

Scalable n-body event prediction

Research Article

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Received 28 November 2011 ; accepted 19 February 2012

Abstract: The general simulation of n-body systems often requires the simulation of pairwise interaction events between the objects. The naive method of simulating these events is an algorithm that polls each pair of objects for an interaction every time step. This algorithm has $O(n^2)$ operations per time step in the number of objects. However, this method scales very well to multiple cores. In this paper, we propose a novel method of pairwise simulation that saves a significant amount of computation time by predicting possible future outcomes rather than reacting to them. We demonstrate the implementation of this method, as well as demonstrate that it has amortized $O(n)$ complexity per time step. We also demonstrate an implementation to allow this algorithm to scale comparably well to multiple cores of a shared memory machine.

Keywords: n-body systems • non-blocking synchronization • discrete event simulation • prediction

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1. Introduction

There is a great deal of interest in discrete event simulation as it applies to video games and real time simulation. One interest in particular is the application of realistic physics in these simulations. This kind of discrete event simulation is traditionally performed on large scale parallel clusters for film and scientific applications, or on massively parallel GPU architectures [3, 7]. Specifically, we wish to explore the simulation and evolution of systems of dynamic and static objects where all the dynamic objects interact with all of the other objects, as happens when the dynamic objects physically collide with other objects. In order to do this, we need to be able to model and react to collisions and interactions between objects.

Although the naive and semi-naive implementations of this kind of simulation are embarrassingly parallel, they are also exceedingly wasteful. The method used to evaluate the interactions between objects is similar to a busy-wait loop,

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where all objects are continually polled, waiting for collisions to occur as the simulation progresses [4, 8]. As we will describe in the next sections, we are interested in using a novel predictive model along with a novel scalable n-body event queue data structure to attempt to make a new kind of economical collision simulation system. This system does not perform wasteful checks when checks are unlikely and maintains much of the parallelism of the naive solution.

2. Problem definition

As input, we are given a description of static objects, dynamic objects, and the geometry defining their boundaries. We are also given initial states like position, acceleration and velocity of the objects in the system, and a set of rules to update those objects. As output, we wish to compute the exact state of each object in the system for each of the $n + 1$ time steps $t_0, t_1, t_2, \dots, t_n$.

For each object o in the system, a subroutine is defined $\text{update}(o, dt)$, which computes the state of the object in the next time step, given the object state in the current time step and a time dt between time steps. For each pair of objects o_1 and o_2 , an additional subroutine is defined $\text{collide}(o_1, o_2)$ which computes the state of both o_1 and o_2 in the next time step, assuming that the objects have collided during the current time step.

2.1. Naive algorithm

The brute force all-pairs solution is the naive solution to this problem as it is commonly implemented in games. This technique is known as a 'detection' technique because it relies on the idea of detecting a collision when it occurs instead of predicting a collision.

Algorithm 1 Naive algorithm

```

for each time step  $t_i$  do
  for each dynamic object  $do$  do
    update( $do, dt$ ) { $dt$  is the constant time step difference}
    for each object  $o$  do
      if check( $do, o$ ) then
        collide( $do, o, dt$ )
      end if
    end for
  end for
end for
end for

```

This technique is embarrassingly parallel, allowing a thread per dynamic object or even a thread per pair with minimal overhead. However, it is extremely wasteful, because there are $O(n^2)$ checks being made every single time step, even if the likelihood of a collision has changed little since the last time step [9].

Each of these $O(n^2)$ checks per time step is computationally cheap individually, but when the number of objects is very great the computational overhead of performing all of them becomes unmanageable. Considering that an extreme majority of the checks will fail due to the fact that collision occurrences are rare, most of them do not need to be computed, and avoiding their computation has the ability to dramatically increase the throughput of our simulation.

2.2. Semi-naive algorithm

A similar algorithm that we refer to as the 'semi-naive' algorithm has also found great use in practical applications [1]. The semi-naive algorithm is the same as the naive algorithm, except that it exploits a bounding volume hierarchy to spatially partition the dynamic objects and pre-filter the collision checks. This algorithm is also wasteful in that most of its checks fail, but only has $O(n \log n)$ checks if the objects are relatively equally distributed throughout the bounding space.

3. Theoretical contribution

3.1. Predictive algorithm

In our desired solution, we take a completely different approach. In contrast to the naive and semi-naive algorithms, our solution avoids computing any unneeded checks on time steps that do not change the solution by *predicting* the intersections before they occur, rather than checking for them when they do not. Our algorithm will use a slightly more complex subroutine `predict(o1,o2)` that returns the time an intersection is likely to occur, and enters it into an event queue that contains the predicted collisions that are likely to occur. When an intersection occurs, it is recomputed and removed from the queue.

Algorithm 2 Predictive Algorithm

```

{initialize events from initial states}
for each time step ti do
  for each dynamic object do do
    update(do, dt) {dt is the constant time step difference}
  end for
  {for each event that occurs in this time step}
  for each e in events.find(ti) do
    for each object o do
      if check(e.o, o) then
        collide(e.o, o, dt) {predict next collision event into queue}
      end if
    end for
    events.remove(e)
  end for
end for
return events

```

It is worth noting that although our solution has the same runtime complexity as the naive solution in the worst case (and, in fact, the initial population of the event queue will need to perform $O(n^2)$ predictions), it is unlikely to compute ANY checks that are unlikely to occur, thus saving tremendous computation time in the average case.

However, our work is not done. This algorithm is difficult to parallelize when compared to the naive algorithm, as it depends implicitly on reading and writing the global future n-body event queue data structure described in Section 3.2. This data structure is a specialized queue in which events take priority over other events about to take place. If we split the evaluation of the simulation into threads, then each thread must somehow synchronize which events it will prioritize and avoid collisions. Furthermore, threads must be able to communicate and respond to future events globally as collisions are predicted into the future. This communication overhead is difficult to parallelize correctly, and will be the focus of our research.

3.2. N-body future event queue

Our algorithm depends on a data structure we call an "N-Body Future Event Queue". This data structure is designed to facilitate the prediction algorithm by storing pairwise events that are probabilistically likely (but not guaranteed) to occur. Such a data structure has the following pseudo-code description:

Data structure 3 Future Event Queue

```

{inserts an event into the queue, possibly superseding any events that happen later}
insert(event)
{removes an event from the queue}
remove(event)
{returns an iterator that iterates over all predicted events that occur after time t}
EventIterator find(time t)
{Returns all objects for which nothing is known about their behavior}
ObjectIterator find_invalid()

```

It is important to take special note here of the operations in the future event queue. The most important differences from it and a standard priority queue or event queue is that the `find()` method can search for all events that occur within a certain time-frame, and that events that get input into the queue are not necessarily persistent, as they may be superseded by events that are inserted later but are queued to occur before. An event A in the queue supersedes an event B iff event A and B have an object in common AND A is scheduled to occur BEFORE B.

Lastly, “find invalid” is important because it is assumed that at some point some event will occur for each object. (If that was not true for a particular object *o*, then simulating *o* is useless). Thus, any objects which are in the scene but not present in the future event queue would indicate an error in our simulation logic. In our example, even objects which will not collide with any other objects must eventually collide with the boundaries of the simulation volume. If no such collisions were known about an object, we must know of this immediately and attempt to re-predict that object’s future. In addition, our queue must implement these algorithms efficiently in order to preserve the desired average-case $O(n)$ that our prediction algorithm facilitates. In particular, all operations must be implemented in $O(n)$ time.

4. Scalable implementation

In this section, we propose a scalable implementation of the predictive algorithm from Section 3, including an implementation of the N-Body Future Event Queue data structure. Our implementation uses barriers and hardware atomic compare-and-swap primitives to guarantee thread-safety. Our scalable implementation works in two phases, which we will discuss here. First, it uses barriers and a hardware compare-and-swap boolean to synchronize the algorithm that utilizes the data structure, a process that is described in Section 4.2. Next, it uses an array with a thread-safe indexing scheme to implement the future event queue which is described in Section 4.4.

4.1. Barriers

A barrier is a construct that is important to many threaded simulations. It is a simple synchronization primitive that only has a single operation: `wait(n)`. A thread that enters `wait(n)` will wait at that point in the control flow of the program until the total number of threads in the `wait(n)` function is *n*. Then, all threads return simultaneously.

Lock-freedom is a *progress condition* and as defined by Herlihy *et al.* [5, 10], a concurrent object is *lock-free* if it guarantees that *some* process in the system makes progress in a *finite* number of steps. To meet the progress condition of lock-freedom, one should not use any form of a mutual exclusion mechanism [2, 5]. In most implementations of a barrier, a locked mutex is used to sleep all the threads, which is unlocked when the number of sleeping threads is equal to *n*. Thus, a barrier implementation does not meet the lock-free progress guarantee. Here we present an alternative optimized implementation of the barrier’s wait routine that is inspired by the implementation techniques of lock-free algorithms [2, 10] and uses a hardware atomic instruction instead. However, our barrier implementation does not meet the lock-free progress guarantee as the system’s progress is still subject to the decisions of the scheduler. `Wait()` can be implemented as the following pseudocode.

Procedure 4 Wait

```

barrier
int num_threads {the number of threads to collect}
wait()
{collect this thread}
fetch_and_sub(&num_threads,1)
{spin until all threads are collected}
while num_threads > 0 do
    noop()
end while
{uncollect this thread}
fetch_and_add(&num_threads, 1)

```

4.2. Scalable timestep sub-phases

In all threaded implementations of time step-based simulation, at least one barrier construct is necessary in order to prevent the system from processing more than one time step at a time. Without an implicit barrier at the beginning or end of the processing for each time step, worker threads could easily get out of sync on the time steps that they are working on, which would produce incorrect calculations for comparisons between threads. The current time step of the simulation must be constant across all threads, and so a barrier is used in order to synchronize that time step. This limitation is true for all threaded time step simulation, our application included.

Therefore, we can consider the timespan of a single time step as a single computational unit, the beginning and end of which all threads are operating in sync. We can ignore all processing that occurs outside of a single time step. For our implementation of the prediction algorithm described in 3, a single time step is broken down into several distinct ‘phases,’ with the overall structure as follows:

Procedure 5 Timestep

```
check_react_collisions();
repredict();
update_simulation();
time_step_sync_barrier.wait();
```

Note the implicit barrier protecting all threads from moving to the next frame before their peers. The implementation of each subroutine are given in Section 4.3.

The primary problem with thread safety in this formulation of the time step is the program errors that can occur in the status of the queue and the determinism of the simulation if multiple threads are in different phases simultaneously. For example, if thread A is updating object j , in the “update simulation stage”, and thread B is writing to object j and i in the “check react collisions” phase, then the exact results the simulation will calculate are non-deterministic based on processor scheduling. Due to the fact that there are 3 phases, and n objects, the total number of possible non-deterministic phase-overlap outcomes is $3 \times 3 \times n = 9n$. This is clearly not a thread-safe solution.

However, as can be seen from Section 4.3, each of the sub-phases can be parallelized somewhat trivially. Additionally, each sub-phase either reads and writes a single array for that phase without inter-thread communication, or reads and writes two arrays with inter-thread communication. Therefore, to allow us to parallelize each phase in isolation, we break up the time step into “sub-phases” using an additional barrier operation, implemented as in Section 4.1.

Procedure 6 Check_React_Collisions

```
{search event queue for events that are occurring now}
for  $e$  in  $events.find(current\_time)$  do
  {if the event happens}
  if check( $e.o1, e.o2$ ) then
    {modify both objects}
    collide( $e.o1, e.o2$ )
  end if
  {the event is passed, so remove it.}
   $events.remove(e)$ 
end for
```

This way, we can guarantee that each thread never has cross-phase non-determinism, reducing the number of potential non-deterministic concurrency cases to account for from $9n$ to $3n$. Furthermore, as we will demonstrate, the update phase and the re-predict phase are embarrassingly parallel and require no cross-thread communication or synchronization, reducing the synchronization concerns to only the implementation of check-react.

4.3. Scalable sub-phases

4.3.1. Update

The update phase is simple and embarrassingly parallel. It also demonstrates our overall technique to distribute work equally to multiple threads. In most of our sub-phases, we evenly distribute one object to perform operations on each

thread, evenly, one at a time by using a hash function that maps object ids to thread ids evenly. $thread(object_id) = object_id \% num_threads + thread_id$

```

for every object  $o$  assigned to  $current\_thread$  do
  update( $o$ )
end for

```

Because updating an object only modifies that particular object, this phase is embarrassingly data-parallel and needs no additional synchronization.

Based on our definition of the problem and our use cases, we assume that the update() procedure has no explicit dependence on any of the other objects in the system, only on some state internal to the object that allows us to move the simulation forward, such as internal velocity and position, and is $O(1)$. This means that the entire Update_Simulation procedure is $O(n)$.

4.3.2. Check-react

Procedure 7 Check_React_Collisions

```

{search event queue for events that are occurring now}
for  $e$  in  $events.find(current\_time)$  do
  {if the event happens and this thread is assigned to it}
  if  $e.o1$  is in  $current\_thread$  and  $check(e.o1, e.o2)$  then
    {modify both objects}
    collide( $e.o1, e.o2$ )
  end if
  {the event is passed, so remove it.}
   $events.remove(e)$ 
end for

```

This phase does the grunt work of the predictive algorithm. During this phase, pending events that were predicted to happen in the current time step are checked to see if they actually occur. This phase requires additional synchronization overhead for several reasons.

1. collide() reads and writes TWO objects, which means it could potentially modify objects that are assigned to other threads non-deterministically.
2. There could be two events in the queue which actually describe the same event but swap the order of the objects, which could mean that a collide() reaction to the event could be triggered twice in the same frame.
3. Multiple threads could check() an object while another object is in the middle of writing to it.

However, we can exploit the structure of our problem in order to guarantee that none of these cases occur. Due to the fact that for two symmetric events, we only want one, we can check to see if one or both of the objects in the set have already collided. We can do this by using a flag bit for each of the objects in the scene to check whether or not the objects have collided yet this frame. Then, we check that bit for the OTHER object in our comparison, to see if the collision has already occurred. Regardless of whether or not the bit is set, we wish to atomically update the status of that bit to true for our current object. The code becomes as follows.

The complexity analysis of this procedure is non-trivial. During Check_React_Collisions, the code in the loop is executed only if there is some collision that is predicted to occur during this time step. However, whether or not any collisions occur depends on a number of factors that cannot be effectively reasoned about mathematically. For the purposes of our analysis in the rest of the section, let k refer to the number of collisions that occur during the current time step.

To analyze this section, we make the reasonable assumption that collide, check and compare_and_exchange all have $O(1)$ complexity, so the complexity of the loop is $O(kR(n))$, where R is the complexity of $events.remove()$, which we will discuss later. In addition, we must consider the complexity F of the $events.find()$ method in the overall procedure, which we will also discuss later. Therefore the final complexity of this procedure is $O(F(n) + kR(n))$, which becomes $O(F(n) + nR(n))$ in the worst case.

Procedure 8 Check_React_Collisions

```

{search event queue for events that are occurring now}
for e in events.find(current_time) do
  {if object reacted yet is false, set reacted to true and do the event}
  if !compare_and_exchange(reacted_yet[e.o1], false, true) then
    {if the event hasn't happened and this thread is assigned to it}
    if e.o1 is in current_thread and check(e.o1, e.o2) then
      {modify both objects}
      collide(e.o1, e.o2)
    end if
    {the event is passed, so remove it.}
    events.remove(e)
  end if
end for

```

4.3.3. Re-predict

The re-prediction phase is slightly more complex, but it is also embarrassingly parallel and scalable, provided that the implementation of our future event queue is thread-safe and scalable as well. It detects simulation errors of the sort described in Section 3.2, and also re-predicts collisions that have been removed in check-react. After finding all of the invalid objects (objects with no known future) that have been recently invalidated by check-react as well as logic errors, it then searches for all known predictions that will occur in the future and puts them into the queue.

Procedure 9 Re-predict()

```

{search event queue for objects that the queue is unaware of}
for oinvalid in events.find_invalid(current_time) do
  {if invalid object is assigned to this thread}
  if oinvalid in current_thread then
    for all dynamic_objects do do
      events.insert(predict(oinvalid, do))
    end for
  end if
end for

```

This algorithm is thread-safe and scalable provided that the implementations of events.insert and events.find_invalid are thread-safe and scalable, because predict is a read-only method that does not write to non-thread assigned data, even though it reads the current state of all dynamic objects in the scene.

The complexity of this procedure also depends on an unknown factor, which is the number of invalid entries in the queue, which we will call k' . In most cases, $k = k'$, because objects are made invalid when removed from the queue in the check-react phase. The complexity of this procedure also depends on the complexity of the events.insert() procedure, which we label as $I(n)$. We assume that predict() can be performed in constant time. Therefore, this procedure runs in $O(F_i(n) + k'nI(n))$ time, where $F_i(n)$ is the complexity of find_invalid() as defined in Section 4.4.

In the case of an empty future queue, this phase is $O(n^2I(n))$, because it has to make predictions on all pairs of objects. However, in order for that to occur in the average case would require a majority of frames to have all objects collide with some other object simultaneously, a case which is exceedingly rare for the simulations we wish to run.

4.4. Future event queue

The future event queue stores N-body events, and is designed to be used in concert with the sub-phase algorithm as described in Section 4.3. Our implementation uses an array to store a series of one-element priority-queues: one for each object in the scene. This array shows the current best-guess prediction of events that are inbound on the object. The advantage of this approach is that our predictive queue does not have to deal with the contention of a standard queue implemented in a list, and it is also space efficient, as it is able to throw away events that supersede other events at insertion time.

4.4.1. insert

The implementation of insert is protected by the fact that an event that is inserted containing object i as the first object in the event will only be inserted from the thread that owns object i , as can be seen from Section 4.3.3. Thus, insert requires no synchronization, as it can be verified that no other thread will read or write to this same location during the react sub-phase.

It is important to note that this implementation can and does remove events before they are checked from the queue, and would not be suitable for any other kind of event queue. However, our application domain says that removing potential events that will not occur from the queue is a feature, not a bug. The repredict phase and the check-react phase work together to ensure the correctness of the algorithm despite the fact that these events can be removed. For example, a pair of objects O1 and O2 may predict a collision may occur at time step 60 in the future. However, an object O3 may predict a collision with O1 at time step 30. This collision may NOT occur due to the fact that something else might occur between now and time step 30, but it is still entered into the queue. Thus, the check-react phase must double-check all collisions in the queue for validity before reacting. However, upon reaction to a collision that event is marked as invalid in the queue. All invalid objects are then recomputed. In our example, the collision predicted at time step 30 removes the collision predicted at time step 60. However, if the first collision fails to occur, then during the repredict phase of time step 30, the collision between O1 and O2 at time step 60 would be re-discovered.

Procedure 10 Insert

```
FutureEventQueue::insert(event e)
if NOT(this.current_events[e.o1].valid) or e.time < this.current_events.time then
    this.current_events[e.o1] = e;
end if
```

This definition of insert runs in $O(1)$ time, as it only needs to access a particular element of the internal event array. Therefore, $I(n) = O(1)$.

4.4.2. remove

Remove is similar to insert, except it simply sets the event to invalid in the array queue.

Procedure 11 Remove

```
FutureEventQueue::remove(event e)
this.current_events[e.o1].invalid;
```

It runs in $O(1)$ time, for the same reason as "insert()", so $R(n) = O(1)$.

4.4.3. find_invalid

The implementation of find_invalid with our array-based queue is simply a linear search for any events that are marked as invalid in the array. However, because of our previous rule that the event array cannot be read or written by a thread not its owner in the same phase, we only return invalid objects from the objects in the current thread. Although this would seem to break the functionality of find_invalid, this behavior simply serves to reinforce the constraint given in 4.3.3 that threads can only re-predict objects they own. It also means that we never re-predict objects across thread ownership boundaries.

Procedure 12 Find_Invalid

```
ObjectIterator FutureEventQueue::find_invalid()
for o in dynamic_objects do
    if o in current_thread AND !this.current_events[o].valid then
        {return object as part of invalid set for this thread.}
        yield o;
    end if
end for
```

As it is implemented as a linear search through the array, Find_invalid() is in $O(n)$.

4.4.4. find

Similar to the implementation of `find_invalid`, the implementation of `find` also is implemented based on a linear search. It finds all valid events and also checks to make sure that those events are happening in the current frame.

Procedure 13 Find

```

EventIterator FutureEventQueue::find(current_time)
for o in dynamic_objects do
  if o in current_thread AND this.current_events[o].valid AND this.current_events[o].time j= current_time then
    { the event that is about to occur }
    yield this.current_events[o];
  end if
end for

```

As it is also implemented as a linear search through the array, `find()` is in $O(n)$.

4.5. Summary of predictive method

The sub-phase barriers and the thread object ownership system work together to ensure the thread-safety of the inputs and outputs to the Future Event Queue. This partnership ensures in all the covered cases that the same object data is never used as a concurrent shared resource for both reading and writing.

4.5.1. Complexity

By examining the results from each of these analyses, we can analyze the complexity of the overall time step by looking at the total complexity for each of the sub-phases described in Section 4.2. From Section 4.3.2 we see that the complexity of the check-react subphase is $O(F(n) + kR(n))$, which becomes $O(O(1) + kO(1)) = O(k)$ after substituting our definitions of F and R from Section 4.4. From Section 4.3.1 the complexity of the update subphase is $O(n)$. Lastly, the complexity of the repredict subphase 4.3.3 is $O(F_i(n) + k'nI(n))$, which also reduces after substitution to $O(O(1) + k'nO(1)) = O(k'n)$. Because the final complexity per time step is the complexity of all of the phases, the final time step complexity is $O(k + n + k'n)$. In the worst case $k' = k = n$, which occurs once during the initialization phase, and could occur with very low probability at any other time, then this reduces to $O(n^2)$. However, experimentally and rationally, $k \ll n$, because the number of collisions that are predicted to occur in any single time step are guaranteed by the physical nature of our problem to be much less than n . As a matter of fact, as seen in Section 6, in our experiments $k = 0$ in 90-98% of the time steps simulated.

5. Experimental setup

In order to test our algorithm, we implemented a particular instance of future event simulation in the detection of collisions between two objects.

In typical instances of this problem in the real world, the type of simulation varies dramatically. In many applications, the objects obey complex interaction patterns such as flocking behavior and user interaction. However, these interactions are almost always built around a simple framework for simulation that obeys Newtonian physics as the primary motion characteristic, with the more complex behaviors simply activating different Newtonian trajectories and velocities [6]. Therefore, without loss of generality, we will simplify our simulation by limiting the object behaviors to simulate Newtonian physical interactions of objects under Earth-like gravity.

For our simulation, since we are using dynamic spheres, we use the following calculations for the collision reaction[?]:

$$c = n \cdot (v_{1i} - v_{2i})$$

$$v_{1f} = v_{1i} - \frac{m_2 \cdot c}{m_1 + m_2} \cdot (1 + e)n$$

$$v_{2f} = v_{2i} + \frac{m_1 \cdot c}{m_1 + m_2} \cdot (1 + e)n$$

where:

v_{1i} = the initial velocity of object 1

v_{2i} = the initial velocity of object 2

m_1 = the mass of object 1

m_2 = the mass of object 2

e = the coefficient of restitution ($e = 1$ for completely elastic)

n = normal unit vector drawn from object 1 to object 2

Real simulations may also involve extremely complicated geometry, such as the convex shapes of human faces, vehicles, and buildings. However, all of these simulations generalize objects by a first pass of a 'bounding volume' for the more complex object. The bounding volume is a simple shape (e.g., a sphere) that forms tight bounds on the object. Real applications solve the first-pass collision checks for the simple 'bounding volume', and only fall back to a more complex collision check on convex shapes if the bounding volumes intersect. Thus, we can solve the problem without loss of generality by solving the problem for simple bounding-volume shapes [7, 11]. In our case, we choose to use dynamic objects made of simple spheres, and static objects made of planes.

In order to program the simulation, we will use the latest version of the C++ standard, as it is a good language for high performance simulation applications. In order to facilitate our 3D geometry and numerical libraries, we will write our own simulation system and geometry routines based on the C++ Numerical Vector library Eigen¹.

We wish to test performance economy as well as parallelizability. For our parallelization tests, we used thread-level parallelism and atomic primitives from the latest version of the C++ standard.

The performance of these systems is typically evaluated in terms of a throughput measurement that measures the number of time steps per second the system is capable of evaluating. Equivalently, one can measure the average wall-clock time that passes while evaluating one time step.

5.1. Naive implementation

The implementation of the naive solution is relatively straightforward, as a naive approach should be. First, all of the dynamic objects must be updated for each time step to their new positions, using their previously calculated velocity vectors. Then, because we are handling the spheres and planes separately, we have two sets of nested for loops to perform the collision checks. The first loops over the spheres and checks each with all of the planes, and the second loops over the spheres and checks each with all of the spheres that are after it in the list. We do this to avoid checking each pair twice (`collide(o1,o2)` would evaluate the same as `collide(o2,o1)`). For the threaded version of this algorithm, we simply run the outermost for loop in parallel:

Algorithm 14 Update

```
for  $i = 0$  to  $num\_spheres$  do
     $dynamic\_spheres[i].update(dt)$ 
end for
```

5.2. Predictive implementation

The scalable predictive implementation has already been described in Section 3. However, we have not described the implementation of the sphere-sphere prediction algorithm, which is crucial to the implementation we performed. This algorithm is in Appendix .

6. Results

The total runtime of the naive algorithm is scale-invariant. That is to say, the size of the volume does not matter; the runtime is bounded by the number of checks being performed, which is constant across varying volume sizes (Figure 1). The total runtime shows quadratic growth due to the increasing number of collision checks as sphere count increases, which also shows quadratic growth. Because of the vast number of collision checks occurring, prediction rates were between 0.0001% and 0.05% correct detection (Figure 2).

¹ Eigen v3, <http://eigen.tuxfamily.org>

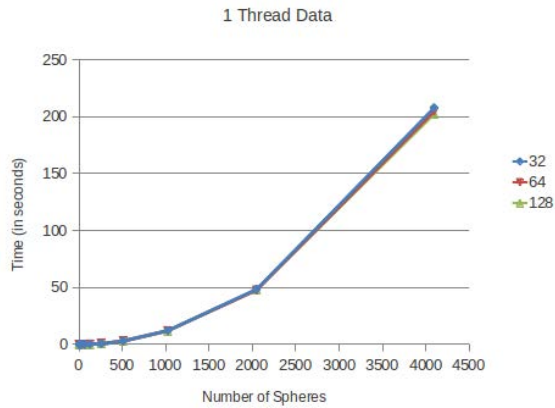


Figure 1: Total time - naive algorithm.

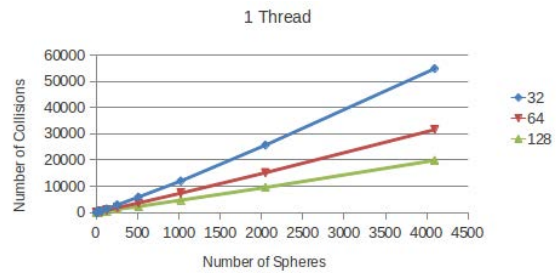


Figure 2: Total collisions - naive algorithm.

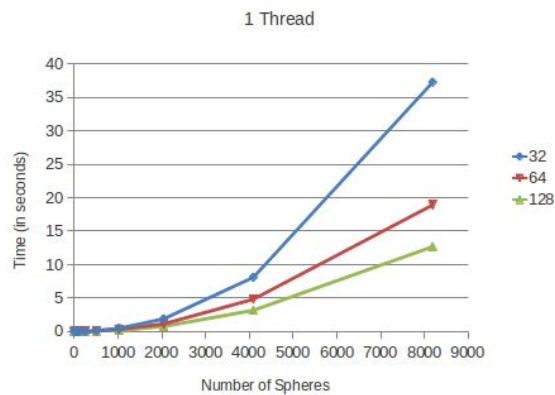


Figure 3: Total time - predictive algorithm.

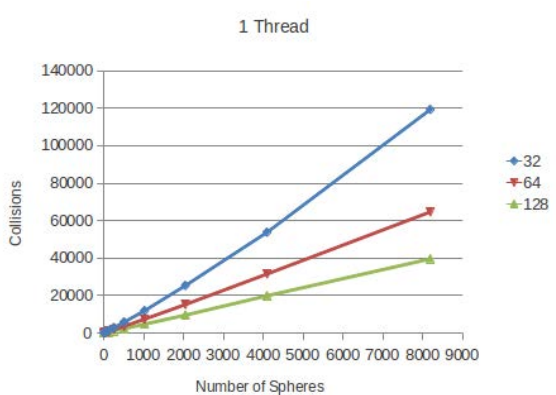


Figure 4: Collisions - predictive algorithm.

For the predictive algorithm, total runtime is not scale-invariant, as predictions rely on calculating physical collisions. As the volume increases, the likelihood of collision decreases (Figure 3). Total runtime is showing growth as was true with the naive algorithm, but at a much slower growth rate than the naive algorithm. This may be a result of extra overhead to perform collision equation calculations in order to reduce the number of unnecessary collision checks. The number of checks performed per time step is linear with respect to the number of objects, as opposed to the quadratic growth of the checks performed in the naive solution (Figure 4). Prediction rates were between 0.3% and 0.7% correct, which although still low, are enormously more efficient when compared to the naive algorithm (Figure 5). These prediction rates are so low because at every time step, the predictive algorithm still iterates over an array which holds future collision information. Every array location maps to a sphere, and each location gets 'checked' once per time step. So an array location could be 'checked' many times before it actually triggers a collision event.

When subjecting the naive solution to thread improvements, multiple threads offer faster run times as the number of spheres increases (Figure 6). Total collisions detected are thread-invariant (as they should be), and scale linearly with the number of spheres in the scene (Figure 7). Total checks are also thread-invariant (as they should be), and scale quadratically with the number of spheres in the scene.

Although single-threaded runs of the predictive algorithm show quadratic growth, it appears that multiple threads of the predictive algorithm begin to exhibit closer to linear growth. As the number of spheres increases, the benefit of multiple threads increases as well (Figure 8). Collisions and checks are also thread-invariant (as they should be), and with the predictive algorithm. Both scale linearly with the number of spheres in the scene (Figure 9).

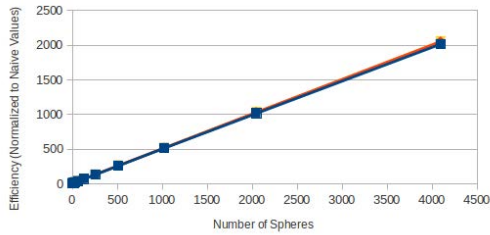


Figure 5: Collisions - efficiency increase.

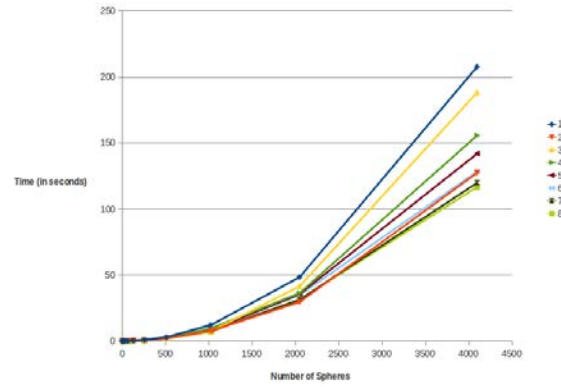


Figure 6: Run time - naive algorithm.

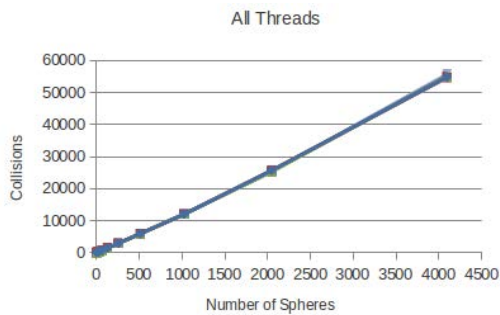


Figure 7: Collisions - naive algorithm.

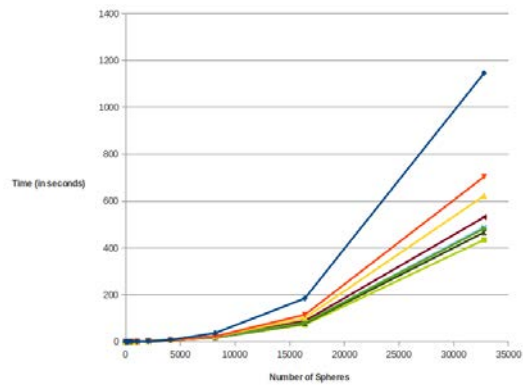


Figure 8: Run time - predictive algorithm.

When comparing the run-times of both algorithms and all thread scenarios, the predictive algorithm appears at this resolution to be linear with respect to the naive algorithm. However, with the addition of a few extra simulations with even more spheres, we know this is not the case (Figure 8). The run-times do begin to exhibit a quadratic growth rate, albeit at a much slower growth. For example, in our data, the predictive algorithm can handle four times as many spheres (shown by the 4096 naive spheres simulation runs just slightly longer than 16384 predictive spheres simulation) to execute in approximately the same runtime as the naive algorithm (Figure 10).

With respect to the naive algorithm, the number of checks skyrocket quickly as the number of spheres increases. In order to more easily visualize the magnitude of this increase, we have shown a log-log plot of the number of spheres for the simulation vs. the number of checks performed by each algorithm. The plot shows that the number of checks performed in the naive algorithm quickly grows to at least 10 orders of magnitude (base 2) more than the predictive algorithm (Figure 11).

Next we show that both algorithms eventually gain a benefit from multi-threading, in comparison with their single threaded counterparts (Figures 12 and 13). In addition to the faster run-times of the predictive algorithm, it also scales better than the naive solution as the number of spheres increases. There is, however, a threshold on the predictive algorithm where there is no threading benefit (in our data, this threshold is between 2048 and 4096 spheres). This could be caused by some kind of overhead in managing the threads that is a significant portion of the total runtime when there are very few spheres (the predictive algorithm runtime between 2048 and 4096 spheres is 2-8 seconds). As the number of spheres increases, that overhead appears to become less and less dominant, and the performance improvement can be seen. At 32768 spheres, the speedup with 8 threads is shown to be 2.6 times faster than the single threaded

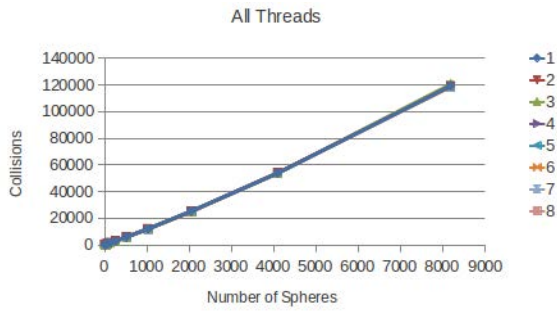


Figure 9: Collisions - predictive algorithm.

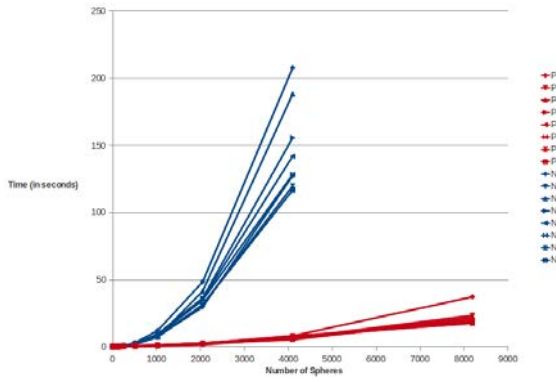


Figure 10: Run time comparisons.

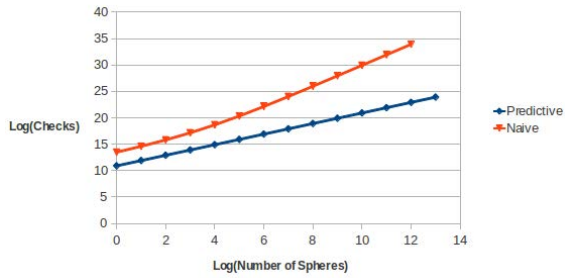


Figure 11: Log (Base 2) of total checks.

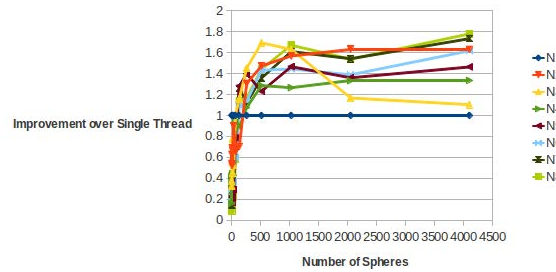


Figure 12: Normalized scalability (naive).

simulation (Figure 13).

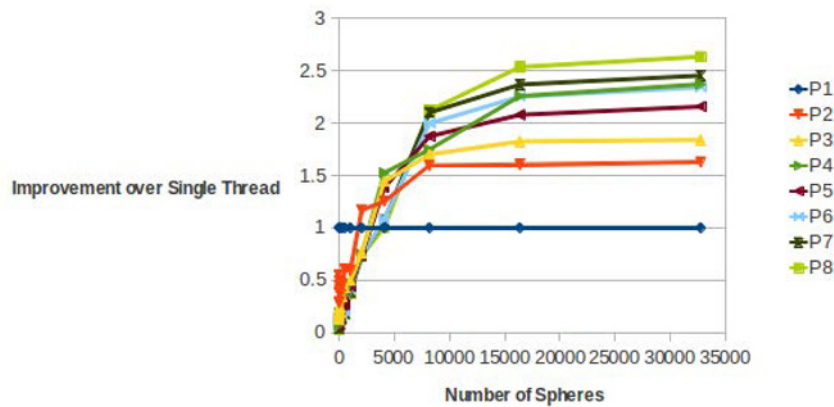


Figure 13: Normalized scalability (predictive).

7. Conclusion

We have presented a novel algorithm to perform discrete event n-body simulations, using predictive event detection instead of polling collision detection. Our algorithm runs in $O(n)$ time in the average-case for a single time step. We have demonstrated that this algorithm has significantly increased single-threaded performance over the naive implementation, and we have presented an implementation that scales well to a multi-core architecture in comparison to the naive algorithm.

Appendix

Prediction algorithm for two spheres

This algorithm is relatively simple. Given two spheres, we can describe their positions, accelerations, and velocities as the equation set.

$$\begin{aligned}\vec{p}_a(t) &= \vec{s}_a + \vec{v}_a t + \vec{a}_a \frac{t^2}{2} \\ \vec{p}_b(t) &= \vec{s}_b + \vec{v}_b t + \vec{a}_b \frac{t^2}{2}\end{aligned}$$

Then, we find when/if the distance between the two spheres is equal to the sum of their radii.

$$\begin{aligned}\|\vec{p}_b(t) - \vec{p}_a(t)\| &= r_a + r_b \\ \|\vec{p}_b(t) - \vec{p}_a(t)\|^2 &= (r_a + r_b)^2\end{aligned}$$

This can be expanded as a quartic polynomial in terms of the difference between their two positions, e.g., \vec{p} is the view of a from b 's frame of reference.

$$\vec{p}(t) = \vec{p}_a(t) - \vec{p}_b = \vec{s} + \vec{v}t + \vec{a} \frac{t^2}{2} \tag{A.1}$$

then, the expanded squared norm of \vec{p} is a quartic polynomial:

$$\begin{aligned}\|\vec{p}(t)\|^2 &= \vec{s} \cdot \vec{s} \\ &+ 2\vec{s} \cdot \vec{v}t \\ &+ (\vec{s} \cdot \vec{a} \\ &+ \vec{v} \cdot \vec{v})t^2 \\ &+ \vec{a} \cdot \vec{v}t^3 \\ &+ \frac{\vec{a} \cdot \vec{a}}{4}t^4 \\ &= (r_a + r_b)^2\end{aligned}$$

When acceleration is constant, then the difference of the two acceleration reference frames \vec{a} is 0, and the above reduces to a quadratic that can be solved using the quadratic formula. When acceleration is not constant, then the above reduces to a quartic equation that can be solved for t as well. In either case, if t has any positive real solutions, then the smallest of these is taken as the predicted collision time. Otherwise, no collision is predicted.

References

- [1] Bittner J., Hierarchical techniques for visibility computations, PhD thesis, Czech Technical University in Prague, 2002
- [2] Dechev D., Pirkelbauer P., Stroustrup B., Lock-free dynamically resizable arrays, In: Shvartsman M. (Ed.), Principles of Distributed Systems, Lect. Notes Comp. Sci., 4305, 142-156, 2006
- [3] Ebisuzaki T., Fukushige T., Funato Y., Makino J., Taiji M., Hachisu I., Sugimoto D., GRAPE: Special Purpose Computer for Gravitational N-body Simulations In: Am. Astron. Soc. Meeting Abstr., 182, ser. B. Am. Aston. Soc., 25, 813, 1993
- [4] Harada T. Real-Time Rigid Body Simulation on GPUs, In: Nguyen H. (Ed.), GPU Gems 3, Addison Wesley Professional, 2007
- [5] Herlihy M., Shavit N., The Art of Multiprocessor Programming, Morgan Kaufmann Publishers, New York, NY, USA, 2008
- [6] Jadbabaie A., Lin J., Morse A.S., Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Transactions on Automatic Control, 48(6), 988-1001, 2003
- [7] Kipfer P., Segal M., Westermann R., UberFlow: a GPU-based particle engine, In: Proceedings of the ACM SIGGRAPH/EUROGRAPHICS Conference on Graphics Hardware, HWWS '04., New York, NY, USA, ACM, 115-122, 2004
- [8] Moore M., Wilhelms, J., Collision detection and response for computer animation, Comp. Graph., 289-298, 1988
- [9] Senin M., Kojekine, N., Savchenko V., Hagiwara I., Particle-based collision detection, 2003
- [10] Shavit N., Data structures in the multicore age, Commun. ACM, 54, 76-84
- [11] Zeller C., Cloth simulation on the GPU, In: ACM SIGGRAPH 2005 Sketches, ACM, New York, NY, USA, 2005