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Recent trends in metaheuristics for stochastic combinatorial optimization

Review Article

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Abstract: This short overview is an addendum to a recent literature survey by Bianchi et al. on metaheuristics for stochastic combinatorial optimization (SCO). It outlines some new developments that occurred in this field during the last few years. Special attention is given to multi-objective SCO as well as to combinations of metaheuristics with mathematical programming.

Keywords: metaheuristics • combinatorial optimization • stochastic optimization • optimization under uncertainty • multi-objective optimization

1. Introduction

This short overview is to be considered as an addendum to a recent survey by Bianchi et al. [1] on metaheuristics for stochastic combinatorial optimization (SCO). The present overview outlines some new developments that occurred in the last few years and were not yet addressed in [1]. Except of a few elementary remarks in Section 2, the basic notions, classifications and ideas of SCO are not explained here; the interested reader is invited to consult [1] (or the survey [2] on the related area of evolutionary optimization in uncertain environments) for information on these fundamentals. It is also assumed that the reader is familiar with the essential ideas of the most important metaheuristic techniques (for an introduction, see, e.g., [3]).
Moreover, the present article does not aim at a comprehensive picture, but rather focuses on some special features. In particular, we concentrate on *static* SCO problems. New approaches in dynamic stochastic programming or Markov Decision Processes will only be touched marginally. Special emphasis, however, will be put on two subfields of SCO that seem to be in rapid evolution at the moment, namely the use of *matheuristic* approaches (combinations of metaheuristics with mathematical programming), and the investigation of solution techniques for *multi-objective* SCO problems. Neither of these two topics has been covered in [1] or [2], although [2] mentions the second one as a topic for future research.
The paper is organized as follows: Section 2 outlines the notion of SCO. In Section 3, new approaches in metheuristics applied to single-objective SCO problems are discussed, whereas Section 4 is devoted to multi-objective problem versions. Section 5 contains concluding remarks.

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2. Stochastic combinatorial optimization

Common to all SCO problems (SCOPs) is the property that the decision is an element of a finite search space S. Depending on where the stochastic influence comes into play, however, the structure of a SCOP can vary. The most common SCOPs are those where the constraints on the decision x are deterministic and the objective function $F(x)$ is an expected value of a random variable. Thus, in the case of a minimization problem, the expected value of a cost function $f(x, \omega)$ depending on decision x and on some stochastic influence ω has to be minimized. In the case of a maximization problem, $f(x, \omega)$ is a utility function.

Another type of SCOPs are *chance-constrained* problems where the random influence enters through the *constraints* into the problem formulation, usually by restricting the expected value of some random variable to some given range, and thus also restricting the set of feasible solutions. Note that a probability can also be expressed as an expected value. A typical example for a chance-constrained problem is a vehicle routing problem under uncertain demand where delivery cost is to be minimized on the constraint that the probability of being unable to serve a customer does not exceed a certain threshold.

The objective function $F(x)$ can also be some operator of $f(x, \omega)$ different from the expected value. E.g., $F(x)$ can be the *variance* of the return from a decision x, measuring the risk connected with x. The constraints ensure then a given lower threshold for the expected return.

Often, SCOPs are complicated by a time structure of decisions and by information that becomes available only step-by-step. The simplest case for such a multi-stage SCOP is a two-stage stochastic program with recourse, where the objective function (expected cost) is composed of a fixed cost term for the first-stage decision and a term reflecting the expected cost of the so-called recourse action (second-stage decision).

When solving a SCOP, a crucial question is how to determine (or at least estimate) the objective function value (e.g., expected cost) assigned to a certain decision x. Basically, there are three ways to do that: (i) In the simplest case, an *analytical* expression for the expected value (or the considered more complex operator) is known. Then, by using this expression, the SCOP can be reduced to a deterministic CO problem. (Surprisingly, also when feasible, this needs not to be the most efficient way to solve the SCOP, as will be discussed for the case of the probabilistic TSP in Section 3.2) (ii) Sometimes, it is not possible to find a closed-form expression for the operator under consideration, but to compute it *numerically*. Also in this case, the SCOP can be reduced to a deterministic CO problem, but the computational effort for computing the objective function values for a large number of considered candidate solutions can be prohibitive, such that standard optimization techniques may fail and specific adaptations may be required. (iii) Very often, neither an analytical nor a numerical way to compute the operator under consideration is available. Then, one has to resort to an estimation of its value by means of *sampling*. E.g., for estimating expected cost, one can draw a certain number of random scenarios and take the average cost over the scenarios as an estimate. Fixed-sample and variable-sample techniques can be distinguished: When working with a fixed sample, the scenarios are drawn in advance, and the set of scenarios is not changed during the (heuristic) optimization procedure. Alternatively, in an iterative heuristic procedure, one may also work with a variable sample which is changed in each iteration, which has the advantage that a larger number of random scenarios can be taken into account during the optimization process.

Sampling is usually done by the *Monte Carlo* technique, i.e., a stochastic simulation run is performed. Therefore, the use of the sampling approach within an optimization procedure amounts to what is often called *Simulation Optimization* in the literature. Also sophisticated stochastic simulation methods from the areas of discrete event simulation or agent-based simulation can be applied in this context.

3. Single-objective SCO problems

3.1. Local-search-based techniques

Birattari et al. [4] tackle a special SCOP by a local search procedure based on empirical estimation of objective function values, i.e., they follow the sampling approach described in Section 2 to estimate the objective function (defined as expected cost). Within this context, they investigate the usefulness of *delta evaluations* of the objective function: contrary to a straightforward approach where for comparing the quality of two neighbor solutions, the entire solutions are evaluated based on the sample, only the part where the two solutions differ is taken into consideration. The delta evaluation method has been applied before in local search heuristics with respect to analytical function determination;

the authors show how, in the stochastic context, it can also be used for the sampling approach. It turns out that in the concrete case of the Probabilistic Traveling Salesman Problem (PTSP), a TSP version where each customer has a (known) probability of requiring service, delta evaluation results in considerable time savings, making the local search algorithm more efficient.

In [5], local search using the delta evaluation method is further improved by incorporating two additional tools from the Stochastic Programming literature, namely adaptive sample sizes and the importance sampling technique. Both have been investigated in a metaheuristic SCO context already before, but they turn out as especially useful in combination with the delta evaluations. Again, the PTSP is used for obtaining experimental evidence.

Iterated Local Search (ILS) repeats the local search process with different initial solutions. An extension of the approach in [4] to ILS is provided in [6].

A metaheuristic that differs from ILS by the ida of working with neighborhoods of different sizes is *Variable Neighborhood Search* (VNS); for a recent survey, see [7]. In [8], an extension of VNS for SCOPs called S-VNS has been proposed and applied to a project portfolio selection problem. Schilde et al. [9] report on the application of S-VNS to a problem in transportation logistics.

3.2. Swarm intelligence techniques

The currently most prominent techniques of Swarm Intelligence seem to be *Ant Colony Optimization* (ACO) and *Particle Swarm Optimization* (PSO); for recent surveys, see [10] and [11], respectively.

In [12], Balaprakash et al. extend the promising results from [4, 5] on the solution of the PTSP by delta evaluations, incorporating the delta evaluation procedures into an ACO framework. Diverse ACO variants in combination with local search are investigated. In the case of the PTSP, analytical formulas for the expected cost of a solution are known, such that empirical estimation would not be necessary. The results in [12], however, show that a sampling-based approach can lead to competitive or even better solutions within given computation time than the use of the analytical expressions. This is especially encouraging for the extension of the technique to problems where the expected cost cannot be computed analytically anymore.

As Balaprakash et al., also Marinakis and Marinaki [13] focus on the PTSP. They designed a (multi-swarm) PSO algorithm for the PTSP and improved it further by hybridizing it with Greedy Randomized Adaptive Search (GRASP) and a local search strategy called Expanding Neighborhood Search (ENS). Their experiments show that the hybrid algorithm outperforms a Tabu Search implementation, classic PSO, classic GRASP, and a combination of GRASP with ENS. Also a comparison with diverse ACO implementations for the PTSP is given. The authors emphasize that this last comparison is only qualitative, because computation times could not be kept equal, but they observed that the new PSO hybrid found new best solutions in a number of cases. Under computation time limits, however, more recent results in [6] seem to indicate a better performance of the ACO-based approaches.

Beyond the specialization to the PTSP, the ACO-based general-purpose SCO technique S-ACO [14] was recently applied in diverse contexts, see, e.g., Vitanov et al. [15] for an application in assembly line planning, and Brailsford et al. [16, 17], for applications in disease prevention. A combination of S-ACO with ideas from racing algorithms, called ACO/F-Race, has been proposed and investigated by Birattari et al. in [18].

3.3. Genetic and memetic algorithms

In the area of *genetic algorithm* (GA) approaches to SCOPs, the focus of research seems to have shifted to some extent to multi-objective problems (see Section 4). In the single-objective domain, recent publications mainly deal with interesting new applications. E.g., Escobar et al. [19] report on the application of a special GA to an electrical power network expansion problem under demand uncertainty and generation uncertainty. The article clearly shows the benefits of suitable metaheuristic SCO solution techniques in engineering.

For the PTSP, Balaprakash et al. [6] also implemented a *memetic* algorithm (i.e., a hybrid between GA and local search) and showed that at least for some classes of instances, the algorithm is competitive with ILS and ACO.

Some recent papers on SCO metaheuristics deal with a theoretical convergence analysis. General convergence results are available not only with respect to several metaheuristic algorithms for deterministic CO problems, but also with respect to some algorithm variants for SCOPs (cf. [20]). In the SCO area, convergence results concerning Simulated Annealing and ACO have already been obtained comparably early, whereas for GAs and related evolutionary algorithms, the extension of convergence guarantees to SCOPs has been achieved only more recently. Examples are Chang [21] who proves convergence of an elitist-based evolutionary computation algorithm for SCO, and Hannah and Powell [22] who show convergence of a sampling-based variant of the Evolutionary Policy Iteration method. Although a convergence result without a concrete statement about convergence rate is only a first step in the theoretical analysis of a metaheuristic, it certainly increases confidence in the application of the considered algorithm: A user will not put too much trust into an algorithm which it cannot be excluded that it will never reach the optimal solution, no matter how much runtime is invested. In a deterministic optimization context, convergence of the best-so-far solution to the optimum can usually already be obtained by incorporating a sufficient amount of random exploration into the algorithm. In the stochastic context, however, the presence of noise makes it typically impossible to identify the best-so-far solution with certainty, such that convergence must be ensured by specific (and usually highly nontrivial) arguments.

3.4. Simulated annealing

The adaptation of the well-known Simulated Annealing (SA) algorithm to stochastic problems has a long tradition. Some more recent techniques using SA are presented in Prudius and Andradottir [23] and Prudius [24]. Branke et al. [25] give a thorough survey on approaches in this area, and they propose two new algorithmic variants of SA for problems with noisy objective functions. The first of them improves the *stochastic annealing* procedure originally introduced by Fink [26]. The second one, called SANE (Simulated Annealing for Noisy Environments), combines features from stochastic annealing with some other ideas, including statistical sampling approaches. SANE distinguishes between the "high temperature" and the "low temperature" phase of the SA algorithm and applies different acceptance criteria for candidate neighbor solutions depending on the temperature. In the low temperature regime, sequential statistical sampling is applied to reach a certain confidence about the correct selection. In an empirical evaluation based on a noisy TSP, the SANE algorithm outperforms the other investigated techniques.

3.5. Matheuristic approaches

The term *matheuristics* stands for a combination of mathematical programming with metaheuristics[1], a young but recently very active area of research (cf. [27]). In the SCO domain, some articles have started to use matheuristic approaches. Let us outline examples.

Till et al. [28] and Tometzki and Engell [29] investigate a batch scheduling problem under uncertain demands and uncertain capacities in the chemical industry. They formulate it as a two-stage stochastic integer program with recourse and propose a solution technique that uses an evolutionary algorithm for the first-stage decision and an exact mixed-integer programming (MIP) solver for the recourse decision. With respect to the specifically chosen evolutionary algorithm, [29] compares a GA with an integer evolution strategy and obtains better results for the latter.

Hvattum and Lokketangen [30] deal with a Stochastic Inventory Routing Problem (SIRP). The influence of uncertainty is represented by a scenario tree. For solving the problem, the authors use the Progressive Hedging Algorithm (PHA) from Stochastic Programming which iteratively "reconciles" partial solutions for special scenarios, making the overall policy implementable in this way, and extend PHA by a GRASP metaheuristic applied for solving subproblems.

Also Crainic et al. [31] adopt a PHA technique, in their case for the solution of a two-stage multicommodity network design problem with stochastic demands. Here (among other alternatives), a *Tabu Search* algorithm is applied for a heuristic solution of subproblems.

Rei et al. [32] address the (two-stage) vehicle routing problem with stochastic demands. The combine Monte-Carlo sampling with the *Local Branching* technique by Fischetti and Lodi [33], a metaheuristic frequently used in the matheuristics literature for solving hard combinatorial or mixed-integer problems. Local Branching searches for the best solution within a certain neighborhood (usually defined by a Hamming distance) of a current feasible solution by means of an

[1] *The notion "matheuristics" refers both to metaheuristic techniques used to speed up the efficiency of an exact algorithm, and to exact techniques incorporated to improve the solution quality of a metaheuristic approach. Overall, the second type continues to be of heuristic nature insofar as when terminating, the algorithm does not come up with* proven *optimality, even in cases where it can be shown that given unlimited runtime, the algorithm will converge to an exact optimum.*

exact MIP solver, continues the search from the new solution, and iterates until a sufficiently good solution has been obtained. Rei et al. show that compared to two alternative techniques, their approach yields solutions of comparable quality within shorter computation time.

Hannah et al. [34] investigate a discrete R & D portfolio optimization problem under uncertainty in energy production and develop a solution algorithm called Stochastic Gradient-Based Portfolio Optimization (SGPO). The algorithm builds on ideas from simultaneous perturbation stochastic approximation and uses a linear approximation of the objective function in order to obtain a series of knapsack problems, which can be tackled by an integer programming solver. For special cases, theoretical convergence to the optimal solution is shown. Comparisons with a SA variant for SCOPs and with the Evolutionary Policy Iteration (EPI) method are reported. Although SGPO is tailored to the problem under consideration, it seems to be generalizable to a much larger class of SCOPs.

Let us mention that for the solution of SCOPs, hybrids of the (exact) branch-and-bound algorithm with heuristic features have already been investigated before; an example is the Stochastic Branch-and-Bound algorithm by Norkin et al. [35, 36]. In [1], these techniques have been subsumed under the more general category "stochastic partitioning methods". Some of them are "matheuristic" in nature and resemble the approaches outlined above.

4. Multi-objective SCO problems

In many areas where decision under uncertainty have to be made, different objectives that cannot be reduced to each other occur. E.g., expected return can be one objective, but a second, possibly conflicting objective measuring risk has also to be taken into account. Of course, a multi-objective consideration of decision problems plays an especially important role in fields where except of economic goals, also other objectives are relevant, such as in medical decision making, in public health, in socioeconomic planning, in environmental policies or in education. Formally, such decision problems can often be represented as multi-objective stochastic optimization problems, and many of them are of combinatorial type.

There are several solution concepts for multi-objective optimization models, such as lexicographic optimization, multi-attribute utility theory (MAUT) and others. We consider here the perhaps most frequently applied solution concept, that of determining *Pareto-optimal* (or: *efficient*) solutions. Whereas other concepts require the decision maker to specify her/his preferences (in some more or less explicit form) in advance, the computation of the set of Pareto-optimal solutions (short: Pareto set) allows her/him to obtain information on the tradeoff between the different objectives *before* being forced to weigh them against each other. Instead of delivering a single "optimal" solution, the system provides the decision maker with a set of reasonable alternatives, among which a final choice can then be made. By a Pareto-optimal solution, one understands a solution that is not *dominated* by any other solution in the search space, where a solution y dominates a solution x if y is at least as good as x in all objectives and better than x in at least one objective.

In Caballero et al. [37], stochastic multi-objective *continuous* optimization problems are investigated. This is different from the SCO situation considered here, but the paper [37] is also relevant in the current context as it outlines the two essential alternative paths to tackle a stochastic multi-objective optimization problem: (i) *Multi-objective approach:* The problem is first reduced to a *deterministic multi-objective* problem which is then solved by techniques of multi-objective optimization. (ii) *Stochastic approach:* The problem is first reduced to a *single-objective stochastic* problem which is then solved by techniques of stochastic optimization. The same two paths can also be followed when solving a multi-objective SCOP.

For the *multi-objective* approach, the simplest way is to draw a fixed sample of random scenarios in order to reduce the stochastic problem to a deterministic (still multi-objective) problem, and then to apply a metaheuristic technique to obtain the set of Pareto-optimal solutions with respect to the average objective function values over the sample. E.g., Claro and de Sousa [38, 39] use a multi-objective hybrid between Tabu Search and Variable Neighborhood Search for the latter purpose.

As in single-objective SCO, however, relying on a fixed sample has the disadvantage that the probability model specified by the drawn sample scenarios is only an approximation to the original probability model, such that even if the applied multi-objective metaheuristic should provide the exact Pareto set of the deterministic problem, it needs not to be the Pareto set of the original problem. Therefore, several articles on multi-objective SCO work with variable samples or with the execution of simulation modules as subprocedures.

Most frequently, techniques for multi-objective SCO in this class adopt modifications of evolutionary algorithms. Let us give some examples. Hughes [40] modifies a multi-objective GA by incorporating sample-based ranking functions. Teich [41] adapts the multi-objective SPEA algorithm by adding a probabilistic dominance concept. A similar technique relying on the well-known NSGA-II algorithm is proposed in Eskandari et al. [42]. Also Ding et al. [43] use a multi-objective GA, combined with a simulation module, when searching for solutions to a multi-objective enterprize networks model. For a supply chain inventory application, Amodeo et al. [44] compare combinations of a discrete-event simulation subprocedure with the multi-objective genetic algorithms SPEA-II and NSGA-II and with a multi-objective PSO procedure. Eskandari and Geiger [45] extend the approach in [40] by an improved statistical differentiation among competing candidate solutions and provide a comprehensive computational study on a benchmark of test problems. For an aircraft manufacturing application, Syberfeldt et al. [46] apply a special multi-objective EA supported by an artificial neural network and combine it with a simulation subroutine.

It is interesting that all these earlier works on metaheuristics for multiobjective SCOPs seem to be based on the paradigm of evolutionary algorithms. In [47], instead of GAs, multi-objective variants of ACO and of SA are extended to the stochastic case, but generally, local-search-based approaches or approaches from swarm intelligence are still not very well studied in this subfield.

Whereas most articles assess the achieved solution quality by experiments, but do no provide theoretical convergence guarantees, [48] introduces an iterative framework procedure called Adaptive Pareto Sampling (APS) for which convergence of the currently proposed solution set to the exact Pareto set can be proven under rather mild conditions. In [49], an application to project portfolio management is given. APS works by exchanging random samples from iteration to iteration and relies on an exact or metaheuristic subprocedure (to be plugged in) for the solution of corresponding deterministic multi-objective problems. Results on the speed of convergence of APS have recently been derived in [50].

The *stochastic approach* has been used in multi-objective SCO mainly by applying the quality indicator concept, which is an elegant way to reduce the given multi-objective problem to a single-objective one: see Basseur and Zitzler [51] or Liefhooge et al. [52, 53].

5. Conclusions

During the last years, the field of metaheuristics for combinatorial optimization under uncertainty has made considerable progress, partially by the introduction of novel techniques such as hybridizations of heuristic with exact methods, but also by extending the scope of addressed problems from simple to more complex (e.g., multi-objective or dynamic) optimization models. Several challenges of modern society in areas as finance, energy, environment, health or socioeconomic development require the application of computer-supported decision technologies that can cope with inherent uncertainty on parameters. Very frequently, moreover, the complexity of decision problems of this kind necessitates the use of heuristic solution techniques. This generates an "application pull" that will hopefully lead to further advances in methodological research. Future developments in the field may possibly concern (just to name a few aspects) new metaheuristics, multi-stage SCOPs, improved sampling techniques, parallel or distributed computing, and innovative applications.

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