening. For the incompressible material, the closed-form solution was obtained in the form of Eq. (41) where

$$\sigma_s^{stat} = \frac{2Y}{3}\left[1 + \frac{\Psi(1-\eta, n)}{\eta^n}\right], \beta_s = 1.5\rho_{sh}, \eta = 1.5Y/E. \quad (47)$$

Application of the compressible model requires numerical solution of differential equations. In order to verify these models Forrestal *et al.* [54] conducted experiments with spherical-nose rods penetrating into semi-infinite steel and aluminum shields. Penetration model for the incompressible material given by Eq. (47) was used along with the results of numerical solutions for compressible material obtained by Luk *et al.* [53]. Forrestal *et al.* [55]performed similar analysis for ogive-nosed rods. Warren and Forrestal [56] suggested a set of SCE models for 6061-T651 aluminum shields which are determined by Eq. (21) where

$$\alpha_s = \tilde{\alpha}_s Y, \gamma_s = \tilde{\gamma}_s\sqrt{\rho_{sh} Y}, \beta_s = \tilde{\beta}_s \rho_{sh}, \quad (48)$$

and the coefficients $\tilde{\alpha}_s$, $\tilde{\gamma}_s$, $\tilde{\beta}_s$ are given for incompressible and compressible media with strain rate effects and without strain rate effects. Warren [57] developed a CCE model for elastic-plastic rate-dependent material with power-law strain hardening assuming that the plastic zone is incompressible while the elastic region is compressible:

$$\sigma_r = \sigma_c^{stat} + \frac{\rho_{sh} V^2}{2} \left[\frac{1}{(1-\nu)\sqrt{1-\lambda^2}} ln\left(\frac{1+\sqrt{1-\lambda^2}}{\lambda}\right) - ln\left(\frac{2(1+\nu)Y}{\sqrt{3}E}\right) - 1\right], + \frac{\tilde{\alpha}}{\sqrt{3}\tilde{m}}\left(\frac{2V}{\sqrt{3}a_{cav}\dot{\varepsilon}_0}\right)^{\tilde{m}}\left[1 - \left(\frac{2(1+\nu)Y}{\sqrt{3}E}\right)^{\tilde{m}}\right] \quad (49)$$

where σ_c^{stat} and η are determined by Eqs. (43) and (45), correspondingly, a_{cav} is radius of the cavity surface, $\widetilde{\alpha}$, $\widetilde{m}$, $\dot{\varepsilon}_0$ are parameters in the stress-strain relation (Chakrabarty [58], Yadav et al. [59]:

$$\sigma = \begin{cases} E\varepsilon & \text{if } \sigma \leq Y_d \\ Y(E\varepsilon/Y)^n + \widetilde{\alpha}(\dot{\varepsilon}/\dot{\varepsilon}_0)^{\widetilde{m}} & \text{if } \sigma > Y_d \end{cases} \quad (50)$$

σ, ε, n are the same as in Eq. (98), $\dot{\varepsilon}$ is a strain rate, $\dot{\varepsilon}_0$ is a reference strain rate,$\widetilde{\alpha}$ is a curve-fitting parameter having units of stress,Y_d is a dynamic yield stress given by the following formula:

$$Y_d = Y + \widetilde{\alpha}(\dot{\varepsilon}/\dot{\varepsilon}_0)^{\widetilde{m}} \quad (51)$$

Masri and Durban [60] derived a closed SCE analytical model for compressible, elastic/perfectly plastic material including strain-hardening that does not require application of best fit procedure to the numerical data:

$$\sigma_r = \sigma_s^{stat} + \beta_s V^2 + \chi_s V^3, \quad (52)$$

where σ_s^{stat} is determined by Eq. (33) and

$$\beta_s = \rho_{sh}\left[\frac{3}{2} - 0.160\frac{(1-2\nu)(13-5\nu)}{(1-\nu)^{5\beta}}\left(\frac{Y}{E}\right)^{1/3}\right], \quad (53)$$

$$\chi_s = -\frac{2}{9}\sqrt{(1-2\nu)\left(\frac{1+\nu}{1-\nu}\right)^5\frac{\rho_{sh}^3}{E}}. \quad (54)$$

This model also includes the case when cavity expansion velocity is much smaller than the speed of sound in a solid, i.e., $V << \sqrt{E/\rho_{sh}}$. The authors note that the term $\chi_s V^3$ can often be omitted when the model is applied to penetration mechanics. Masri and Durban [61] proposed the CCE model for incompressible, elastic-plastic, linear strain-hardening materials which is determined by Eq. (42) with

$$\sigma_c^{stat} = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{E}{\sqrt{3}Y}\right)\right] + \frac{\pi^2}{18}\eta_{hard},$$
$$\beta_c = \frac{\rho_{sh}}{2}ln\left(\frac{E}{4\sqrt{3}Y}\right), \quad (55)$$

The following stress-strain equation is used:

$$\sigma = \begin{cases} E\varepsilon & \text{if } 0\varepsilon \leq Y/E \\ \overline{\eta}_{hard}E\varepsilon + (1-\overline{\eta}_{hard})Y & \text{if } \varepsilon > Y/E \end{cases} \quad (56)$$

where $\overline{\eta}_{hard} = \eta_{hard}/E$, η_{hard} is a constant hardening modulus,$0.001 < \overline{\eta}_{hard} < 0.01$. Masri and Durban [62] suggested a CCE model for plastic orthotropic strain hardening materials that is described by Eq. (52). In order to simplify application of the model for penetration mechanics, the model was approximated by Eq (41). Similar procedure was applied for a SCE model. On the basis of the numerical simulations of expansion of spherical and cylindrical cavities in elastic-perfectly plastic materials (aluminum, steel, lead) Rosenberg and Dekel [63] proposed approximate SCE and CCE models. The SCE model is described by Eq. (41) with σ_s^{stat} given by Eq. (33) and $\beta = 1.1875\ \rho_{sh}$. The formula for CCE model is given by Eq. (21) where

$$\alpha_c = \frac{Y}{\sqrt{3}}\left[1 + ln\left(\frac{\sqrt{3}E}{6(1-\nu)Y}\right)\right],$$
$$\gamma_c = 0.29\rho_{sh}, \beta_c = 0.29\rho_{sh}. \quad (57)$$

Employing similar approach, Rosenberg and Dekel [64] suggested a SCE model that is determined by Eq. (52) with σ_s^{stat} given by Eq. (33) and

$$\beta_s = K\left[1.5 - (1.3 - 0.53\nu - 1.19\nu^2)\left(\frac{Y}{K}\right)^{1/3}\right],$$
$$\chi_s = -K\left[[2/3 - \zeta(\nu)]\left(\frac{Y}{K}\right)^{1/3}\right]. \quad (58)$$

where K is the bulk modulus, and ζ is equal to 2.36, 2.237, 2.066, 1.88 for ν equal to 0.1, 0.2, 0.3, 0.4, respectively. Rosenberg and Dekel [65] analyzed results of numerical simulations of penetration of rigid long rods having ogive, spherical, conical and flat nose shapes into semi-infinite aluminum and steel shields having different strengths. They found that for impact velocities, v_{imp} , smaller than some critical velocity, v_c , a deceleration (resistance force) acting on the rods is practically constant and is independent of the impact velocity, while for the velocities larger than v_c a velocity dependent term must be added to the static resistance term. Mathematically, the model by Rosenberg and Dekel [65] can be described as follows. Cavity expansion approach implies the following expression for the impactor's resistance force, :

$$\frac{D}{\pi R^2} = \sigma^{stat} + \chi\rho_{sh}v^2, \quad (59)$$

where σ^{stat} is static normal stress determined from a SCE or CCE model, χ depends on a nose shape of a rod. Instead of the model given by Eq. (59), the Rosenberg and Dekel [65] use the following model:

$$\frac{D}{\pi R^2} = \begin{cases} \sigma^{stat} & \text{if } v \leq v_c \\ 0.5(\sigma^{stat} + \chi\rho_{sh}v^2) & \text{if } v > v_c \end{cases} \quad (60)$$

where

$$v_c = \sqrt{\sigma^{stat}/(\chi\rho_{sh})} \quad (61)$$

4.3. Dynamic cavity expansion models

Hopkins [31] noted that "important aspects of this type of theory were first considered by R. Hill" referring to Hill's unpublished report of 1948, and suggested the following dynamic SCE model for incompressible elastic-plastic material:

$$\sigma_r = \sigma_r(y, \dot{y}, \ddot{y}) = \sigma_s^{stat} + \sigma_s^{dyn}, \quad (62)$$

where

$$\sigma_s^{dyn} = \sigma_s^{dyn}(y, \dot{y}, \ddot{y}) = \rho_{sh}(y\ddot{y} + 1.5\dot{y}^2), \quad (63)$$

y is the radius of the hole, σ_s^{stat} is determined by Eq. (29). On the basis of Eq. (63), Goodier [66] proposed the following formula for the normal stress on the surface of a hemispherical-nose impactor:

$$\sigma_n = \sigma_s^{stat} + \sigma_s^{dyn}(R, v, \dot{v})\cos\phi = \sigma_s^{stat} + \rho_{sh}(R\dot{v} + 1.5v^2)\cos\phi, \quad (64)$$

where R is the radius of the hemisphere. The equation of motion of impactor reads (Bernard and Hanagud [35]):

$$(m + m_{add})\dot{v} = -\pi R^2(\sigma_s^{stat} + \rho_{sh}v^2), \quad m_{add} = \pi\rho_{sh}R^3. \quad (65)$$

Clearly the term m_{add} in Eq. (65) can be neglected if $m_{add} << m$ and, consequently, the term $\rho_{sh}R\dot{v}\cos\phi$ in Eq. (64) can be omitted. Then a quasi-dynamic SCE model can be considered whereby a solution of the equation of motion of impactor is used for determining penetration characteristics (Bernard and Hanagud [35]; Forrestal et al. [67]). Bernard and Hanagud [35] also extended the approach suggested by Goodier [66] to impactors having non-hemispherical nose shapes. Sagomonyan [68] considered CCE in incompressible elastic-plastic material and proposed the following model:

$$\sigma_r = \sigma_r(y, \dot{y}, \ddot{y}) = \alpha_c + \beta_c\dot{y}^2 + \eta_c y\ddot{y}, \quad (66)$$

where

$$\alpha_c = \frac{Y}{2}[1 + ln(1+\omega)], \omega = \frac{2E}{3Y},$$
$$\beta_c = \frac{\rho_{sh}}{2}\left[ln(1+\omega) - \frac{\omega}{\omega+1}\right], \eta_c = \frac{\rho_{sh}}{2}ln(1+\omega) \quad (67)$$

The latter equations and Eq. (14) yield the following formula for normal stress on the surface of a projectile:

$$\sigma_n = \alpha_c + (\beta_c\Phi'^2 + \eta_c\Phi\Phi'')v^2 + \eta_c\Phi\Phi'\dot{v}, \quad (68)$$

where the function $\Phi(x)$ describes the shape of the impactor nose (see Fig. 3). Sagomonyan [68] considered conical-nosed impactors in more detail (see also Ben-Dor *et al.* [69]). Generally, Eq. (68) implies a linear differential equation of motion of a projectile that can be solved in quadratures. Sagomonyan [70] developed a model describing expansion of a cylindrical cavity inside an incompressible elastic plastic medium starting from the non zero radius, and outlined the applications of this model for investigating penetration of impactors having a plane bluntness that is accompanied by plug formation. Penetration was considered as two simultaneous processes, namely, expansion of the cavity in the shield and motion of the plug. Ben-Dor *et al.* [22] extended this approach and determined the closed form solutions for the BLV of an arbitrary body of revolution without any additional simplifications. Aptukov *et al.* [71] derived an approximate solution for the expansion of a cylindrical hole in a compressible elastic plastic medium in the form of Eq. (66) with

$$\beta_c = \hat{\beta}_c/v_*^2, \eta_c = \hat{\eta}_c/v_*^2, \quad (69)$$

where the values of the coefficients α_c, $\hat{\beta}_c$, $\hat{\eta}_c$ for several metals are given in Table 1, while v_* is equal to $5040 m/s$, $5160 m/s$ and $4650 m/s$ for aluminum, titanium and steel, correspondingly.

Aptukov [72] determined an approximate solution for the expansion of a spherical hole in a compressible elastic plastic medium in the form of Eq. (66) and proposed formulas for calculating the coefficients of the model. Warren and Forrestal [56] suggested a SCE model that includes power-law a strain hardening and strain-rate sensitivity of an incompressible shield material:

$$\sigma_r = \sigma_s^{stat} + \rho_{sh}(1.5V^2 + y\dot{V}) + \frac{2\tilde{\alpha}}{3\tilde{m}}\left(\frac{2V}{y\dot{\varepsilon}_0}\right)^{\tilde{m}}, \quad (70)$$

Table 1. Parameters α_c, $\hat{\beta}_c$, $\hat{\eta}_c$

Parameter	Aluminum,Y(GPa)			Titanium,Y(GPa)			Steel,Y(GPa)		
	0.1	0.3	0.5	0.4	0.6	0.8	0.8	1.0	1.2
α_c (GPa)	0.41	1.10	1.70	1.49	2.12	2.7	2.94	3.57	4.18
$\hat{\beta}_c$ (GPa)	76	71	64	122	118	112	170	162	157
$\hat{\eta}_c$ (GPa)	55	50	48	88	85	83	121	118	117

where σ_s^{stat} is determined by Eq. (47). Li et al. [73] and Chen et al. [74, 75] considered the class of SCE models of the following type:

$$\sigma_r = \alpha_s + \xi_s V + \beta_s V^2 + \varsigma_s \dot{V} \quad (71)$$

where coefficients α_s, ξ_s, β_s, ς_s are depend on shield material. They derived also a dimensionless formula for the DOP as a function of three parameters for general convex shaped projectiles.

5. Momentum and Energy Balance Approach

Based on momentum and energy balances Recht and Ipson [76] proposed the following formula for the residual velocity for a relatively thin plate ($b/L_{imp} < 0.5$, $b/d < 0.5$) penetrated by a blunt fragment:

$$v_{res} = a(v_{imp}^2 - v_{bl}^2)^{1/2}, v_{imp} \geq v_{bl}, \quad (72)$$

or

$$\widetilde{v}_{res} = a(\widetilde{v}_{imp}^2 - 1)^{1/2}, \widetilde{v}_{imp} \geq 1, \quad (73)$$

where

$$a = 1/\left(1 + \frac{m_{plug}}{m}\right) = 1/\left(1 + \frac{\rho_{sh} b d_{plug}^2}{\rho_{imp} L_{imp} d^2}\right), \quad (74)$$

$$\widetilde{v}_{imp} = v_{imp}/v_{bl}, \widetilde{v}_{res} = \widetilde{v}_{res}/v_{bl}, \quad (75)$$

m_{plug} and d_{plug} are mass and diameter of a plug, correspondingly. For the BLV, Recht and Ipson [76] suggested the following formula:

$$v_{bl} = \frac{4\zeta_1 \rho_{sh} b^2 \psi \eta}{L_{imp} d}\left[1 + \sqrt{\left(1 + \frac{L_{imp}\rho_{imp}}{b\rho_{sh}}\right)\left(1 + \frac{\varsigma_2 d}{4\rho_{sh} b \psi^2 \eta}\right)}\right] \quad (76)$$

where

$$\psi = \frac{1}{\rho_{imp} C_{imp}} + \frac{1}{\rho_{sh} C_{sh}}, \quad (77)$$

C_{sh}, C_{imp} are the longitudinal acoustic-wave velocities in the projectile and shield materials, respectively, ς_1, ς_2 are constants depending on chosen units for parameters, η is a coefficient related to the dynamic shear strength if the shield material properties are assumed to be constant. Recht and Ipson [76] demonstrated that their model successfully predicts penetration of mild steel plate with $\eta = 1.76 \cdot 10^5 psi = 1.21 MPa$ by a cylindrical projectile. In the case of a thick plate penetrated by a cylinder, Recht and Ipson [76] suggested the following formula for the parameter a in Eqs. (72) and (73):

$$a = 1/\sqrt{\left(1 + \frac{\hat{m}_{plug}}{m}\right)\left(1 + \frac{\rho_{sh} b d_{plug}^2}{\rho_{imp} L_{imp} d^2}\right)}, \quad (78)$$

where $\hat{m}_{plug}$ is the experimental value of the ejected plug mass. The model suggested by Recht and Ipson [76] for penetration by a sharp impactor without plug formation is based on energy conservation law that can be written as follows:

$$0.5 m v_{imp}^2 - 0.5 m v_{res}^2 = W \quad (79)$$

where W is a work performed during perforation. Assumption that W does not depend on the impact velocity implies Eqs. (72) and (73) with $a = 1$:

$$v_{res} = (v_{imp}^2 - v_{bl}^2)^{1/2}, v_{imp} \geq v_{bl}, \quad (80)$$

$$\widetilde{v}_{res} = (\widetilde{v}_{imp}^2 - 1)^{1/2}, \widetilde{v}_{imp} \geq 1, \quad (81)$$

and the following formula for the BLV:

$$v_{bi} = \sqrt{2W/m} \quad (82)$$

Formula for the work, W_{cav}, that is required for expanding the hole in a ductile material to the radius R reads (for details see Zaid et al. [77] and Corbett et al. [9]):

$$W_{cav} = \pi k_w R^2 b Y, \quad (83)$$

where k_w equals to 2.0 (Bethe [78]), 1.92 (Hill [79]), 1.33 or 0.5, depending on the deformation mode (Taylor [80]). Associating with the energy loss of rigid, point-nosed impactor, W , and assuming that $W = W_{cav}$ one can obtain formulas for the residual velocity and the BLV by substituting W_{cav} from Eq. (83) instead of W in Eq. (82). Thomson [81] proposed a model that takes into account a dynamic component of the work required for perforation of a thin plate. This model can be written as follows (Sodha and Jain [82], Nixdorff [83]):

$$W(h) = 0.5\pi bY\Phi^2(h) + \pi b\rho_{sh}v^2 I(h), \tag{84}$$

where h is the instantaneous penetration depth of a projectile in the plate after beginning penetration with a constant velocity v, W is the power that is needed to overcome the resistance of the plate, function $\Phi(x)$ determines the shape of a sharp nose of the impactor (see Fig. 2a-b), and

$$I(h) = \int_0^h [\Phi\Phi_{xx} + 2\Phi_x^2]\Phi\Phi_x dx. \tag{85}$$

Following a quasi-dynamic approach let us assume that Eq. (84) is valid for $0 \le h \le L$, where L is the length of the nose of impactor. Energy conservation equation yields the following formula for the instantaneous impactor velocity v (Sodha and Jain [82]):

$$W(h) = 0.5m(v_{imp}^2 - v^2) \tag{86}$$

Substituting $v = v_{res}$, $h = L$ and $\Phi(L) = R$ in Eqs. (84) and (86) yields:

$$v_{res} = \sqrt{\frac{mv_{imp}^2 - \pi bYR^2}{m + 2\pi b\rho_{sh}I(L)}},\, v_{bl} = R\sqrt{\pi bY/m},\, v_{imp} \ge v_{bi} \tag{87}$$

Gupta et al. [84] considered a thin plate and took into account the work required for forming a crater by ogive nosed projectile and energy of the radial stretching in the following form:

$$W = \pi R^2 b\left[0.5Y + 0.62\rho_{sh}\left(\frac{v_{imp}R}{L}\right)^2\right] + \frac{\pi bYw_c^2(1+2kl)exp(-2kl)}{4\sqrt{1-v+v^2}} \tag{88}$$

where l is length of the crack. Gupta et al. [84] used the following empirical correlation between the radial plate deflection, w, the central deflection of the plate, w_c, and the distance from the point of impact, $\widetilde{r}$ (Calder and Goldsmith [85]; Levy and Goldsmith [86]):

$$w = w_c \exp(-k\widetilde{r}), \tag{89}$$

where constant k can be evaluated from the profile of the plate. The residual velocity and the BLV can be determined from Eqs. (79) and (82), respectively. Zhang and Mu [87] considered perforation of a thin shield by a rigid sphere or a hemispherical nose projectile and developed an analytical model describing perforation in a dicing mode. Their approach is based on the estimation of energy dissipation during three stages of penetration: bulging, fracture, and hole enlargement. Pol et al. [88] suggested a model whereby the work required for perforation of a thin plate by ogive-nose projectile comprises the work required for plastic deformation, the work for transferring the matter to new position, and the work spent for bending of the petals. Zaid and Paul [89] and Paul and Zaid [90] investigated normal penetration of pointed and truncated cone-nose and ogive-nose projectiles into thin ductile plate using momentum balance and obtained a number of relationships which allow estimating characteristics of penetration. In particular, their analysis yields the following formula for a pointed cone-nose projectile with a semi-vertex angle ϑ:

$$v_{res} = v_{imp}(1 - \pi b\rho_{sh}R^2 sin\vartheta/m). \tag{90}$$

Brown [91] applied a quasi-dynamic theory to describe penetration of a truncated cone-nose projectile into plastically deformed thin plate and derived formula for energy of the projectile in order to estimate the thickness of the shield that allows projectile containment. Woodward [92] proposed a model for cone-nose projectiles taking into account ductile and dishing failure modes that yields the following formula for the minimum perforation energy:

$$W = \frac{\pi bYR^2}{2}\left[1 + 0.5\pi\left(\frac{b}{R}\right) + 0.867\left(\frac{b}{R}\right)^2\right]. \tag{91}$$

Woodward [93] also suggested the following formulas for W which can be applied for thick and thin shields, correspondingly:

$$W = 2\pi R^3 Y\left(\frac{b}{R} + \frac{1}{3\tan\vartheta} - \sqrt{3}\right). \tag{92}$$

$$W = \frac{2\pi b^3 Y}{\sqrt{3}\tan\vartheta}, \tag{93}$$

where the difference between thick and thin plates is determined by the value of $b = \sqrt{3}R$. Forrestal et al. [94]

and Rosenberg and Forrestal [95] devised an engineering model for shields that exhibit ductile hole-growth which was applied for conical-nose rods penetrating into aluminum plate having a finite thickness:

$$v_{bl} = R\sqrt{2\pi b\sigma_c^{stat}/m},\ v_{res} = v_{bl}\sqrt{(v_{imp}/v_{bl})^2-1},\quad v_{imp} \geq v_{bl}, \tag{94}$$

$$\sigma_r = \sigma_c^{stat} = \frac{Y}{2}\left[1 + ln\left(\frac{2E}{(5-4\nu)Y}\right)\right], \tag{95}$$

where σ_c^{stat} corresponds to static CCE model for elastic-perfectly plastic material with compressible plastic zone and the Tresca yield criterion (Forrestal [49]; Satapathy [34]). The second equation in Eqs. (94) for v_{res} recovers formula suggested by Recht and Ipson [76]. The approach used by Landkof and Goldsmith [96] and Wierzbicki [97] for thin plates is based on energy balance comprising the energies required for crack propagation, petal bending, and plate dishing. Wierzbicki [97] emphasizes that the main advantage of his model is determining these three energies simultaneously while Landkof and Goldsmith [96] assume that these fractional energies are independent of each other. Wierzbicki [97] proposed the following formula for the BLV of a plate penetrated by a sharp conical-nose impactor:

$$v_{bl} = 4.22\left(\frac{\sigma_u Y}{1+n}\right)^{0.5}\left(\frac{\delta_b}{b}\right)^{0.1}\left(\frac{R\rho_{sh}}{m}\right)^{0.7} b^{0.8} \tag{96}$$

where n is the exponent in the power-type stress-strain law, δ_b is the crack tip opening displacement parameter (CTOD). For simplicity, it can be assumed that $\delta_b/b = 1$. Masri [98] suggested a simple model for penetration resistance of pointed nose projectiles, D, that is based on power balance between the power supplied by a projectile during steady-state penetration and elastic-plastic power required for expanding the penetration hole:

$$D = \frac{\pi d^2}{4}\widetilde{D}, \widetilde{D} = \left(\frac{2S}{Ld}\right)\left[\hat{\alpha}\sigma_s^{stat} + \left(\frac{2L}{d}\right)\frac{k_{fr}Y}{\sqrt{3}}\right] \tag{97}$$

where S and L are the meridional cross-sectional area and the length of a projectile nose, correspondingly, d is a projectile shank diameter, k_{fr} is a friction factor (Durban and Fleck [99]), $0 \leq k_{fr} \leq 1$, $\hat{\alpha}$ is an empirical parameter, σ_s^{stat} is static spherical cavitation pressure. For the hardening power law with parameter n (Chakrabarty [58]),

$$\sigma = \begin{cases} E\varepsilon & \text{if } \varepsilon \leq Y/E \\ Y(E\varepsilon/Y)^n & \text{if } \varepsilon > Y/E \end{cases} \tag{98}$$

Masri, 2010 proposed the following approximate formula for σ_s^{stat}:

$$\sigma_s^{stat} = \frac{2Y}{3}\times \left\{1 - ln2 - ln(1-\nu) + \frac{1}{n}\left[\left(\frac{2E}{3Y}\right)^n - 1\right] + 0.73n\left(\frac{2E}{3Y}\right)^n\right\} \tag{99}$$

In particular, formula for penetration resistance force for ogive-nose projectile reads:

$$\widetilde{D} = f(K_{CRH})\left[\hat{\alpha}\sigma_s^{stat} + \sqrt{4K_{CRH}-1}\frac{k_{fr}Y}{\sqrt{3}}\right] \tag{100}$$

where

$$f(K_{CRH}) = 1 + 2K_{CRH}\left[\frac{\sin^{-1}(\omega)}{\omega} - 1\right],\quad \omega = \frac{\sqrt{4K_{CRH}-1}}{2K_{CRH}}, K_{CRH} = \frac{R_{og}}{2R}, \tag{101}$$

K_{CRH} is a caliber radius head of ogive-nose projectile, R_{og} is a radius of a circular arc of an ogive. Solving the equation of projectile motion yields the following formula for the depth of penetration (DOP):

$$H = \frac{\widetilde{m}}{2\widetilde{D}}v_{imp}^2, \widetilde{m} = m/(0.25\pi d^2). \tag{102}$$

Using the results of experiments from the literature, Masri [42] concluded that empirical parameter $\hat{\alpha}$ is practically independent of the shield material characteristics but depends on the shape of the projectile nose, whereas $\hat{\alpha}$ varies in a narrow range and is slightly smaller than for ogive nose projectiles. It was found that experimental data on penetration into aluminum shields by spherical-nose projectiles can be fairly well described (up to some critical impact velocity) by the following formula:

$$H = \frac{\widetilde{m}}{\pi\sigma_s^{stat}}v_{imp}^2, \tag{103}$$

which is obtained from Eqs. (99)-(102) for $K_{CRH} = 0.5$, $\hat{\alpha} = 1$ and $k_{fr} = 0$. Penetration of ogive-nose projectile with $K_{CRH} = 0.5$ into aluminum shields can be described by these equations with $\hat{\alpha} = 1$ and $0.2 \leq k_{fr} \leq 0.6$;

$k_{fr} = 0.4$ is suggested as a representative value for all experiments. Using energy balance approach Srivathsa and Ramakrishnan [100, 101]derived a ballistic performance index to estimate and compare the ballistic quality of metals. This index is a function of common mechanical properties of the material and the impact velocity of a projectile. In their subsequent study Srivathsa and Ramakrishnan [102] presented these indexes in the form of maps. Using energy and momentum balance equations and assuming that shield is thin and that the absorbed energy is small in comparison with the impact energy, Giere [103] derived the following formula:

$$v_{res}/v_{imp} = m_{plug}/(m_{plug} + m_{imp}). \tag{104}$$

6. Oversimplified models

6.1. Shields having finite thickness

Chen and Li [104, 105] started from a quasi-dynamic SCE model of the type given by Eqs. (17) and (3) with $\alpha_s = \widetilde{\alpha}_s Y$, $\beta_s = \widetilde{\beta}_s \rho_{sh}$ where $\widetilde{\alpha}_s$ and $\widetilde{\beta}_s$ are dimensionless constants depending on the chosen model of shield material (such models are discussed above). They showed that this quasi-dynamic SCE model yields the following formulas for the residual velocity and the BLV:

$$v_{res}^2 = (v_{imp}^2 - v_{bl}^2)\exp\left(-\frac{\pi b}{2dN}\right),$$
$$v_{bl}^2 == \frac{\widetilde{\alpha}_s Y N_1}{\widetilde{\beta}_s \rho_{sh} N_2}\left[\exp\left(\frac{\pi b}{2dN}\right) - 1\right], \tag{105}$$

where

$$N_1 = 1 + 2\mu_{fr} K_{CHR}^2 (\pi - 2\phi - \sin 2\phi), \tag{106}$$

$$N_2 = N^* + \mu_{fr} K_{CRH}^2 \left[\frac{\pi}{2} - \phi - \frac{1}{3}\left(2\sin 2\phi + \frac{\sin 4\phi}{4}\right)\right], \tag{107}$$

$$N^* = \frac{8K_{CRH} - 1}{24K_{CRH}^2},$$
$$\phi = \sin^{-1}\left(1 - \frac{1}{2K_{CRH}}\right), K_{CRH} \geq \frac{1}{2} \tag{108}$$

for ogive-nose projectiles, and

$$N_1 = 1 + \mu_{fr} cot\vartheta, N_2 = \frac{1 + \mu_{fr} cot\vartheta}{1 + cot^2\vartheta} \tag{109}$$

for conical-nose projectiles. For sharp and slender projectiles when the parameter

$$N = \frac{m}{\widetilde{\beta}_s \rho_{sh} d^3 N_2} \tag{110}$$

is sufficiently large, Chen and Li [104] proposed the following simplified version of formulas in Eq. (105):

$$v_{res}^2 = v_{imp}^2 - v_{bl}^2,\ v_{bl}^2 = \frac{\pi b \widetilde{\alpha}_s Y N_1 d^2}{2m}. \tag{111}$$

Forrestal and Warren [106] started from a quasi-dynamic CCE model of the type described by Eq. (18) with $\sigma_\tau = 0$, $\alpha_c = \sigma_c^{stat}$, $\beta_c = \widetilde{\beta}_c \rho_{sh}$ where $\widetilde{\beta}_c$ is a dimensionless constant depending on the chosen model of shield material, σ_c^{stat} is determined by Eq. (43). This model yields the following formulas for the residual velocity and the BLV:

$$v_{res} = \sqrt{v_{imp}^2 - v_{bl}^2}\exp(-C),\ v_{bl} =$$
$$= \left(\frac{\sigma_c^{stat}}{\widetilde{\beta}_c \rho_{sh} N_*}\right)^{1/2}\sqrt{\exp(2C) - 1}, \tag{112}$$

where

$$C = \frac{\widetilde{\beta}_c b \rho_{sh} N_*}{(L_0 + k_{shape} L)\rho_{imp}},$$
$$N_* = 8K_{CRH}^2 ln\left(\frac{2K_{CRH}}{2K_{CRH} - 1}\right) - 4K_{CRH} - 1 \tag{113}$$

for ogive-nose projectiles, and $N_* = \tan^2\vartheta$ for conical-nose projectiles. Forrestal and Warren [106] assumed the following formula for the mass of a projectile:

$$m = \frac{\pi d^2 \rho_{imp}}{4}(L_0 + k_{shape} L), \tag{114}$$

where

$$k_{shape} = 4K_{CRH}^2 - \frac{4}{3}K_{CRH} + \frac{1}{3} -$$
$$\frac{4K_{CRH}^2(2K_{CRH} - 1)}{\sqrt{4K_{CRH} - 1}}\sin^{-1}\left(\frac{\sqrt{4K_{CRH} - 1}}{2K_{CRH}}\right) \tag{115}$$

for ogive-nose projectiles, and $k_{shape} = 1/3$ for conical-nose projectiles. If the nose of a projectile is sufficiently sharp then keeping several first terms in Taylor series expansion of the exponent in Eq. (112) yields the following simplified formulas for the residual velocity and the BLV:

$$v_{res} = \sqrt{v_{imp}^2 - v_{bl}^2}(1 - C + 0.5C^2)^{1/2}, \quad (116)$$

$$v_{bl} = \sqrt{\frac{2\sigma_c^{stat} b}{(L_0 + k_{shape}L)\rho_{imp}}} \left(1 + C + \frac{2}{3}C^2\right)^{1/2} \quad (117)$$

The latter equations with $C = 0$ can be also considered as a version of a simplified penetration model. Forrestal and Warren [106]; Forrestal et al. [107]; Børvik et al. [108] analyzed the accuracy of the model given by Eqs. (116) and (117) using experimental results. Chen and Li [105] compared the models suggested by Chen and Li [104] and by Forrestal and Warren [106] and concluded that there are the differences in their predictions are quite small. Note that Chen and Li [105] used a simplified (as compared with Eq. (116)) formula for the residual velocity (Forrestal and Warren [106]):

$$v_{res} = \sqrt{v_{imp}^2 - v_{bl}^2}(1 - 2C + 2C^2)^{1/2}, \quad (118)$$

This formula is obtained by squaring Eq. (112) and keeping only the first three terms in Taylor series expansion of $\exp(-2C)$.

6.2. Semi-infinite shields

Forrestal and Warren [106] started from a quasi-dynamic SCE model given by Eq. (41) and Eq. (47). This model yields the following formula for the DOP of ogive-nose projectiles:

$$\frac{H}{L_0 + k_{shape}L} = \frac{\rho_{imp}}{3\rho_{sh}N^*} ln\left(1 + \frac{3\rho_{sh}N^*}{2\sigma_s^{stat}} v_{imp}^2\right), \quad (119)$$

where k_{shape} and N^* are determined by Eq. (115) and (108), respectively. Assuming that $1.5\rho_{sh}N^* v_{imp}^2/\sigma_s^{stat}$ is much smaller than 1 Eq. (119) can be written in two simplified forms by replacing the logarithm by one or two first terms in the Taylor series expansion:

$$\frac{H}{L_0 + k_{shape}L} = \frac{\rho_{imp} v_{imp}^2}{2\sigma_s^{stat}}, \quad (120)$$

$$\frac{H}{L_0 + k_{shape}L} = \frac{\rho_{imp} v_{imp}^2}{2\sigma_s^{stat}} \left(1 - \frac{3\rho_{sh}N^*}{4\sigma_s^{stat}} v_{imp}^2\right) \quad (121)$$

Comparison with the results of experiments showed that Eq. (121) is very accurate for striking velocities smaller than $1300 m/s$. The model similar to Eq. (120) was also proposed by Chen and Li [109].

7. Plugging and multi-stage models

Most of models taking into account plug formation, for instance, those suggested by Awerbuch [110]; Awerbuch and Bodner [111]; Chen and Li [104, 112]; Goldsmith and Finnegan [113]; Hsueh [114], [115]; Jenq et al. [116]; Krishna Teja Palleti et al. [117]; Liss et al. [118]; Ning et al. [119]; Shoukry et al. [120]; Ravid and Bodner [121, 122]; Teng and Wierzbicki [123]; Wu and Batra [124] (most of which are surveyed by Corbett et al. [9]) generally do not allow deriving explicit formulas for determining the residual velocity or the BLV. Two models considered below allow determining the residual velocity and the BLV.

7.1. Basic simplified model

The simplest model of plugging for a cylindrical impactor (see e.g., Sagomonyan [70]; Holt et al. [125]) is based on the assumption that a cylindrical plug with a radius $R = r$, height b and mass

$$m_{plug} = \pi \rho_{sh} r^2 b \quad (122)$$

is formed at the beginning of penetration, and that this plug moves together with a projectile under the action of the resistance force associated with the plug:

$$D_{plug}(h) = 2\pi\tau_s r(b - h), h \le b, \quad (123)$$

where h is the instantaneous depth of penetration and τ_s is a constant shear stress. If a projectile penetrates at the depth $h = b$ then perforation occurs. More sophisticated models of plug formation when the plate is penetrated by cylindrical impactor were suggested by Sagomonyan [126]; Sagomonyan et al. [127]. Ben-Dor et al. [22, 128, 129] considered impactors with arbitrary generatrix having a flat bluntness, and took into account the resistance force at the lateral surface of the impactor, $D_{lat} = D_{lat}(h, \dot{h}, \ddot{h})$. In this case, the impactor is subjected to the resistance forces D_{plug} and D_{lat} until $h \le b$ (see Fig. 4a) and the resistance force D_{lat} when the plug breaks apart at the depth $h = b$ (see Fig. 4b). Therefore, equations describing motion of a projectile read:

$$(m + m_{plug})\ddot{h} + D_{lat}(h, \dot{h}, \ddot{h}) + D_{plug}(h) = 0, 0 \le h < b \quad (124)$$

$$m\ddot{h} + D_{lat}(h, \dot{h}, \ddot{h}) = 0, b \le h \le b + L \quad (125)$$

Considering $w = v^2 = \dot{h}^2$ as a function of the independent variable h, Eqs. (124) and (125) can be replaced by the following equation:

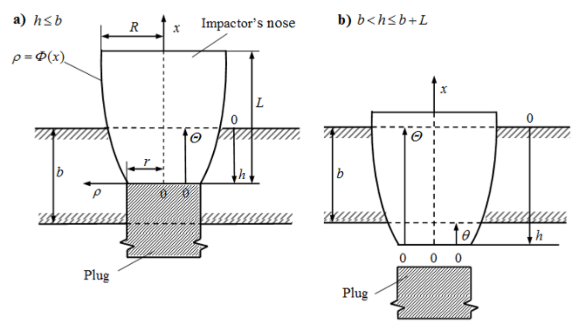

Figure 4. Model of penetration with plug formation.

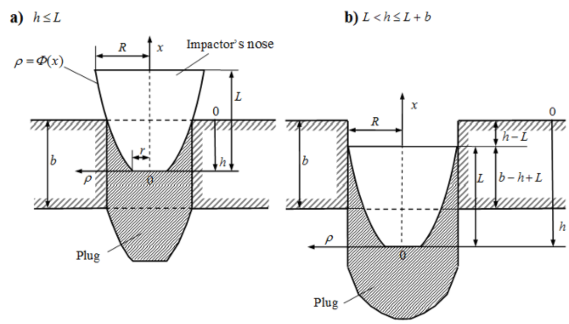

Figure 5. Slezkin's penetration model with plug formation

$$\frac{1}{2}[m+m_{plug}\delta(h)]\frac{dw}{dh}+D_{lat}\left(h,\sqrt{w},\frac{1}{2}\frac{dw}{dh}\right)+D_{plug}(h)\delta(h)=0, \quad (126)$$

where function $\delta(h)$ is determined as follows:

$$\delta(h)=\begin{cases}0 & \text{if } h\le 0\\ 1 & \text{if } 0\le h\le b\\ 0 & \text{if } 0>b\end{cases} \quad (127)$$

In most analytical approximate models D_{lat} can be written in the following form:

$$D_{lat}(h,\dot{h},\ddot{b})=\chi_0(h)+\chi_1(h)\dot{h}^2+\chi_2(h)\ddot{h}, \quad (128)$$

where functions χ_0,χ_1,χ_2 depend upon h,upon a shape of impactor and upon characteristics of material of a shield. Substituting D_{lat} from Eq. (128) into Eq. (126) yields the following linear ordinary differential equation:

$$\frac{1}{2}[m+m_{plug}\delta(h)+\chi_2(h)]\frac{dw}{dh}+\chi_1(h)w+D_{plug}(h)\delta(h)+\chi_0(h)=0, \quad (129)$$

which can be solved analytically to obtain formulas for the BLV and the residual velocity.

7.2. Slezkin's model

In 1944 N. A. Slezkin (see Grigoryan [17]; Sagomonyan [130]) suggested a simple model based on the assumption that only shear associated with plug formation is the cause of the resistance force. In the following we describe this model in the generalized form applicable for an arbitrary impactor (body of revolution) with flat bluntness.

The two-stage model is illustrated in Fig. 5 where a projectile and a plug move together as a unitized projectile-plug body. The expression for the resistance force acting on this body reads:

$$D=\begin{cases}2\pi b\tau_s\Phi(h) & \text{if } 0\le h\le L\\ 2\pi R\tau_s(b+L-h) & \text{if } L<h<L+b\end{cases} \quad (130)$$

where a mass of a plug is assumed to be given by the following formula:

$$m_{plug}=\begin{cases}\pi b\rho_{sh}\Phi^2(h) & \text{if } 0\le h\le L\\ \pi R^2 b\rho_{sh} & \text{if } L<h<L+b\end{cases} \quad (131)$$

Motion of this unitized projectile-plug body having a variable mass at the first stage of penetration ($0\le h\le L$) is described by the following equation:

$$\frac{d}{dt}[(1+k_1\Phi^2)v]=-k_2\Phi \quad (132)$$

where t is time and $k_1=\pi b\rho_{sh}/m$, $k_2=2\pi b\tau_s/m$. Motion of the composite body having a constant mass that comprises the impactor and the plug, at the second stage of penetration ($L\le h\le L+b$) is described by the following equation:

$$(1+k_1R^2)\frac{dv}{dt}=-k_2\frac{R}{b}(b+L-h) \quad (133)$$

The expression for the residual velocity when the impact velocity is sufficiently high for perforating the shield reads:

$$v_{res}=\frac{\sqrt{(1+k_1r^2)^2v_{imp}^2-2k_2(J_1+k_1J_3)-k_2Rb(1+k_1R^2)}}{(1+k_1R^2)} \quad (134)$$

Formula for the BLV is obtained by substituting $v_{res}=0$ and $v_{imp}=v_{bl}$ into Eq. (134).

8. Some other models and related problems

To the best of our knowledge, the first model in the penetration mechanics which belongs to the class of the LIMs and is not based on the cavity expansion approach was suggested by Nishiwaki [131]:

$$\sigma_n = a_0 + a_2 u^2 v^2 \tag{135}$$

where a_0 is the "static contact pressure" and $a_2 = \rho_{sh}$. This model takes into account impactor-shield friction and variation of the impactor plate contact surface during penetration. Using his experimental results for conical impactors penetrating into aluminum shields, Nishiwaki [131] arrived at the conclusion that a_0 is proportional to the thickness of the perforated plate. He developed a relationship between the impact velocity and the residual velocity of a cone shaped impactor. In the following we consider in more detail a study by Vitman and Stepanov [132] which is not readily available for Western readers and which is often cited in publications in Russian as a classic study that pioneered the idea of applying LIM of the kind given by Eq. (135) in modeling of high speed penetration. Vitman and Stepanov [132] investigated experimentally penetration of cone nose projectiles with impact velocities up to into various metal shields. They found that the ratio of the resistance force to the shank area exhibits a linear dependence on the squared impactor velocity, provided that a penetration depth exceeds the length of the conical nose. Analysis of these linear correlations for projectiles having different cone angles allowed to make the following observations: (i) the slope of these lines is approximately the same as the slope of the drag force dependence for a conical projectile moving in a gaseous medium and calculated using the Newton's model (Hayes and Probstein 133); (ii) all straight lines obtained in the experiments on penetration into shields manufactured from the same material intersect in the same point. These two observations imply that the formula for a drag force for a conical impactor may be written as follows:

$$D_{cone} = \pi R^2 (a_0 + a_2 \sin^2 \vartheta v^2), \tag{136}$$

where $a_2 = \rho_{sh}$, $a_0 = \widetilde{H}_{sh}$ is the "dynamic hardness of the metal for impact velocities of $\widetilde{v} \sim 10 m/s$", ϑ is the half apex angle of a cone. Some values of $\widetilde{H}_{sh}$ adopted from the study by Vitman and Ioffe [134] are as follows: $350MPa, 910MPa, 910MPa$ and $1330MPa$ for aluminum, soft steel, copper and duralumin, respectively. Initially, Vitman and Stepanov [132] used the following correlation:

$$a_0 = \widetilde{H}_{sh} (v/\widetilde{v})^{\alpha} \tag{137}$$

but subsequently they decided that the dependence of a_0 on the impactor velocity can be neglected since $0 < \alpha << 1$. Let us assume that local interaction force between a projectile and a shield is equal to a force acting on a surface of a tangent cone at this location provided that projectile velocity and shield material are the same in both cases. It is significant that Eq. (136) for cone-nose impactors implies a model given by Eq. (135) with $a_2 = \rho_{sh}$ and $a_0 = \widetilde{H}_{sh}$ for sharp impactors having different shapes. Hence the result obtained using the Vitman and Stepanov [132] formula is often referred to as a LIM although describing local interaction between a shield and a projectile was not the main goal of their study. Different versions of the above assumption are known in aerodynamics as "methods of tangent cones" (Chernyi [135]; Hayes and Probstein [133]). Similar approach in penetration mechanics was used by Recht [13]. Vitman and Stepanov [132] analyzed results of experiments with cone nosed and cylindrical projectiles and noticed that drag force acting on penetrating striker is practically constant in the range of relatively small impact velocities for most metals. Rosenberg and Dekel [46, 65] (see Section 4.2) developed an approach based on this peculiarity of penetration. Golubev and Medvedkin [136, 137] studied penetration of a rigid rods with conical and hemispherical heads into a thick plate manufactured from mild low-carbon steel at velocities of up to $600m/s$. On the basis of the obtained experimental data they proposed a three-stage penetration model. At the initial stage of penetration ($h \leq R$), the linear dependence between the resistance to penetration and impactor velocity is used. At the third stage ($h > 2R$), the resistance force is assumed to be constant. At the intermediate stage ($R < h \leq 2R$), the resistance force is determined by linear interpolation between the values of the resistance force at the initial and final stages. Landgrov and Sarkisyan [138] proposed the following model:

$$\sigma_n = E(\hat{\sigma}_0/E)^{1-uv/c}, c = \sqrt{E\rho_{sh}} \tag{138}$$

where $\hat{\sigma}_0$ is experimentally determined parameter that is close to the value of the Mayer hardness. Yarin et al. [139] studied penetration of a rigid projectile (ovoid of Rankine) into an elastic-plastic shield. This particular shape of a projectile was selected because it implied a reasonably simple velocity field that exactly satisfied the continuity equation and the condition of impenetrability of the projectile. Although equation of motion of projectile can be generally solved numerically, analytical formulas for the DOP, the residual velocity and the BLV were derived by making some additional assumptions. Particularly, for deep penetration into a semi-infinite shield, the DOP is given by the following formula:

$$H = \frac{3m + \pi \rho_{sh} R^3}{6\pi Y R^2 ln\left(4^{5/9}\frac{3G}{4Y}\right)} v_{imp}^2 \qquad (139)$$

where G is shear modulus of the material of the shield. Yarin et al. [139] also described the procedure that allows applying the suggested approach to a projectile having a tip of arbitrary shape. Levy and Goldsmith [86] considered normal impact on thin plates by hemispherical tip cylindrical hard-steel projectiles and derived expressions for the resistance force, D, as a function of time,t. The authors note that "the theory is based on a lumped-parameter system that includes the radial variation in axial target displacement". For a non-perforation case (the impact velocity is less than the BLV) the theory implies the following relationship:

$$D(t) = \frac{1.33 m^{2.5}\sqrt{Yb}}{(m+m_{eq})^2} v_{imp} / \exp\left(\frac{1.33\sqrt{mYb}}{m+m_{eq}} t\right), \quad m_{eq} = \frac{\pi \rho_{sh} b}{2k^2} \qquad (140)$$

where k is the same as in Eq. (89). For perforation case (the impact velocity is larger than the BLV) the sub-model "requires either measured information from the plug separation process or corresponding reasonable geometric and kinematic assumptions for a solution in the perforation domain". Detailed analysis of the model by Levy and Goldsmith [86] may be found in Corbett *et al.* [9]. Wen [140, 141] suggested a model where normal stress on the projectile-metal shield contact surface is a linear function of the impact velocity. Shaw [142] proposed a model of the type given by Eq. (135) with $a_0 = \sigma_r$ and $a_2 = \rho_{sh} G(v_{imp})$, where σ_r is determined in Eq. (27) while G is a function that strongly depends on the impact velocity in addition to the shield material properties. The function is determined on the basis of experiments; it is a decreasing function that tends to a constant value of 0.5 when impact velocity is very large. Shaw [142] gave two examples of approximations for $G(v_{imp})$, for 0.4 GPa aluminum and 0.8 GPa aluminum, correspondingly:

$$G = 0.0019 v_{imp}^4 - 0.0373 v_{imp}^3 + 0.2860 v_{imp}^2 - 1.0059 v_{imp} + 1.8603, \qquad (141)$$

$$G = 0.0092 v_{imp}^4 - 0.1369 v_{imp}^3 + 0.7877 v_{imp}^2 - 2.1346 v_{imp} + 2.8697. \qquad (142)$$

Sagomonyan [130] applied a "model of normal sections" for modeling penetration of cones with non small vertex angle. This approach was used also by Bagdoev et al. [143]. In the studies of Partom [144] and Littlefield et al. [145], cavity expansion models were modified to account for the finite size of the shield in the direction normal to the direction of penetration. A number of solutions based on elastic-plastic isotropic models of shield material were obtained by Vantsyan [146]; Bagdoev and Vantsyan [147, 148]; Rahmatulin and Bagdoev [149]. Bagdoev and Vantsyan [150, 151]; Bagdoev et al. [143, 152, 153]; Asatryan et al. [154] (see also Sagomonyan [70]) modeled translational penetration into anisotropic media while Bagdoev and Vantsyan [155] found a solution for a problem of penetration into an elastic transversely isotropic medium when the impactor rotates or twists with constant angular velocity. Huang et al. [156, 157] applied cavity expansion theory for calculating the DOP in the case when impact velocity is smaller than the BLV and a projectile only partially penetrates into a plate. They found that for relatively low velocities nose shape of the projectile has a stronger influence on the penetration depth while the reverse dependence is observed for relatively high velocities. Dienes and Miles [158] proposed a membrane model that requires numerical calculation of an integral for determining the BLV of thin plate. Tirosh et al. [159] suggested a closed-form assessment of residual cavity depth left after a rigid hemispherical nose cylinder impacts a rigid-plastic shield. Teland and Moxnes [160] conducted numerical simulations using a hydrocode in order to compare analytical results from cavity expansion models with the results obtained by "exact" numerical calculations. Wijk et al. [161]; Wijk [162] suggested a model that can be described as follows. Drag force acting on a projectile is assumed to be a sum of a drag force acting on the nose of a projectile and a friction drag force acting on body of a projectile. Initially the nose drag increases with penetration depth until it attains a constant value which corresponds to lateral displacement of shield material along the trajectory of a projectile. Projectile body friction force also increases at the initial stage of penetration and becomes constant when the trailing edge of the projectile passes through the front face of the shield. It is assumed that when the leading edge of the projectile is close to the rear surface of the shield, the remaining bulk of shield material ahead of the projectile crushes and forms fragments. At this penetration depth the force required for crushing is equal to the force required for continuing lateral displacement of shield material, and fragments are ejected with the same velocity as the projectile.

9. Concluding remarks

In this study we presented more or less comprehensively all widely used and some not well known analytical models which were suggested for describing high-speed penetration into metal shields. This survey is characterized by the following distinguishing features: (i) includes an unprecedented large number of models; (ii) presents models suggested during recent years; (iii) analyzes models which have been originally published in Russian and are not well known in the West either because of the language barrier or because these studies were classified. We believe that there it is important to expose these models so that scientific community can use them. The performed analysis shows that until now prevailing tendency among researchers is to improve cavity expansion models for non-thin shields in order to determine dependencies of coefficients in simple models (mainly, two- and three-term LIMs) vs. mechanical properties of shield material. Such models allow conducting analytical studies of various applied problems in case of normal impact and numerical investigations in case of oblique impact. In descriptions of their own models the authors usually include arguments in favor of a particular model which are based on theoretical considerations, some limited experimental results or "exact" calculations. Nevertheless, it must be emphasized that these arguments or comparisons with other models have mainly only illustrative character. Devising consistent procedures for comparing different models and establishing ranges of their validity is still a subject of ongoing research.

References

[1] Børvik, T., Langseth, M., Hopperstad, O.S., Malo, K.A. (1998). Empirical equations for ballistic penetration of metal plates. Fortifikatorisk Notat No. 260/98. The Norwegian Defence Construction Service, Central Staff - Technical Division, Oslo, Norway.

[2] Backman, M., Goldsmith, W. (1978). The mechanics of penetration of projectiles into targets. Int. J. of Engineering Science, 16(1): 1-99.

[3] Jonas, G.H., Zukas, J.A. (1978). Mechanics of penetration: analysis and experiment. Int. J. of Engineering Science, 16(1): 879-903.

[4] Neilson, A.J. (1985). Empirical equations for the perforation of mikd steel plates. Int. J. of Impact Engineering, 3(2): 137-142.

[5] Brown, S.J. (1986). Energy release protection for pressurized systems. Part 2. Review of studies into impact/terminal ballistics. Applied Mechanics Reviews, 39(2), Part 1: 177-202.

[6] Anderson, C.E., Jr., Bodner S.R. (1988). Ballistic impact: the status of analytical and numerical modeling. Int. J. of Impact Engineering, 11(1): 33-40.

[7] Aptukov, V.N. (1990). Penetration: Mechanical aspects and mathemanical modeling (Review). Strength of Materials, 22(2): 230-240.

[8] Amde, A.M., Mirmiran, A., Walter, T.A. (1996). Local damage assessment of metal barriers under turbine missile impacts. J. of Structural Engineering, 122(1): 99-108

[9] Corbett, G.G., Reid, S.R., Johnson, W. (1996). Impact loading of plates and shells by free flying projectiles: a review. Int. J. of Impact Engineering, 18(2): 141-230.

[10] Ben-Dor, G., Dubinsky, A., Elperin, T. (2005). Ballistic impact: recent advances in analytical modeling of plate penetration dynamics. A Review. ASME Applied Mechanics Reviews, 58: 355-371.

[11] Aly, S.Y., Li, Q.M. (2008). Critical impact energy for the perforation of metallic plates. Nuclear Engineering and Design, 238(10): 2521-2528.

[12] Zukas, J.A. (1982). Penetration and perforation of solids. In: Zukas J.A., Nicholas, T., Swift, H.F., Greszczuk, L.B., Curran, D.R., eds. Impact Dynamics. J. Wiley and Sons, New York, pp. 155-214.

[13] Recht, R.F. (1990). High velocity impact dynamics: Analytical modeling of plate penetration dynamics. In: Zukas, J.A., ed. High Velocity Impact Dynamics. John Wiley and Sons, New York, pp. 443-513.

[14] Zukas, J.A., Walters, W.P. (1990). Analytical models for kinetic energy penetration. In: Zukas J.A., ed. High Velocity Impact Dynamics. John Wiley and Sons, New York, pp. 405-441.

[15] Bulson, P.S. (1997). Explosive Loading of Engineering Structures. A History of Research and a Review of Recent Developments. E&FN Spon, London.

[16] Bangash, M.Y., Bangash, H.T. (2006). Explosion - Resistant Buildigs: Design, Analysis, and Case Studies. Springer, Heidelberg, Berlin.

[17] Grigoryan, V.A. (2006). Particular Problems of Terminal Ballistics. MGTU imeni N.E. Baumana, Moscow (in Russian).

[18] Carlucci, D.E., Jacobson, S.S. (2008). Ballistics. Theory and Design of Guns and Ammunition. CRC Press.

[19] Bangash, M.Y.H. (2009). Shock, Impact and Explosion: Structural Analysis and Design. Springer, Berlin.

[20] Szuladziñski, G. (2010). Formulas Mechanical and Structural Shock and Impact. CRC Press, Bosa

Roca.

[21] Ben-Dor, G., Dubinsky, A., Elperin, T. (1997a). Area rules for penetrating bodies. Theoretical and Applied Fracture Mechanics, 26(3): 193-198.

[22] Ben-Dor, G., Dubinsky, A., Elperin, T. (2006). Applied High-Speed Plate Penetration Dynamics. Springer, Dordrecht.

[23] Ben-Dor, G., Dubinsky, A., Elperin, T. (1997). Optimal 3D impactors penetrating into layered targets. Theoretical and Applied Fracture Mechanics, 27(3): 161-166.

[24] Ben-Dor, G., Dubinsky, A., Elperin, T. (1998). On the ballistic resistance of multi layered targets with air gaps. Int. J. of Solids and Structures, 35(23): 3097-3103.

[25] Alekseeva, E.V., Barantsev, R.G. (1976). Local Method of Aerodynamic Calculations in Rarefied Gas. Leningrad State University Press, Leningrad (in Russian).

[26] Bunimovich, A.I., Dubinsky, A. (1995). Mathematical Models and Methods of Localized Interaction Theory. World Scientific, Singapore.

[27] Miroshin, R.N., Khalidov, U.A. (1991). Locaized Interaction Theory. Leningrad State University Press, Leningrad (in Russian).

[28] Miroshin, R.N., Khalidov, U.A. (2002). Local Methods in Continuum Mechanics. St.-Petersburg University Press, St.-Petersburg (in Russian).

[29] Ben-Dor, G., Dubinsky, A., Elperin, T. (2012). Shape optimization of high-speed penetrators: a review. Central European Journal of Engineering, 2(4): 473-482

[30] Ben-Dor, G., Dubinsky, A., Elperin, T. (2012). Investigation and optimization of protective properties of metal multi-layered shields: A Review. International Journal of Protective Structures, 3(3): 275-291.

[31] Hopkins, H.G. (1960). Dynamic expansion of spherical cavities in metals. In: Hill, R. and Sneddon, I.N., eds. Progress in Solid Mechanics, 1. Pergamon Press, Oxford, pp. 84-164.

[32] Yu, H.S. (2000). Cavity Expansion Methods in Geomechanics. Kluwer, Dordrecht.

[33] Teland, J.A. (1999). A review of analytical penetration mechanics. Report FFI/RAPPORT 99/01264. Norwegian Defence Research Establishment (FFI), Kjeller, Norway.

[34] Satapathy, S., (1997). Application of cavity expansion analysis to penetration problems. Report IAT.R 0136. The University of Texas at Austin, Institute for Advanced Technology, Austin, TX.

[35] Bernard, R.S., Hanagud S.V. (1975). Development of a projectile penetration theory. Report S-75-9. Penetration Theory for shallow to moderate depths (Report 1 of Series). U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS.

[36] Sagomonyan, A.Ya. (1960). Penetration of sharp bodies of revolution into soils. Doklady Akademii Nauk SSSR, 134(6): 1320-1323 (in Russian).

[37] Rahmatulin, K.A., Sagomonyan, A.Ya., Alekseev, N.A. (1964). Soils Dynamics. Moscow State University Press, Moscow (in Russian).

[38] Yankelevsky, D.Z., Adin, M.A. (1980). A simplified analytical method for soil penetration analysis. Int. J. for Numerical and Analytical Methods in Geomechanics, 4(3): 233-254.

[39] Bishop, R.F., Hill, R., Mott, N.F. (1945). The theory of indentation and hardness tests. Proceedings of the Physical Society, 57(321), Part 3: 147-155.

[40] Hill, R. (1950). The Mathematical Theory of Plasticity. Clarendon Press, Oxford.

[41] Masri, R., Durban, D. (2007). Cylindrical cavity expansion in compressible Mises and Tresca solids. European J. of Mechanics. A/Solids, 26(4): 712-727.

[42] Masri, R., Cohen T., Durban, D. (2010). Enlargement of a circular hole in a thin plastic sheet: taylor-bethe controversy in retrospect. Quarterly J. of Mechanics and Applied Mathematics, 63(4): 589-616.

[43] Cohen, T., Masri, R., Durban, D. (2010). Ballistic limit predictions with quasi-static cavitation fields. Int. J. of Protective Structures, 1(2): 235-255.

[44] Rosenberg, Z., Dekel, E. (2009). On the deep penetration and plate perforation by rigid projectiles. Int. J. of Solids and Structures, 46(24): 4169-4180.

[45] Rosenberg, Z., Dekel, E. (2010). Revisiting the perforation of ductile plates by sharp-nosed rigid projectiles. Int. J. of Solids and Structures, 47(22-23): 3022-3033.

[46] Rosenberg, Z., Dekel, E. (2012). Terminal Ballistics. Springer.

[47] Forrestal, M.J., Okajima, K., Luk, V.K. (1988). Penetration of 6061 T651 aluminum target with rigid long rods. J. of Applied Mechanics, 55(4): 755-760.

[48] Forrestal, M.J., Luk, V.K. (1988). Dynamic spherical cavity expansion in a compressible elastic plastic solid. J. of Applied Mechanics, 55(2): 275-279.

[49] Forrestal, M.J. (1986). Penetration into dry porous rock. Int. J. of Solids and Structures, 22(12): 1485-1500.

[50] Forrestal, M.J., Luk, V.K., Brar, N.S. (1990). Perforation of aluminum armor plates with conical nose projectiles. Mechanics of Materials, 10(1-2): 97-105.

[51] Forrestal, M.J., Romero, L.A. (2007). Comment on "Perforation of aluminum plates with ogive-nose steel rods at normal and oblique impacts" (Int. J. of

Impact Engineering 1996; 18: 877-887). Int. J. of Impact Engineering, 34(12): 1962-1964.

[52] Luk, V.K., Amos, D.E. (1991). Dynamic cylindrical cavity expansion of compressible strain hardering materials. J. of Applied Mechanics, 58(2): 334-340.

[53] Luk, V.K., Forrestal, M.J., Amos, D.E. (1991). Dynamic spherical cavity expansion of strain hardening materials. J. of Applied Mechanics, 58(1): 1-6.

[54] Forrestal, M.J., Brar, N.S., Luk, V.K. (1991). Perforation of strain hardening targets with rigid spherical nose rods. J. of Applied Mechanics, 58(1): 7-10.

[55] Forrestal, M.J., Luk, V.K., Rosenberg, Z., Brar N.S. (1992). Penetration of 7075-T651 aluminum targets with ogival nose rods. Int. J. of Solids and Structures, 29(14/15): 1729-1736.

[56] Warren, T.L., Forrestal, M.J. (1998). Effect of strain hardening and strain rate sensitivity on penetration of aluminum targets with spherical nosed rods. Int. J. of Solids and Structures, 35(28-29): 3737-3753.

[57] Warren, T.L. (1999). The effect of strain rate on the dynamic expansion of cylindrical cavities. J. of Applied Mechanics, 66(3): 818-821.

[58] Chakrabarty, J. (2010). Applied Plasticity. Springer, New York.

[59] Yadav, S., Chichilli, D.R., Ramesh, K.T. (1995). The mechanical response of a 6061-T6 Al/Al203 metal matrix composite at high rates of deformation. Acta Metallurgica et Materialia, 43(12): 4453-4464.

[60] Masri, R., Durban, D. (2005). Dynamic spherical cavity expansion in an elastoplastic compressible mises solid. J. of Applied Mechanics, 72(6): 887-898.

[61] Masri, R., Durban, D. (2006). Dynamic cylindrical cavity expansion in an incompressible elastoplastic medium. Acta Mechanica, 181(1-2): 105-123.

[62] Masri, R., Durban, D. (2009). Deep penetration analysis with dynamic cylindrical cavitation fields. Int. J. of Impact Engineering, 36(6): 830-841.

[63] Rosenberg, Z., Dekel, E. (2008). A numerical study of the cavity expansion process and its application to long-rod penetration mechanics. Int. J. of Impact Engineering, 35(3): 147-154.

[64] Rosenberg, Z., Dekel, E. (2009). Analytical solution of the spherical cavity expansion process. Int. J. of Impact Engineering, 36(2): 193-198.

[65] Rosenberg, Z., Dekel, E. (2009). The penetration of rigid long rods - revisited. Int. J. of Impact Engineering, 36(4): 551-564.

[66] Goodier, J.N. (1965). On the mechanics of indentation and cratering in solid targets of strain-hardening metal by impact of hard and soft spheres. In: Proc.of 7th Hypervelocity Impact Symp. (17-19 November, 1964, Tampa, FL), vol. 3, pp. 215-260.

[67] Forrestal, M.J., Tzou, D.Y., Askar, E., Longcope, D.B. (1995). Penetration into ductile metal targets with rigid spherical nose rods. Int. J. of Impact Engineering, 16(5/6): 699-710.

[68] Sagomonyan, A.Ya. (1975). Plate piercing by slender solid projectile. Moscow University Mechanics Bulletin, 30(5-6): 26-31.

[69] Ben-Dor, G., Dubinsky, A., Elperin, T. (2000). Analytical solution for penetration by rigid conical impactors using cavity expansion models. Mechanics Research Communications, 27(2): 185-189.

[70] Sagomonyan, A.Ya. (1988). Dynamics of Barriers Perforation. Moscow State University Press, Moscow (In Russian).

[71] Aptukov, V.N., Murzakaev, A.V., Fonarev, A.V. (1992). Applied Theory of Penetration. Nauka, Moscow (in Russian).

[72] Aptukov, V.N. (1991). Expansion of a spherical cavity in a compressible elasto plastic medium. II. Effect of inertial forces. Temperature effects. Strength of Materials, 23(12): 1269-1274.

[73] Li, X.L., Liu, C.L., Huang, F.L., Wu, H.J., Chen, Y.Z., Chen, X.W. (2007). On the normal perforation of thick metallic plates by a sharp nose rigid projectile. In: Proc. of the 23rd Int. Symp.on Ballistics (16-20 April, 2007, Tarragona, Spain), vol. 2, pp. 1173-1180.

[74] Chen, X.W., Li, X.L., Deng, K.W. (2008). Damping function in the penetration/perforation dynamics of rigid projectiles. In: Jones, N. and Brebbia, C.A., eds. Structures Under Shock and Impact X. WIT Press, Southampton, pp. 263-272.

[75] Chen, X.W., Li, X.L., Huang, F.L., Wu, H.J., Chen, Y.Z. (2008). Damping function in the penetration/perforation struck by rigid projectiles. Int. J. of Impact Engineering, 35(11): 1314-1325.

[76] Recht, R.F., Ipson, T.W. (1963). Ballistic perforation dynamics. J. of Applied Mechanics, 30(3): 384-390.

[77] Zaid, A.I.O., El Kalai, A., Travis, F.W. (1973). An examination of the perforation of mild steel plate by a flat ended cylindrical projectile. Int. J. of Mechanical Science, 15(2): 129-143.

[78] Bethe, H.A. (1941). Attempt of a theory of armor penetration. Report. Ordnance Laboratory, Frankford Arsenal, Philadelphia, PA.

[79] Hill, R. (1950). A theory of the plastic bulging of a metal diaphragm by lateral pressure. Philosophical Magazine. Series 7, 41(322): 1133-1142.

[80] Taylor, G.I. (1948). The formation and enlargement of a circular hole in a thin plastic sheet. Quarterly J. of Mechanics and Applied Mathematics, 1(1): 103-

124.

[81] Thomson, W. T. (1955). An approximate theory of armor penetration. J. of Applied Physics, 26(1): 80-82.

[82] Sodha, M.S., Jain, V.K. (1958). On physics of armor penetration. J. of Applied Physics, 29(12): 1769-1770.

[83] Nixdorff, K. (1987). On the efficiency of different head shapes to perforate thin targets. Transactions of the Canadian Society for Mechanical Engineering, 11(2): 109-112.

[84] Gupta, N.K., Ansari, R., Gupta, S.K. (2001). Normal impact of ogive nosed projectiles on thin plates. Int. J. of Impact Engineering, 25(7): 641-660.

[85] Calder, C.A., Goldsmith, W. (1971). Plastic deformation and perforation of thin plates resulting from projectile impact. Int. J. of Solids and Structures, 7(7): 863-881.

[86] Levy, N., Goldsmith, W. (1984). Normal impact and perforation of thin plates by hemispherically tipped projectiles - 1. Int. J. of Impact Engnineerng, 2(3): 209-229.

[87] Zhang, T.G., Mu, J.C. (2000). Discing around perforations of thin plates and tubes by hemispherical-nosed missiles or spheres. Key Engineering Materials, 177-180: 75-80.

[88] Pol, M.H., Bidi, A., Hoseini, A.V., Liaghat, G.H. (2009). Analysis of normal penetration of ogive-nose projectiles into thin metallic plates. World Academy of Science, Engineering and Technology, 38: 239-242.

[89] Zaid, M., Paul, B. (1957). Mechanics of high speed projectile perforation. J. of the Franklin Institute, 264(2): 117-126.

[90] Paul, B., Zaid, M. (1958). Normal perforation of a thin plate by truncated projectiles. J. of the Franklin Institute, 265(4): 317-335.

[91] Brown, A. (1964). A quasi-dynamic theory of containment. Int. J. of Mechanical Science, 6(4): 257-262.

[92] Woodward, R.L. (1978). The penetration of metal targets by conical projectiles. Int. J. of Mechanical Science, 20(6): 349-359.

[93] Woodward, R.L. (1978). The penetration of metal targets which fail by adiabatic shear plugging. Int. J. of Mechanical Science, 20(9): 599-607.

[94] Forrestal, M.J., Rosenberg, Z., Luk, V.K., Bless, S.J. (1987). Perforation of aluminum plates with conical nosed rods. J. of Applied Mechanics, 54(1): 230-232.

[95] Rosenberg, Z., Forrestal, M.J. (1988). Perforation of aluminum plates with conical nosed rods additional data and discussion. J. of Applied Mechanics, 55(1): 236-238.

[96] Landkof, B., Goldsmith, W. (1993). Petalling of thin, metallic plates during penetration by cylindro-conical projectiles. Int. J. of Solids and Structures, 21(3): 245-266.

[97] Wierzbicki, T. (1999). Petalling of plates under explosive and impact loading. Int. J. of Impact Engineering, 22(9-10): 935-954.

[98] Masri, R. (2010). Estimation of deep penetration resistance with spherical specific cavitation energy. Int. J. of Protective Structures, 1(3): 391-408.

[99] Durban, D., Fleck, N.A. (1992). Singular plastic fields in steady penetration of a rigid cone. J. of Applied Mechanics, 59(4): 706-710.

[100] Srivathsa, B., Ramakrishnan, N. (1997). On the ballistic performance of metallic materials. Bulletin of Material Science, 20(1): 111-123.

[101] Srivathsa, B., Ramakrishnan, N. (1998). A ballistic performance index for thick metallic armour. Computer Simulation Modelling in Engineering, 3(1): 33-39.

[102] Srivathsa, B., Ramakrishnan, N. (1999). Ballistic performance maps for thick metallic armour. J. of Materials Processing Technology, 96(1-3): 81-91.

[103] Giere, A.C. (1964). Some energy and momentum considerations in the perforation of plates. AIAA J., 2(8): 1471-1472.

[104] Chen, X.W., Li, Q.M. (2003). Perforation of a thick plate by rigid projectiles, Int. J. of Impact Engineering, 28(7): 743-759.

[105] Chen, X.W., Li, Q.M. (2011). Comparative analysis of perforation models of metallic plates by rigid sharp-nosed projectiles. Int. J. of Impact Engineering, 38(7): 613-621.

[106] Forrestal, M.J., Warren, T.L. (2009). Perforation equations for conical and ogival nose rigid projectiles into aluminum target plates. Int. J. of Impact Engineering, 36(2): 220-225.

[107] Forrestal, M.J., Børvik, T., Warren, T.L. (2010). Perforation of 7075-T651 aluminum armor plates with 7.62 mm APM2 bullets. Experimental Mechanics, 50(8): 1245-1251.

[108] Børvik, T., Forrestal, M.J., Warren, T.L. (2010). Perforation of 5083-H116 aluminum armor plates with ogive-nose rods and 7.62 mm APM2 bullets. Experimental Mechanics, 50(7): 969-978.

[109] Chen, X.W., Li, Q.M. (2002). Deep penetration of a non deformable projectile with different geometrical characteristics. Int. J. of Impact Engineering, 27(6): 619-637.

[110] Awerbuch, J. (1970). A mechanical approach to projectile penetration. Israel J. of Technology, 8(4):

375-383.
[111] Awerbuch, J., Bodner, S.R. (1974). Analysis of the mechanics of perforation of projectiles in metallic plates. Int. J. of Solids and Structures, 10(6): 671-684.
[112] Chen, X.W., Li, Q.M. (2003). Shear plugging and perforation of ductile circular plates struck by a blunt projectile. Int. J. of Impact Engineering, 28(5): 513-536.
[113] Goldsmith, W., Finnegan, S.A. (1971). Penetration and perforation processes in metal targets at and above ballistic velocities. Int. J. of Mechanical Science, 13(10): 843-866.
[114] Hsueh, D.-W. (1988). Normal perforation of a thin infinite plate by a flat-headed cylindrical projectile. Applied Mathematics and Mechanics, 9(12): 1113-1119.
[115] Hsueh, D.-W. (1989). Normal perforation of a thin infinite plate by a flat-headed cylindrical projectile. Int. J. of Impact Engineering, 8(2): 133-139.
[116] Jenq, S.T., Goldsmith, W.,. Kelly, J.M. (1988). Effect of target bending in normal impact of a flat-ended cylindrical projectile near the ballistic limit. Int. J. of Solids and Structures, 24(12): 1243-1266.
[117] Krishna Teja Palleti, H.N., Gurusamy, S., Kumar, S., Soni, R., John, B., Vaidya, R., Bhoge, A., Naik, N.K. (2012). Ballistic impact performance of metallic targets. Materials and Design, 39: 253-263.
[118] Liss, J., Goldsmith, W., Kelly, J.M. (1983). A phenomenological penetration model of plates. Int. J. of Impact Engineering, 1(4): 321-341.
[119] Ning, J., Song, W., Wang, J. (2005). Study of the perforation of stiffened plates by rigid projectiles. Acta Mechanica Sinica. 21(6): 582-591.
[120] Shoukry, M., Nair, S., Kalpakjian, S. (1991). Effect of shear deformation on the dynamic plastic bending of metallic plates during normal penetration. Int. J. of Engineering Science, 29(9): 1035-1052.
[121] Ravid, M., Bodner, S.R. (1983). Dynamic perforation of viscoplastic plates by rigid projectiles. Int. J. of Engineering Science, 21(6): 577-591.
[122] Ravid, M., Bodner, S.R. (1994). Penetration into thick targets - refinement of a 2D dynamic plasticy approach. Int. J. of Impact Engineering, 15(4): 491-499.
[123] Teng, X., Wierzbicki, T. (2005). Dynamic shear plugging of beams and plates with an advancing crack. Int. J. of Impact Engineering, 31(6): 667-698.
[124] Wu, Y., Batra, R.C. (1996). An engineering penetration/perforation model of hemispherical nosed rigid cylindrical rods into strain-hardening targets. Computers and Structures, 58(1): 51-58.
[125] Holt, W.H., Mock, W., Jr., Soper, W.G., Coffey, C.S., Ramachandran, V., Armstrong, R.W. (1993). Reverse ballistic impact study of shear plug formation and displacement in Ti6Al4V alloy. J. of Applied Physics, 73(8): 3753-3759.
[126] Sagomonyan, A.Ya. (1977). On problem of barrier perforation by cylinder. Vestnik Moskovskogo Universiteta. Matematika Mekhanika, 5: 111-118 (in Russian)
[127] Sagomonyan, A.Ya., Gendugov, V.M., Sagomonyan, E.A. (1982). On the problem of perforation of a barrier. Moscow University Mechanics Bulletin, 37(1-2): 14-18.
[128] Ben-Dor, G., Dubinsky, A., Elperin, T. (2000). Optimization of the shape of a penetrator taking into account plug formation. Int. J. of Fracture, 106(3): L29-L34.
[129] Ben-Dor, G., Dubinsky, A., Elperin, T. (2007). Shape optimization of impactors against a finite width shield using a modified method of local variations. Mechanics Based Design of Structures and Machines, 35(2): 113-125.
[130] Sagomonyan, A.Ya. (1974). Penetration: Penetration of Solids into Compressible Continuous Media. Moscow State University Press, Moscow (In Russian).
[131] Nishiwaki, J. (1951). Resistance to the penetration of a bullet through an aluminium plate. J. of Physics Society of Japan, 6(5): 374-378.
[132] Vitman, F.F., Stepanov, V.A. (1959). Effect of strain rate on resistance of metals to deformation at impact velocities of 100-1000 m/s. In: Nekotoryje Problemy Prochnosti Tvjordogo Tela. USSR Academy of Science, Moscow-Leningrad, pp. 207-221 (in Russian). [English translation: Vitman, F.F., Stepanov, V.A. Influence of Strain Rate on Resistance to Deformation of Metals at Impact Velocities of 100 to 1000 m/sec. Foreign Technology Division Wright-Patterson AFB, Ohio, 1964 (AD0605234)].
[133] Hayes, W.D., Probstein, R.F. (1959). Hypersonic Flow Theory. Academic Press. New York & London.
[134] Vitman, F.F., Ioffe, B.S. (1948). A simple method of determining dynamic hardness of metals using a double cone. Zavodskaja Laboratorija 14(6): 727-732 (in Russian).
[135] Chernyi, G.G. (1969). Introduction to Hypersonic Flow. Academic Press, New York.
[136] Golubev, V.K., Medvedkin, V.A. (2000). Penetration of a rigid shell into a steel obstacle at moderate impact velocities. J. of Applied Mechanics and Technical Physics, 41(1): 32-34.
[137] Golubev, V.K., Medvedkin, V.A. (2001). Penetration

of a rigid rod into a thick steel plate at elevated velocities. Strength of Materials, 33(4): 400-405.
[138] Landgrov, I.F., Sarkisyan, O.A. (1984). Piercing plastic material barriers with a rigid punch. J. of Applied Mechanics and Technical Physics, 25(5): 771-773.
[139] Yarin, A.L., Rubin, M.B., Roisman, I.V. (1995). Penetration of a rigid projectile into an elastic-plastic target of finite thickness. Int. J. of Impact Engineering, 16(5/6): 801-831.
[140] Wen, H.M. (2002). Predicting the penetration and perforation of targets struck by projectiles at normal incidence. Mechanics of Structures and Machines, 30(4): 543-577.
[141] Wen, H.M. (2002). Penetration and perforation of thick metallic targets under impact by missiles. Chinese J. of High Pressure Physics, 16(2): 94-104 (in Chinese).
[142] Shaw, A. (2012). Penetration of rigid objects into semi-infinite compressible solids. Mechanics of Materials, 50: 22-35.
[143] Bagdoev, A.G., Vantsyan, A.A., Khachatryan, B.K., Khachatryan, L.A. (1997). Investigation of dynamic problem of penetration of rigid indentor into anisotropic medium on the basis of hypothesis of normal cross sections. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 50(1): 44-52 (in Russian).
[144] Partom, Y. (1996). Static cavity expansion model for partially confined targets. Report IAT.R 0092. The University of Texas at Austin, Institute for Advanced Technology, Austin, TX.
[145] Littlefield, D.L., Anderson, C.E., Jr., Partom, Y., Bless, S.J. (1997). The penetration of steel targets finite in radial extent. Int. J. of Impact Engineering, 19(1): 49-62.
[146] Vantsyan, A.A. (1981). Determining depth of penetration of slender body into metals. Doklady Akademii Nauk Armjanskoj SSR, 72(2): 95-101 (in Russian).
[147] Bagdoev, A.G., Vantsyan, A.A. (1981). Penetration of slender bodies into elastic media. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 34(1): 3-14 (in Russian).
[148] Bagdoev, A.G., Vantsyan, A.A. (1981). Penetration of slender body into metals and soils. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 34(3): 25-38 (in Russian).
[149] Rahmatulin, K.A., Bagdoev, A.G. (1982). The penetration of thin rigid body in initially elastic media. Int. J. of Engineering Science, 20(1): 101-111.
[150] Bagdoev, A.G., Vantsyan, A.A. (1983). Penetration of slender body into elastic anisotropic media. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 36(6): 23-30 (in Russian).
[151] Bagdoev, A.G., Vantsyan, A.A. (1987). Investigation of penetration of slender rigid body into transversely isotropic medium. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 40(4): 3-6 (in Russian).
[152] Bagdoev, A.G., Vantsyan, A.A., Grigoryan, M.S. (1988). Influence of anisotropic properties of metal laminated specimen on penetration. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 41(6): 28-34 (in Russian).
[153] Bagdoev, A.G., Vantsyan, A.A., Grigoryan, M.S. (1989). Investigation of singularity of stresses in anisotropic plastic medium for cone penetration. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 42(4): 52-57 (in Russian).
[154] Asatryan, V.L., Bagdoev, A.G., Vantsyan, A.A. (1996). Solution of a dynamic problem of penetration of rigid cone into initially elastic transversely isotropic medium. Izvestija Akademii Nauk Armjanskoj SSR. Mekhanika. 49(4): 77-85 (in Russian).
[155] Bagdoev, A.G., Vantsyan, A.A. (1989). Penetration of a thin body into a transversely isotropic medium with twisting. Mechanics of solids, 24(2): 182-184.
[156] Huang, G.-Y., Wu, G., Feng, S.-S., Zhou, T. (2011). Theory and experiment study of normal penetration of slender nose projectile into steel target. Applied Mechanics and Materials, 80-81: 137-142.
[157] Huang, G.-Y., Feng, S.-S., Guang, W., Li, S.-P. (2011). Performance of different projectile nose shapes in normal penetrating armor targets. Advanced Materials Research, 308-310: 1420-1425.
[158] Dienes, J.K., Miles, J.W. (1977). A membrane model for the response of thin plates to ballistic impact. J. of the Mechanics and Physics of Solids, 25(4): 237-256.
[159] Tirosh, J., Tylis, A., Davidi, G. (2009). Foreign object damage: Penetration of rigid projectiles into solids. Mechanics of Materials, 41(5): 535-544.
[160] Teland, J.A., Moxnes, J.F. (2003). Analytical cavity expansion penetration models compared with numerical simulations. Report FFI/RAPPORT 2003/00934. Norwegian Defence Research Establishment (FFI), Kjeller, Norway.
[161] Wijk, G., Hartmann, M., Tyrberg, A. (2005). A model for rigid projectile penetration and perforation of hard steel and metallic targets. Report FOI-R-1617-SE. Swedish Defence Reasearch Agency (FOI), Tumba, Sweden.
[162] Wijk, G. (2005). Initially increasing penetration resistance. Friction and target size effects in connection with rigid projectile penetration and perforation of steel and metallic targets. Report FOI-

R-1631-SE. Swedish Defence Reasearch Agency (FOI), Tumba, Sweden.